How Good is Opportunistic Routing? – A Reality Check under Rayleigh Fading Channels

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ABSTRACT

Considerations of realistic channel dynamics motivate the design of a new breed of opportunistic schemes, such as opportunistic transmission, scheduling and routing. Compared to their deterministic counterpart, opportunistic schemes have been shown to achieve optimal performance. In this paper, we propose analytical models that characterize the performance of routing protocols in lossy wireless networks with Rayleigh fading channels. We investigate the efficiency of a few representative deterministic and opportunistic routing protocols under both light-loaded and saturated traffic conditions. This work deviates from existing work that assumes idealistic packet reception, and is thus of practical relevance. Our study shows that opportunistic routing significantly outperforms its deterministic counterpart through limited experimental/simulation studies. In deterministic routing schemes, tuning of parameters based on node density is critical to achieve optimal performance.

Categories and Subject Descriptors
C.2.1 [Computer-communication Networks]: Network architecture and design—distributed networks, wireless communication

General Terms
Algorithms, theory

1. INTRODUCTION

Recently, considerations of realistic channel dynamics motivate the design of a new breed of opportunistic schemes, such as opportunistic transmission, scheduling and routing. Compared to their deterministic counterpart, opportunistic schemes have been shown to increase network throughput, and/or reduce delay in limited experimental/simulation scenarios. In this paper, we propose analytical models that characterize the performance of routing protocols in lossy wireless networks with Rayleigh fading channels. We investigate the efficiency of a few representative deterministic and opportunistic routing protocols under both light-loaded and saturated traffic conditions. This work deviates from existing work that assumes idealistic packet reception, and is thus of practical relevance. Our study shows that opportunistic routing significantly outperforms its deterministic counterpart through limited experimental/simulation studies. In deterministic routing schemes, tuning of parameters based on node density is critical to achieve optimal performance.

The thesis of this paper is to investigate the performance of different routing schemes in lossy wireless networks with Rayleigh fading channels. In contrast to many existing scaling law analysis in multihop wireless networks, which assumes idealistic, lossless channel condition and collision-free schedules, our analysis explicitly accounts for the effects of channel fading on packet reception, and thus is of more practical relevance. Using stochastic geometry techniques, we analyze key metrics that are central to the routing behavior of deterministic routing protocols, namely VC-based routing and k-nearest neighbor routing, and an ideal form of opportunistic routing protocol. Results from theoretical analysis are corroborated by Monte-Carlo simulations. The analytical models enable us to study the impact of different parameter settings, which provides guidelines to network practitioners and system designers.

Our work builds upon the techniques of existing work [5, 8] and advances the state of the art in several aspects. First, we derive a rigorous analytical framework to study the performance of routing protocols. A product form is developed to characterize the average contribution of concurrent transmitters to packet reception probability in saturated traffic scenarios, which can be of independent interest. Second, the loss model considered in this paper takes into account the realistic conditions in wireless channels and current network practice including fading channels, link level retransmission, hidden nodes. Finally, to our best knowledge, this is the first analytical work that explores the parameter space of deterministic and opportunistic routing schemes.

The rest of the paper is organized as follows. In Section 2, we summarize the physical layer and network models assumed in this paper. Three routing strategies studied and their associated path and hop efficiency expressions are presented in Section 3. In Section 4 and 5, derivation for the light load and saturated traffic is presented, with numerical and simulation results discussed in Section 6. Finally, we conclude the paper in Section 7.

2. PRELIMINARIES

In this section, we establish the physical and network layer models assumed in this paper.

2.1 Physical layer model

In this paper, we consider that nodes are individually power-constrained, i.e., their transmission power cannot exceed \( P_{\text{max}} \). At the physical layer, all wireless links are assumed to be baseband channels corrupted by circularly symmetric, complex-valued additive white Gaussian noise with power spectral density \( \frac{1}{2}N_0/\text{Hz} \) as well as additive interference from other transmitting nodes in the network. All channels are modeled as compound channels consisting of a deterministic path-loss channel with exponent \( \alpha \) cascaded with a slow fading channel. The fading channels are assumed to be

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independently and identically-distributed (i.i.d.) with unit mean for every distinct pair of nodes.

