Entropy and error: density limit for magnetic data storage

R. F. L. Evans,$^1$ R. W. Chantrell,$^1$ U. Nowak,$^2$ A. Lyberatos,$^3$ and H.-J. Richter$^4$

$^1$Department of Physics, The University of York, York, YO10 5DD, UK
$^2$Fachbereich Physik, Universität Konstanz, 78457 Konstanz, Germany
$^3$Department of Materials Science, University of Crete, Heraklion 71003, Greece
$^4$Hitachi Global Storage Technologies, 3403 Yerba Buena Road, San Jose, CA 95135

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Magnetic data storage is pervasive in the preservation of digital information and the rapid pace of computer development requires ever more capacity. Increasing the storage density for magnetic hard disk drives requires a reduced bit size, previously thought to be limited by the thermal stability of the constituent magnetic grains. The limiting storage density in magnetic recording is investigated treating the writing of bits as a thermodynamic process. A 'thermal writability' factor is introduced and it is shown that storage densities will be limited to 15 to 20 TBit/in$^2$ unless technology can move beyond the currently available write field magnitudes.

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As a technology, magnetic recording has been in existence since the invention of magnetic tape recording in the 1920s and 1930s. Since the early 1980’s, and the introduction of metallic thin film recording media, the industry has seen a rapid increase in storage density; up to the TByte storage available in today’s PC hard drives. Because technology has kept pace with demand, magnetic information storage is now ubiquitous. Having been around for some 60 years, magnetic recording is running into difficulties imposed by physical limitations. It has long been wondered for how long these technology developments can be maintained, and consideration of this question is the aim of the present letter.

Magnetic recording relies on the storage of information on media comprised of grains of a material with a high magnetocrystalline anisotropy. The grains can be considered as bistable systems capable of representing bits of information in terms of the polarity of the grains. Stability of the information is provided by an anisotropy energy barrier $KV$ where $K$ is the anisotropy constant and $V$ the grain volume. It has long been realised that the phenomenon of ‘superparamagnetism’ (SPM) defines the upper limit of information storage density in magnetic recording. SPM arises due to thermal activation which can cause spontaneous switching of the magnetisation over the anisotropy energy barrier, with a consequent loss of recorded information. The physics of SPM is encapsulated in the Arrhenius-Neel law\cite{1}

$$\tau^{-1} = f_0 \exp(-\Delta E/kT),$$

where $f_0$ is a characteristic frequency having a value around $10^{10}$Hz, which relates the relaxation time $\tau$ for thermally activated transition to the energy barrier $\Delta E$. Setting $\tau$ equal to some characteristic measurement time $t_m$ Bean and Livingston\cite{2} derived a criterion separating thermally stable (TS) from thermal equilibrium or superparamagnetic (SPM) behaviour as

$$\Delta E_c = kT \ln(t_m f_0).$$

Taking $t_m = 100$s gives the usual criterion for SPM behaviour as $\Delta E_c = 25kT$. In the case of magnetic recording information should be stable for at least 10 years, which leads to an established criterion of $KV/kT > 60$ for media design. The first study of the possible limits of recording density was made by Charap et al\cite{3}. This study predicted an upper limit of 36 Gb/in$^2$ and, remarkably, current technology has already achieved densities over one order of magnitude beyond this value. The reason for this lies in advances in the ‘non-magnetic’ aspects of recording technology, including error detection and correction and the mechanical actuator systems used to position the read and write sensors, which were not anticipated by the authors of ref\cite{3}. The question is: does there exist a physical upper limit to recording density which cannot be exceeded by improved technology? Here we argue that the limitation is essentially determined by the maximum tolerable Bit Error Rate and certain materials parameters which, critically, includes the saturation magnetisation of the recording medium.

Currently information densities are at a level which allows one bit to be stored on a number of grains. For reasons of signal to noise ratio the grain size is forced to reduce. Consequently there is a need for increased anisotropy to ensure thermal stability of recorded information. However, the increased $K$ leads to the requirement of increased switching fields. Because of the limitations of current write head technology the limits of switching are reached at relatively low $K$ values. This is referred to as the medium ‘trilemma’\cite{4}. A number of schemes to overcome the write field problem have been proposed, most notably the techniques of Heat Assisted Magnetic Recording (HAMR)\cite{5} and exchange spring media\cite{6}.

