Planning of Multi-Carrier Broadband Wireless Systems with Ideal Power Control over Frequency Selective Channels

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Abstract—Planning procedures for multi-carrier wireless systems based on the SINR evaluated over one sub-carrier do not allow to account for multi-carrier transmission features where coded bits in the same block are spread over different sub-carriers. Due to channel frequency selectivity, sub-carriers can be subjected to different propagation conditions. The decoder in the receiver can compensate for errors even when some sub-carriers are received below the SINR target level. This fact should be accounted for during cell planning so to avoid wrong dimensioning of network parameters. In order to extend the classical planning approach to the multi-carrier systems we adopt the effective exponential SINR parameter for planning. The proposed procedure is general and it is based on the simulated statistics of the effective SINR. For evaluation purposes the two proposed procedures are validated on multi-carrier systems. It is included.

Planning procedures for multi-carrier systems based on the SINR measured over one sub-carrier [2]-[5] allow to greatly simplify planning since classical cellular networks methodologies can be adopted. However these methods do not allow to explicitly account for the fact that bits in the block are spread over more sub-carriers undergoing different (uncorrelated) fading effects and that the decoder can compensate for block errors even when some sub-carriers are received with SINRs below the target level. Neglecting this fact during planning can lead to wrong dimensioning of the cell network parameters. In order to extend the single SINR planning procedures to multi-carrier systems, we consider the link error prediction methods for multi-carrier links that have been recently proposed in the literature, [6]-[7]. Link performance is evaluated in terms of block error probability which is a decreasing function of the effective SINR (indicated in the following as $SINR_{eff}$) which is a “combination” of the SINRs of the $M$ sub-carriers used for the transmission of the block. The exponential formulation for the $SINR_{eff}$ is considered in this paper. In [8] it is demonstrated that the achieved accuracy of this link error prediction method is within few tenths of a dB for most of the cases even when interference is included. In order extend the single SINR procedure to the multi-carrier case we assume that when a data block is erroneously received an outage event has occurred. Thus the cellular network parameters should be selected so to guarantee the block error probability is below the target for a certain percentage of time or, equivalently, that the $SINR_{eff}$ is above the signal-to-noise ratio target for the same percentage of time. Starting from this
The main goal of cell planning is to determine the cell radius so that the SINR for a (generic) user located at the cell border is greater or equal than a specified target for a specified percentage of time. When the only degrading source is the AWGN noise, indicating with $N$ the noise power, the coverage radius should be selected so that:

$$\frac{C}{N} \geq \left( \frac{C}{N} \right)_{\text{target}},$$

where $C$ is the (average) received power which is a function of the coverage radius $R$; the dependence of $C$ on $R$ is accounted for by the selected propagation model. As shown in (1) $R$ depends on the target signal-to-noise ratio, $(\frac{C}{N})_{\text{target}}$, that, for AWGN channel can be easily determined from the curves showing the probability of bit error as a function of $C/N$. When fast fading is introduced, the planning criterion can be reformulated in terms of the link outage probability i.e. the radius $R$ should be such that:

$$\text{Prob}\left\{ \frac{C\alpha}{N} \leq \left( \frac{C}{N} \right)_{\text{target}} \right\} \leq \zeta_0,$$

where $\alpha$ is the fast fading term and $\zeta_0 \in (0,1)$ is the performance outage index\(^1\). Commonly $\zeta_0 = 0.01$ or $\zeta_0 = 0.05$ are considered as acceptable figures. Starting from (2) it is well known that $R$ can be obtained solving the equation:

$$\frac{C}{N} = \left( \frac{C}{N} \right)_{\text{target}} \Gamma_{\alpha}(\zeta_0),$$

with equal sign, where $\Gamma_{\alpha}(\zeta_0)$ is the fast fading margin corresponding to the performance target $\zeta_0$. The margin $\Gamma_{\alpha}(\zeta_0)$ can be obtained given the statistics of the fast fading variable, $\alpha$.

Finally, when interference $I$, due to re-using cells and/or to adjacent channels etc., is included in the calculations, equation (2) takes the form:

$$\text{Prob}\left\{ \frac{C\alpha}{N + I} \leq \left( \frac{C}{N} \right)_{\text{target}} \right\} \leq \zeta_0,$$

and the radius $R$ is obtained solving the following equation:

$$\frac{C}{N} = \left( \frac{C}{N} \right)_{\text{target}} \Gamma_{\alpha}(\zeta_0) \Gamma_f(\zeta_0),$$

where $\Gamma_f(\zeta_0)$ is the interference margin that can be obtained given the (measured or simulated) statistics of the interference $I$.