We assume that to successfully decode messages at rate $R$ bps, the received signal-interface-plus-noise ratio (SINR) needs to be greater than a threshold value $\beta$. For instance, according to hardware specifications of the IEEE 802.11 WiFi cards, at $R = 1$ Mbps, $\beta = 4dB$. Formally, let the received SINR at node $j$ from node $i$ be, $\gamma_{ij} = \frac{S_{ij}}{B N_{ij} + \sum_{k \in \mathcal{T} \setminus \{i\}} I_{kj}}$, where $B$ represents the common bandwidth for all links on the network, $S_{ij}$ is the received power. Under Rayleigh fading channels, $S_{ij}$ is exponentially distributed with mean $\overline{S}_{ij} = P_i l(d_{ij})$, where $l(\cdot)$ is the path loss function defined as follows,

$$l(d) = \left\{ \begin{array}{ll} A d^{-\alpha} & d > d_0 \\ A d_0^{-\alpha} & d \leq d_0 \end{array} \right.$$  

$P_i$ is the power transmitted by node $i$, $d_{ij}$ is the distance between nodes $i$ and $j$, $T$ is the set of transmitters, $I_{kj}$ is the interference at node $j$ from node $k$, $d_0$ is the near field distance. Therefore, the probability of correct packet reception is $p_{r_{ij}} := P(\gamma_{ij} \geq \beta)$.

To simplify notations without loss of generality, we assume $N_i = N_0$ and $P_j = P$, $\forall j$. Denote $I_j = \sum_{k \in \mathcal{T} \setminus \{i\}} I_{kj}$. Under the Rayleigh fading channel, the probability of correct reception has been shown in [5] as follows:

$$p_{r_{ij}} = P[\gamma_{ij} \geq \beta] = P[S_{ij} \geq \beta (I_j + B N_{ij})] = E_{I_j} \left\{ \exp \left[ -\frac{\beta (I_j + B N_{ij})}{S_{ij}} \right] \right\} = \exp \left( -\frac{\beta B N_{ij}}{P_{r_{ij}}} \right) \cdot \prod_{k \in \mathcal{T} \setminus \{i\}} \frac{1}{1 + \frac{\beta}{P_{r_{ij}}}} \left( \frac{d_{ij}}{d_0} \right)^{l(d_{ij})} \cdot \exp \left( -\frac{\beta B N_0}{P_{r_{ij}}} \right) \cdot \prod_{k \in \mathcal{T} \setminus \{i\}} \frac{1}{1 + \frac{\beta}{P_{r_{ij}}}} \left( \frac{d_{kj}}{d_0} \right)^{l(d_{kj})} \right),$$

where $d_{ij}$/$\forall k, j \in T$ is the Euclidean distance between node $k$ and $j$. Interestingly, the probability of correct reception is decomposable, with the first term corresponding to the contribution of noise and the second term for interference.

2.2 Network model

We construct a random extended network by placing nodes according to a Poisson point process $\Phi$ of density $\lambda$ on a 2-D plane $\mathbb{R}^2$. In the subsequent analysis, we focus on a “typical node in the field.” This eliminates the evolved analysis due to boundary effects. Clearly, for all practical purposes, a wireless network should have finite size. When a restriction of area is required (e.g., for connectivity), we let $B(n)$ denote the box $[0, \sqrt{n}]^2$ and let the process $\Phi_n := \Phi \cap B(n)$ represent the restriction of $\Phi$ to $B(n)$.

We assume that packet transmissions are subject to independent fading in both space and time. Retransmissions are performed if none of the intended receiver can successfully decode the packet.

2.3 Geometric notations

For ease of presentation, we summarize some of the geometric notations in Table 1.

### 3. ROUTING STRATEGIES

In this section, we present and analyze a number of representative routing strategies. In particular, we consider two determinis-

tic routing schemes, namely, Voronoi cell (VC)-based shortest distance routing¹, and $k$-nearest neighbor routing. In both schemes, given a topology, the routing path (and thus all the relay nodes) is fixed. We further consider a form of opportunistic routing similar to the scheme proposed by [1, 2]. In opportunistic routing, the traveled distance and the relay node at each hop depend on packet receptions, and are non-deterministic.

To characterize the efficiency of a routing scheme, adopting the notations in [5], we define the path efficiency $\rho$ as the average ratio of the Euclidean distance between the end nodes and the actual distance traveled. At each hop, let $s$ be the distance covered by a transmission. If the transmission is unsuccessful, $s = 0$. One can also define hop efficiency $\eta$ as the average ratio of actual progress towards the destination and $s$. Since the distance covered by each transmission is independent and identically distributed under our physical and network model, it is easy to show that hop efficiency is the same as path efficiency. As a convention, in the rest of the paper, we use subscripts to distinguish various routing schemes.