We proceed by calculating the Bit Error Rate (BER) induced by thermal fluctuations during the write process. Consider the ultimate recording system in which
one magnetic grain is sufficient to store a binary '1' or '0' (see supplementary Fig. 1). Our approach is to consider the equilibrium magnetization \( m_c \) in the recording context. In conventional recording there are a number of grains per bit so \( m_c \) has the meaning of an ensemble average magnetisation. Here, values of \( m_c \) less than unity represent the probability of a non-reversed grain in a bit, which gives rise to a dc noise. Consider now the situation of recording one bit of information per grain. Since we are now dealing with individual grains, \( m_c \) must be interpreted differently; in terms of the probability \( p_{sw} \) that the magnetisation is switched into the correct state by the field during the attempt to write the information, specifically, \( p_{sw} = (m_c + 1)/2 \). Since the Bit Error Rate is essentially the probability of wrongly recording the bit, we have simply that \( BER = 1 - p_{sw} = (1 - m_c)/2 \). Considering for simplicity the case of a system with perfectly aligned easy anisotropy axes, it is shown in the supplementary information, using a master equation approach that

\[
m_c = \tanh\left(\frac{\mu_0 H}{kT}\right) \tag{3}
\]

where \( \mu = M_s V \) is the magnetic moment of the grain with \( M_s \) the material saturation magnetisation and \( V \) the particle volume. In the (relevant) limit of \( \frac{\mu_0 H}{kT} \gg 1 \) we have the result that

\[
BER = \exp\left(-\frac{2\mu_0 H_{wr}}{kT}\right) \tag{4}
\]

where \( H_{wr} \) is the field available from the write transducer.

It is shown in the supplementary information that to achieve a given BER, we require a minimum value of \( \frac{\mu_0 H}{kT} \) of

\[
\left(\frac{\mu_0 H_{wr}}{kT}\right)_{min} = \frac{1}{2} \ln(1/BER). \tag{5}
\]

where \( H_{wr} \) is the field available from the write transducer, where \( \mu = M_s V \) is the magnetic moment of the grain with \( M_s \) the material saturation magnetisation and \( V \) the particle volume (see supplementary information).

Eq. 5 introduces a new factor in the writability of stored information. The writability is normally considered in terms of the field needed to switch the magnetisation. However, it is clearly necessary, in addition, to maintain a large value of \( \frac{\mu_0 H_{wr}}{kT} \) in order to avoid thermally driven switching failures and to achieve the required BER; we define this as the thermal writability of the medium.

We now introduce the requirement of thermal stability of the stored information, which is specified as \( \frac{K V}{T} \geq 60 \), where \( K \) is the anisotropy constant of the recording medium. This leads to an expression relating the main physical parameters to the BER:

\[
BER = \exp\left(-\frac{120 M_s \mu_0 H_{wr}}{K}\right). \tag{6}
\]

Importantly, Eq. 6 implies that the required BER increases with increasing \( K \) as shown in Fig. 1. The penalty for increased storage density is evidently an increased BER requirement arising from purely thermodynamic effects. Eq. 6 highlights important new physical factors relating to ultra-high density recording: it shows that maintaining high saturation magnetisation of the medium is required for thermal writability as the grain size decreases.

Apparent the design of ultra-high density recording techniques is a quadrilemma rather than the trilemma which underlies current media and system design. In addition to decreasing the volume of the bits and increasing \( K \) (thereby leading to the requirements for write assist) the value of \( M_s \) must remain high in order to maintain writability as the grain sizes decreases. A further factor is the nature of the role of the write field. It is known to be important to keep the write field as large as possible, and generally speaking this is considered to result from the need to lower the energy barriers sufficiently to switch the magnetisation. However, our model suggests that increasingly large fields are important in maintaining a tolerable BER because of the requirement for thermal writability.

In order to evaluate the likely achievable areal densities we proceed from Eq. 7. Solving for the volume and assuming that the Areal Density (\( AD \)) is given approximately by \( V^{-2/3} \alpha \), where \( \alpha = 0.5 \) is the areal packing fraction of the storage islands, it is straightforward to show that

\[
AD = \left(\frac{2M_s \mu_0 H_{wr}}{kT \ln((BER)^{-1})}\right)^{2/3} \alpha. \tag{7}
\]

Note that the areal density is now determined by two
conditions. Firstly it cannot be larger than determined by the thermal stability criterion \( KV/kT > 60 \). Using the approximation \( AD = V^{-2/3} \alpha \) this gives

\[
AD_{ts} = \left( \frac{K}{60kT} \right)^{2/3} \alpha. \tag{8}
\]

Secondly, the \( AD \) must also be less than the value determined by the allowed BER, given by eq. [7]. Consequently the achievable areal density is the minimum of \( AD_{ts} \) and the value determined from Eq. [7].