3.1 VC-based shortest distance routing

**Route selection.** In VC-based routing, the deployment region is partitioned into Voronoi cells of side length $\rho$. Given a traffic flow over the wireless network, if the destination node is not in the same cell as the source node, multihop routing is performed by drawing a line segment $L$ from the source node to the destination node. For each Voronoi cell that intersects with the line segment, a node is randomly selected as a relay node. For such a scheme to work, the size of a cell should be chosen sufficiently large such that with high probability each cell contains at least one node. The sufficient condition for this to hold in asymptotic regime is given in [4] for a Poisson point process $\mathcal{P}$ of unit density on a 2-D plane $\mathbb{R}^2$ restricted to $B(n)$, namely, $\rho = \sqrt{200 \log(n)/\pi}$ as $n \to \infty$. From this equation, we see a larger deployment area implies larger cells. To get around this problem, we note that according to Eq. (2), the probability of packet reception is always non-negative albeit small over long distance. Thus, if there does not exist any relay node in a cell, a direct transmission to the destination node is made. Furthermore, without loss of generality, we limit the shape of VC cells to be squares of side length $\rho$.

Intuitively, there exists a tradeoff in the choice of $\rho$ in VC-based shortest distance routing. A large $\rho$ leads to a high probability of finding a relay node in the adjacent cell; but at the same time a

<table>
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<th>Table 1: Geometric notations</th>
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<tr>
<td>$B(x, Z)$</td>
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<td>$v_{iz}$</td>
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<td>$B(x, r, \phi)$</td>
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<td>$v'(x, t, Z, \phi)$</td>
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¹Given a set of points $S$ in the plane, called the Voronoi sites, each site $s$ has a Voronoi cell $V(s)$ consisting of all points closer to $s$ than to any other site.
longer distance needs to be traversed in each hop and thus more packet errors. Such tradeoff will be explored in the analysis.

**Path efficiency $\kappa_{\text{vec}}$.** Consider the source and destination is separated by distance $s$. The straight line that connects the end nodes forms angle $\psi$ with respect to the horizontal line. The number of hops taken using the VC-based shortest distance routing is thus upper bounded by $\frac{1}{\phi}(|\cos(\psi)| + |\sin(\psi)|)$. Therefore, the path efficiency is

$$
\kappa_{\text{vec}} \leq \frac{s}{\frac{1}{\phi}(|\cos(\psi)| + |\sin(\psi)|)E(s_{\text{vc}})} = \frac{\rho}{(|\cos(\psi)| + |\sin(\psi)|)E(s_{\text{vc}})}
$$

(3)

where $E(s_{\text{vc}})$ is the average distance in a successful transmission.

**3.2 $k$-nearest neighbor routing**

**Route selection.** In the $k$-nearest neighbor routing, relay nodes along the path between the source and destination are fixed. Each intermediate node, picks the closest $k$th neighbor in its $\phi$ circle sector towards the destination as the next hop, where $\phi$ is a protocol parameter. If no such neighbor exists, the packet is sent to the destination node directly.

**Path efficiency $\kappa_{kn}$.** To derive path efficiency, we examine the hop efficiency of each transmission.

**Lemma 1.** The hop efficiency $\eta(\phi)$ of selecting a relay node in a sector $\phi$, defined as the ratio of the progression towards the destination and the actual distance traveled, is given by $\eta(\phi) = \frac{\phi}{\pi} \sin(\frac{\phi}{2})$.

**Proof.** Due to shift- and rotation invariants of homogeneous Poisson point processes, it is reasonable to assume angle to the relay node in each hop to be uniformly distributed in the $\phi$ sector.

Consider a transmission from a node $A$ at distance $d_A$ from the destination. Let $x$ and $\theta$ be the distance and angle (with respect to the direction connecting node $A$ and the destination) to the intended receiver $B$ according to the routing mechanism. Let $d_B$ be B’s distance to the destination. We have

$$d_A - d_B = r - \sqrt{x^2 + r^2 - 2xr \cos(\theta)}$$

$$= r(1 - \sqrt{1 + \frac{x^2}{r^2} - 2x \frac{x}{r} \cos(\theta)$$

$$\approx r(1 - \frac{x}{r} \frac{x}{r} \cos(\theta))$$

$$= x \cos(\theta)$$

The approximation in the third line is due to Taylor series expansion when $r \gg x$. The average progress towards the destination is thus given by:

$$E(d_A - d_B) = E[x]E[\cos(\theta)]$$

$$= E[x] \int_{\phi/2}^{\phi} \cos(\theta) d\theta$$

$$= E[x] \frac{\phi}{2} \sin(\frac{\phi}{2})$$


**3.3 Opportunistic routing**

**Route selection.** In opportunistic routing [1, 2], routes are not predefined. Instead, forwarding decision is made among nodes with successful packet receptions. In practice, a coordination scheme is required to select a single forwarder in each progression. Such coordination can be achieved using control message exchanges. For example, in [1], the authors propose to use “batch list” in packet headers to indicate correct packet reception. The node closest to the destination uses a small delay timer before forwarding the packet, and thus tends to take precedence. In [2], a network coding based solution is proposed, where random mixing is performed at each relay node. To further avoid redundant relays, a source node can select a set of preferred relay candidates based on knowledge of link quality. To make our analysis independent of a particular implementation, we optimistically assume that coordination comes at no cost. Furthermore, in each hop, the node closest to the destination in a sector $\phi$ is used as relay among all nodes that have successfully received the packet.

**Path efficiency $\kappa_{opp}$.** Using similar arguments as in $k$-nearest neighbor routing, we have $\kappa_{opp}(\phi) = \eta_{opp}(\phi) = \frac{\phi}{\pi} \sin(\frac{\phi}{2})$.

**4. LOSSY NETWORKS IN LIGHT LOAD**

When the network traffic load is light and the distances between concurrent transmitters are relatively large, interference is negligible. The probability of correct packet reception at distance $d_{ij}$ between node $i$ and $j$ can be approximated by the first term in Eq.(2), i.e.,

$$p_{r_{ij}} \approx p_{r_{ij}}^N = \exp \left( -\frac{\beta BN_s}{P_{tr}(d_{ij})} \right).$$

(4)

We observe that in the light-loaded scenario, packet errors are a function of the distance between the transmitter and receiver and noise floor. In what follows, we first determine the average distance covered in each transmission using different routing schemes. Combined with the path efficiency results in Section 3, one can easily determine other quantities of interest, including energy efficiency and end-to-end latency.

**4.1 VC-based shortest distance routing**

**Average distance in each transmission $E(s_{\text{vc}})$.** Given a Poisson point process, the distribution of a node’s location is uniform on the condition that it falls in a cell. Let $x_i$ denote the location of the $i^{\text{th}}$ relay node and $C_i$ denote the Voronoi cell that contains $x_i$. Let $|C_i| = \int_{C_i} dx_i$, the size of the cell and $d_{i,j+1} = |x_i - x_{i+1}|$, i.e., the Euclidean distance between $x_i$ and $x_{i+1}$. The average distance in the transmission from the $i^{\text{th}}$ to $(i+1)^{\text{th}}$ relay node $x_i$ and $x_{i+1}$ is

$$E(s_{\text{vc}}) \approx (1 - e^{-\lambda|C_i+1|}) \int_{C_{i+1}} \frac{d_{i,j+1}}{|C_{i+1}|} p_{r_{i,j+1}}^N d_{i,j+1}$$

(5)

The first term is due to the probability that at least one node exists in the cell $C_{i+1}$. If no node exists in the cell $C_i$, direct communication with the destination node is needed. However, the probability of successful transmissions in this case is very small and yield on average close to zero progress. Therefore, the above expression gives a close approximation of the average distance in each transmission.

**4.2 $k$-nearest neighbor routing**

**Average distance in each transmission $E(s_{\text{kn}})$.** We first compute the distance to the $k^{\text{th}}$ nearest neighbor.

**Lemma 2.** (Distance to the $k^{\text{th}}$ nearest neighbor (5), Proposition 3)): The probability density of the distance to the $k^{\text{th}}$ nearest neighbor...
neighbor in a sector $\phi$ is

$$p_{R_k}(r) = r^{2k-1} \left( \frac{\lambda \Delta \phi}{2} \right)^k \frac{2}{(k-1)!} e^{-\frac{r^2 \lambda \Delta \phi}{2}}$$  \hspace{1cm} (6)

In particular, when $k = 1$, $p_{R_1}(r) = r \lambda \Delta \phi e^{-\frac{r^2 \lambda \Delta \phi}{2}}$, which follows Rayleigh distribution with mean $\sqrt{\pi/2} \lambda \Delta \phi$.