As an illustrative example, Fig. 2 shows calculation of Areal Densities for the (reasonable) case of a write field \( \mu_0H_{wr} = 1\) T. Values of \( AD_{ts} \) are shown for FePt and SmCo, two high-\( K \) materials. Consider first the case of no heat assist. The conclusion from these calculations is that very large areal densities are possible, but that the BER needed to achieve maximum density increases with decreasing \( M_s \), which becomes an important factor in the quadrilemma now seen to govern media and system design. Taking first FePt, with a value of \( M_s = 12 \times 10^5 \) A/m the \( AD_{ts} \) value is achieved with a BER of \( 10^{-9} \), consistent with today’s technology. However, for a value of \( M_s = 6 \times 10^5 \) A/m the \( AD_{ts} \) value requires a significant increase of BER to around \( 3 \times 10^{-5} \). Increasing the anisotropy value imposes even more stringent requirements on the BER. Here calculations are given to a limiting value of BER=\( 10^{-2} \), which would present an enormous technical challenge. For the \( AD_{ts} \) value corresponding to SmCo, achieving the maximum areal density requires a BER value of \( 7 \times 10^{-4} \) for \( M_s = 12 \times 10^5 \) A/m. For \( M_s = 6 \times 10^5 \) A/m \( AD_{ts} \) cannot be reached; the areal density is limited by the BER (assuming that the highly challenging figure of \( 10^{-2} \) is achievable) to about 51 TBit/in\(^2\).

These figures suggest that extremely high densities are possible with magnetic recording. However, if one takes account of the fact that heat assist is expected to be necessary to write on high anisotropy media a different picture appears. To extend the calculations to HAMR we introduce the temperature dependence of \( M \) using the approximation proposed by Arrott

\[
M_s(T) = M_s(T=0)(1 - (T_{wr}/T_c)^2)^{1/2}, \tag{9}
\]

with \( T_c \) the Curie temperature of the material, and \( T_{wr} \) the writing temperature, giving

\[
AD_{ha} = \left( \frac{2M_s(T)\mu_0H_{wr}}{kT\ln([BER]^{-1})} \right)^{2/3} \alpha \tag{10}
\]

as the BER limited areal density for heat assisted recording. In Fig. 2 results are included assuming a heat assist with a temperature of \( T_{wr} = 740\)K and \( T_c = 750\)K. It can be seen that there is a dramatic reduction in the BER limited areal density, and that the thermal stability limit, even for FePt is not reached. The results for the case with heat assist are, of course, the most realistic since some form of write-assist is necessary for recording on high anisotropy media and heat assistance is seen as the most likely solution. The important finding of the current work is that heat assist transforms the areal density \( 60kT \) from being thermal stability limited to BER limited, leading to much lower limiting values.

Our findings indicate that the reduction in grain volume has not one but two important consequences. The first of these is the thermal stability requirement which leads to the necessity of large \( K \) values and the resultant problems with writability and the well-known ‘trilemma’ for media design. However, the reduction in grain volume also lowers the value of \( \mu_0\mu_0H_{wr}/kT \), which must be compensated for by increasing the value of the saturation magnetisation \( M_s \). Consequently recording media design in fact becomes a ‘quadrilemma’ (see supplementary Fig. 2). Thus the future of magnetic recording requires moving to a new paradigm in which the thermal writability introduced here is treated on an equal footing to the conventional writability, which is essentially the requirement of a sufficiently large field to switch the magnetisation state.

In summary, we present arguments which suggest that the areal density of magnetic recording is likely to be limited by the achievable BER due to purely thermodynamic effects. The theoretical approach suggests that the optimisation of recording density must be considered as a quadrilemma including the requirement of maintaining a saturation magnetisation sufficiently large to ensure writability. Our calculations suggest that between 15 and 20 TBit/in\(^2\) may be achievable with heat assisted writting.
Finally, we note that the calculations given here are made under the assumption of a write field of 1T, which is realistic for the inductive technology used in today’s write transducers. Although much effort has been put into studies of high moment materials, significant improvements are not currently foreseen. However, it is clear from the expressions given here that a significant increase in write field would greatly extend magnetic recording technology. In this context an important development is the discovery of optomagnetic reversal.\footnote{10} This uses circularly polarised laser light to drive the reversal and involves estimated fields of up to 20T arising from the inverse Faraday effect. If such fields could be brought to bear in ‘all optical magnetic recording’, magnetic recording technology could overcome the BER thermodynamic limit predicted here and achieve the ultra-high densities expected for the stability limit of high anisotropy materials such as SmCo$_5$, leaving hard drives as the recording systems of choice into the middle of the 21st century.

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