Without loss of generality, we assume the transmitter is at the origin. The average distance covered by each transmission is thus computed as,

$$E(s_{kn}) = \int_0^{\infty} x p_{R_k}(x) p_r^N dx$$

$$= \int_0^{\infty} 2 \lambda \Delta \phi \frac{2}{(k-1)!} e^{-\frac{r^2 \lambda \Delta \phi}{2}} dx$$  \hspace{1cm} (7)

4.3 Opportunistic routing

Average distance in each transmission $E(s_{op})$. Recall $\Phi$ represents points of the 2-D Poisson point process. To avoid tedious notations, we also use $\Phi(\cdot)$ to represent the number of points in the area in one realization. Denote by $F(x, y)$ the event that node $y$ cannot decode node $x$'s transmission, i.e., $P\{F(x, y)\} = 1 - p_r x y$. The probability distribution of $s_{op}$ is derived as follows.

$$P\{s_{op} < t\} = \lim_{Z \to \infty} \sum_{n \in \mathbb{N}} \mathbb{P}\left\{ \sum_{i=1}^{n} \int_{0}^{t} F(x, Y_i) | \Phi(B(x, t, Z, \phi)) = n \right\}$$

$$= \lim_{Z \to \infty} \int_{0}^{\infty} e^{-\lambda \Delta \phi (t, \phi)} \prod_{1 \leq i \leq n} \mathbb{P}\{F(X_i, Y_i)\} dy_1 \ldots dy_n$$

$$= \lim_{Z \to \infty} e^{-\lambda \Delta \phi (t, \phi)} \prod_{1 \leq i \leq n} \mathbb{P}\{F(X_i, Y_i)\} dy_1 \ldots dy_n$$

$$= e^{-\lambda \Delta \phi (t, \phi)} \prod_{1 \leq i \leq n} \mathbb{E}\{P_r x y\}$$  \hspace{1cm} (8)

Therefore, the expected progress in each step is

$$E(s_{op}) = \int_0^{\infty} P\{s_{op} > t\} dt$$

$$= \int_0^{\infty} (1 - e^{-\lambda \Delta \phi (t, \phi)} \prod_{1 \leq i \leq n} \mathbb{E}\{P_r x y\}) dt$$

5. LOSSY NETWORKS WITH SATURATED TRAFFIC

In a wireless network, in addition to varying channels, concurrent transmissions can cause loss of packets. Intuitively, when the network is highly loaded, packet losses are predominantly the result of corruption. As the load grows, the effect of contention becomes more evident resulting in collisions. In this section, we consider the network operates in the saturated regime, namely, all flows are sending (homogeneously) at the maximum rate as allowed by the network. Typically, scheduling or a contention resolution mechanism is adopted to avoid excessive interference. To control the degree of interference in a heavy-loaded network, in this paper, we consider that carrier sensing multiple access (CSMA) is used to arbitrate access to the medium. CSMA is a fundamental building block in many medium access protocols such the IEEE 802.3, 802.11 and 802.15. In CSMA, a node $i$ refrains from transmission if $\exists j$ s.t. $S_{ji} > P_0$, where $P_0$ is the carrier sensing threshold. Recall that $S_{ji}$ is a random variable exponentially distributed with mean $P(d_{ji})$. Therefore, the probability for node $i$ to refrain from transmission due to node-$j$ transmission is the following:

$$P_{c,i}(j, i) = \exp(-K \max(d_0, d_{ji})^\alpha),$$  \hspace{1cm} (9)

where $K = \frac{P_0}{P}$.

5.1 Transmission schedules under CSMA MAC

In [8], the authors model the distribution of concurrent transmitters as the result of CSMA using the Matern’s point processes [11]. The Matern’s hard core process $\Phi$ of radius $Z$ associated with the homogeneous Poisson process $\Phi$ is a non-independent thinning of $\Phi$ built as follow: each point of $\Phi$ is attributed an independent mark which is uniformly distributed in the interval $[0, 1]$. A point $x$ of $\Phi$ is selected (or retained) in the Matern thinning if its mark $m(x)$ is smaller than that of any other point of $\Phi$ in the open disc of $B(x, Z)$ centered in $x$ and of radius $Z$. Formally, $\Phi_1 = \{ x \in \Phi | m(x) < m(y), y \in N(x) \}$. For a CSMA MAC, the definition of the Matern thinning process is modified such that the neighborhood $N(x)$ should include all stations that can carrier sensing $x$. In this paper, we apply the Matern approximation to analyze the transmission probabilities and interferences in the network. The techniques we use are similar to the stochastic geometry approach adopted in [8]. It should be noted that the Matern thinning process models the effect of CSMA in “spacing” out the transmitters. Packet transmissions are still subject to corruption due to poor channel quality and collisions due to hidden terminal. In the subsequent sections, we take a unified approach in modeling corruption and collisions as the result of low SINR.

5.1.1 Transmission probability $p$

**PROPOSITION 1.** [8] In a saturated network, all nodes are constantly backlogged with packets to transmit. Then, the probability that a node is retained is:

$$p \approx 1 - e^{-\lambda c}, \quad \text{with} \quad c = 2\pi \int_{r^+} e^{-K \max(d_0, r)^\alpha} dr$$  \hspace{1cm} (10)

5.1.2 Probability of concurrent transmissions of two nodes within distance $r$

Conditioning on the fact that $\Phi$ has two points at $A$ and $B$ with $d(A, B) = r$ and that $B$ is retained, the probability of retaining $A$:

$$h(r) \approx 2 \frac{1 - e^{-h(b(r))}}{b(r)} (1 - e^{-K \max(d_0, r)^\alpha})$$  \hspace{1cm} (11)

where $c$ is defined in Eq (10) and

$$b(r) = 2c - \int_0^{2\pi} e^{-K \max(d_0, r)^\alpha + r^2 + 2r^2 \cos(\theta)^\alpha} d\theta$$  \hspace{1cm} (12)

Clearly, $h(r)$ asymptotically goes to $p$ as $r$ gets larger since concurrent transmissions from distant nodes become independent. On the other hand, the $p$-persistent Aloha scheme can be modeled by setting $h(r) = p$.  

2The Matern approximation ignores the dependence of two-hop neighbors, and thus underestimates the number of active transmitters. However, this effect is somehow negated by contention avoidance. The Matern approximation is shown to be a good estimate for the behavior of the IEEE 802.11 in saturated regime [8].
5.2 Expected progress in a single transmission

Without loss of generality, we assume that the transmitter \( i \) is at the origin. The analysis of the expected progress in a single transmission is complicated, compared to the light-loaded case, due to the correlation between the choice of next hop relay and the amount of interference at the receivers. Let \( \Phi_i \) be the Matern thinning process of the original Poisson point \( \Phi \) given that node \( i \) is retained.

Before analyzing the performance of routing protocols in saturated traffic regime, we derive a product form to compute the expected progress in a single transmission. Let \( \rho_{i,j} \), \( \delta_{i,j} \), and \( \beta_{i,j} \) denote node-\( l \) average contribution to packet reception at the receiver node.

**Proposition 2.** Conditional on a total of \( n \) nodes in \( B(0, Z) \),

\[
\rho_{l,i,j} = \prod_{l=1, \ldots, n; l \neq i, j} \varphi_{l,i,j},
\]

where

\[
\varphi_{l,i,j} = \frac{\lambda(h(d_{ij}))}{1 + \beta(\delta_{ij}) + 1 - h(d_{ij})}
\]

By averaging over all outcomes of Matern thinning process that retains \( i \), we have

\[
\rho_{l,i,j} = \prod_{i=1, \ldots, n; j \neq i} \frac{h(d_{ij})}{1 + \beta(\delta_{ij}) + 1 - h(d_{ij})}
\]

The above equality holds under the assumption the nodes are retained independently when node \( i \) is transmitting.

**VC-based shortest distance routing.** Derivation of the VC-based routing under saturated traffic scenario is similar to that of the light-loaded case with the exception that we need to account for the average contribution of each node to packet reception at the receiver node. In the derivation, we first limit the domain of the considera-

\[
E_Z(s_{vc}) = \int \cdots \int d_{ij} p_{ij}^{l} r_{ij}^{l}(1 - h(d_{ij})) \frac{e^{-\lambda v_{ij} r_{ij}^{l} d_{ij}}}{|C_j||C_l|} dx_{ij} dx_{l}
\]

where \( \lambda = 1 - e^{-\lambda |C_j|} \) is the probability of having at least one node in a cell. The above integral can be easily evaluated using the Gaussian quadrature method ([3], Chapt. 5.3). Finally, \( E(s_{vc}) = \lim_{Z \to \infty} E_Z(s_{vc}) \).

**k-nearest neighbor routing.** Conditional on the \( k \)th-nearest neighbor is at distance \( r, r + \delta r \), there are \( k - 1 \) neighbors in \( B(0, r, \delta r) \) and \( n - k \) nodes in \( B_{r}(0, r, \delta r) \). The average distance covered by each transmission is thus,

\[
E_Z(s_{kn}) = \int_{Z} \cdots \int_{Z} \cdots \int_{Z} n! \lambda^n v^n \rho^n \frac{e^{-\lambda v}}{\lambda^n v^n} dx_{1} dx_{2} \cdots dx_{n}
\]

The above equation is rather complex to evaluate numerically as all transmitters in the field contribute to the packet loss probability term. To make the analysis tractable, we make the relaxation
that failure events that packets are not received correctly at each location are independent.

\[
P\{s_{op} < t\} = \lim_{Z \to \infty} \sum_{n \in \mathbb{N}} \mathbb{P}\left\{ \bigcap_{i=1}^{n} F_Z(x, Y_i) | \Phi(B(x, t, Z, \phi)) = n \right\} \times \mathbb{P}\{ \Phi(B(x, t, Z, \phi)) = n \} 
\geq \lim_{Z \to \infty} \sum_{n \in \mathbb{N}} \frac{\lambda^n}{n!} e^{-\lambda Z} \prod_{i=1}^{n} \mathbb{P}\{ F_Z(x, Y_i) | \Phi(B(x, t, Z, \phi)) = n \} 
\approx \lim_{Z \to \infty} e^{-\lambda Z} \int_{y_i \in B(x, t, Z, \phi)} \left( \frac{\mathbb{P}\{ F_Z(x, y_i) \} - 1}{d_{yi}} \right) dy_i 
\geq \lambda \int_{y_i \in B(x, t, \infty, \phi)} \left( \mathbb{P}\{ F_Z(x, y_i) \} - 1 \right) dy_i 
= \lambda \int_{y_i \in B(x, t, \infty, \phi)} \left( \mathbb{P}\{ F_Z(x, y_i) \} - 1 \right) dy_i
\]

where \( F_Z(x, y) \) denotes the event that node \( x \) cannot reach \( y \). In Eq. (16), the first approximation is due to the independence assumption. This underestimates the overall probability. The second approximation drops the conditional term and will lead to an underestimation. As part of our future work, we plan to derive a better approximation to Eq. (16). The event of \( F_Z(x, y) \) can be caused by two reasons, i.e., i) \( y \) is transmitting, and ii) \( y \) cannot decode node \( x \)'s transmission.

\[
P\{ F_Z(x, y) \} = h(d_{xy}) + (1 - h(d_{xy})) \left[ 1 - P_{r_{xy}}^N \right] 
\times \sum_{m \in \mathbb{N}} \frac{\lambda^m}{m!} e^{-\lambda Z} \int_{z_1, ..., z_m} \prod_{i=1}^{m} \Phi(B(x, z_i)) dz_1 ... dz_m
\]

Finally, the expected progress in a single hop can be computed as follows,

\[
E(s_{op}) = \int_0^\infty P\{s_{op} > t\} dt \tag{16}
\]

When \( h(r) \to 0 \) (i.e., no two nodes can transmit at the same time), it is easy to see that the performance of opportunistic routing under saturated load case degenerates to the lightly loaded case.

### 6. NUMERICAL RESULTS

In this section, we present the numerical results from the analysis in the previous section. We also conduct Monte Carlo (MC) simulations to verify the correctness of the analytical models. For each setting, we generate 1000 random scenarios in the MC simulations. In each MC simulation, nodes are randomly placed according to Poisson distribution. Packet receptions are subject to channel fading and/or interference. The parameters used in the simulations and numerical results are summarized in Table 2, which are derived from existing WiFi cards.

#### 6.1 Validation of analytical models

In this section, we first validate the analytical results using Monte Carlo simulations. From the parameter settings, the average distance of a successful transmission (loosely called transmission range) is around 250 meters when senders and receivers are placed arbitrarily. This is consistent with the IEEE 802.11 specifications. In this set of experiments, the node density \( \lambda \) is fixed to 0.001, or equivalently around 100 nodes in a 1000x1000m² area. The parameter \( \phi \) in the \( k \)-nearest neighbor routing and opportunistic routing is set to be \( \pi/6 \) unless otherwise specified.

Figure 1–2 give the results for the average progress in each transmission with different routing schemes in the light-loaded and saturated traffic cases. The solid curves show the theoretical results, and the dashed ones are the assembly average of MC simulations. For the opportunistic routing scheme, we show the cumulative density function (CDF) from the simulations and the theoretical prediction of the mean value (the straight line.) We see the theoretical results match well with the simulations for the light-loaded case. In the saturated case, due to approximations introduced for ease of computation, the theoretical results deviate from the simulation result slightly. However, it is important to note that the trend predicted by the analytical results agrees with simulation results. In both cases, MC simulations are very time consuming and even with 1000 random scenarios, one can still observe roughness in the mean values. Also evident from the figures, opportunistic routing can achieve a much larger progress in each hop in both lightly and saturated loaded case.

#### 6.2 Impact of protocol parameters and node density

In this section, we use the proposed theoretical model to study the impact of node density and angle \( \phi \). The metric used is the average effective progress in a single transmission, defined as the product of average progress \( s \) and hop efficiency \( \eta \). Due to the lack of space, a subset of the results are presented.

Figure 3 shows the average effective progress in VC-based routing. From Figure 3(a), we see that node density does not have a significant effect on the effective progress for large cell sizes. When the cell side length is small, some cells may be empty. Therefore, smaller density implies smaller average progress. As the cell size becomes bigger, the void probability diminishes. The progress of VC-based routing only depends on the channel model as in Eq. (5). Similar observation can be made in the saturated load case in Figure 3(b). The effective progress under saturated load condition is significantly less (around 1/2) due to interferences from current transmitters.

With \( k \)-nearest neighbor routing (Figure 4), when \( k \) is small, the effective progress increases as \( \phi \) decreases. This is because for smaller \( \phi \), the \( k \)-th neighbor is likely to be at a farther distance. However, for large \( k \), a large \( \phi \) is beneficial as the \( k \)-th neighbor is more likely to be close enough, which can decode packet transmissions successfully. Compared to light-loaded scenarios (Figure 4(a)), smaller progress (around 1/2) is made under saturated load case. Furthermore, the optimal \( k \)'s for different angles \( \phi \)'s are much less in the interference dominated regime.

Figure 5 shows the results of opportunistic routing. When \( \phi \) is fixed, larger node density \( \lambda \) leads to a larger effective progress. Even though the successful reception probability is low at far dis-

### Table 2: Parameter settings in the Numerical Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise ( E_{N0} )</td>
<td>-90dBm</td>
</tr>
<tr>
<td>Sensing threshold ( P_0 )</td>
<td>-85dBm</td>
</tr>
<tr>
<td>Decoding threshold ( \beta )</td>
<td>4dBm</td>
</tr>
<tr>
<td>Path loss exponent</td>
<td>3</td>
</tr>
<tr>
<td>Data rate</td>
<td>1Mbps</td>
</tr>
<tr>
<td>Antenna gain, absorption ( A )</td>
<td>0.0016</td>
</tr>
<tr>
<td>( K )</td>
<td>1.98 \times 10^{-8}</td>
</tr>
</tbody>
</table>
tance, there are likely more nodes when the density is higher. The effect of \( \phi \) is more evident in Figure 5(a). The concavity of effective progress as a function of \( \phi \) comes from the fact that \( \phi \) plays two roles. A larger \( \phi \) leads to more relay candidates. However, it can also result in lower path/hop efficiency. The best \( \phi \) appears to be around 50 – 80 degree for the set of node densities experimented in light-loaded scenarios. Opportunistic routing in saturated load scenario appears to benefit from a larger \( \phi \). This observation is confirmed by MC simulations. Finally, comparing Figure 5(b) with Figure 3–4(b), we observe that significant gains are attainable with opportunistic routing even in saturated traffic cases. This in fact contradicts our initial intuition that the gain of opportunistic routing will diminish in interference dominated regime due to the presence of strong interferers around non-transmitting nodes.

7. CONCLUSION

In this paper, we use stochastic geometry technique to analyze the performance of different routing protocols under realistic channel fading model. Our analysis and simulation show that under both light-loaded and saturated traffic, opportunistic routing outperforms significantly compared to its deterministic counterpart. Furthermore, performance of deterministic routing schemes is sensitive to the selection of parameters. However, such results should be taken with a grain of salt as the control overhead for coordination in opportunistic routing is not modeled.

8. REFERENCES


Figure 3: VC-based routing with different node density

Figure 4: k-nearest neighbor routing with $\lambda = 0.001$

Figure 5: Opportunistic routing as a function of angle $\phi$ and density $\lambda$