Neglecting, for the moment, shadowing effects the generic equation to be solved for the calculation of $R$ is:

$$\frac{C}{N} = \left( \frac{C}{N} \right)_{\text{min}},$$

where $(\frac{C}{N})_{\text{min}}$ is the minimum required target signal to noise ratio including margins against fast fading and interference. For example, from (3), $(\frac{C}{N})_{\text{min}} = \left( \frac{C}{N} \right)_{\text{target}} \Gamma_{\alpha}(\zeta_0)$ in the presence of fast fading (FF) only.

In general, the calculation of $(\frac{C}{N})_{\text{min}}$ is not straightforward especially when broadband multi-carrier systems are considered. An example the $(\frac{C}{N})_{\text{min}}$ obtained from (3) cannot be directly applied to a multi-carrier system since it is based only on the statistics of the SINR evaluated over a single sub-carrier.

When multi-carrier system is considered coded bits belonging to the same block are transmitted over $M$ different sub-carriers. Due to frequency selectivity sub-carriers are subjected to uncorrelated propagation effects. Thus, indicating with $P_{e,\text{block}}$ the probability of block error, in general it can be written that [6], $P_{e,\text{block}} = f(SINR_1, SINR_2, ..., SINR_M)$ and the function $f(\cdot)$ depends on the coding strategy adopted for transmission\(^2\). For our purposes we assume that when the $P_{e,\text{block}}$ is above a specified performance threshold an outage event occurs. Thus in the multi-carrier case the cell network parameters should be selected so that:

$$\text{Prob}\{P_{e,\text{block}} \geq P_0\} \leq \zeta_0,$$

and $P_0$ is the outage performance target.

The direct solution of (7) is in general very difficult (if not impossible) and (time consuming) simulations may represent the only viable solution. In the following we introduce a numerical procedure that can be helpful to solve the planning problem indicated in (7) in a more effective way. This procedure allows to put into evidence the performance improvement due to the spreading of the bits of the coded data block over $M$ sub-carriers.

\(^1\)Considerations can be easily extended to the fractional area parameter which is commonly used by telecom operators for cellular planning.

\(^2\)An analytical expression for $f(\cdot)$ is in general very difficult even if not impossible to achieve.
A. Shadowing effects on planning

Before concluding this Section, it is necessary to outline the role of the shadowing on planning. Ideal power control allows to compensate for path loss and shadowing. Thus the received power level $C$ at the cell edge is:

$$ C = P_{max}G L(R)S, \quad (8) $$

where $P_{max}$ is the maximum transmitted power, $G$ is the overall transmitter-receiver antenna gains, $L(R)$ is the path loss at the cell border and $S$ is the shadowing random variable. Due to shadowing, the cell radius $R$ should be selected so that:

$$ \text{Prob} \left\{ \frac{P_{max}G L(R)S}{N} \leq \left( \frac{C}{N} \right)_{\min} \right\} \leq \zeta_0. \quad (9) $$

Equation (9) can be easily re-written by introducing the shadowing margin $\Gamma_S$ so that the planning equation in (6) can be further generalized as:

$$ \frac{C}{N} = \frac{P_{max}G L(R)}{N} = \left( \frac{C}{N} \right)_{\min} \cdot \Gamma_S. \quad (10) $$

and $(C/N)_{\min}$ includes both fast fading and interference margins.

III. PLANNING PROCEDURE

As indicated in [6]-[9] the link error performance of a multi-carrier systems can be accurately predicted by introducing the effective SINR which re-maps the set of received SINRs on the $M$ sub-carriers in a unique value $\text{SINR}_{eff}$ such that:

$$ P_{e, block, AWGN}(\text{SINR}_{eff}) = P_{e, block}, \quad (11) $$

where $P_{e, block, AWGN}(\text{SINR}_{eff})$ is the probability of block error evaluated over AWGN channel for the selected decoding strategy.

From [7] the $\text{SINR}_{eff}$ formulation providing accuracy within few tenths of dBs under a wide range of modulation schemes, coding rates and channel types is the exponential SINR mapping i.e. the $\text{SINR}_{eff}$ is:

$$ \text{SINR}_{eff} = -\beta \ln \left( \frac{1}{M} \sum_{m=1}^{M} e^{-\text{SINR}_m/\beta} \right), \quad (12) $$

where $\beta$ is a parameter optimized to provide the best matching of the $P_{e, block}$ and $P_{e, block, AWGN}$ in (11) in accordance to the coding scheme and the number of the coded bits in a block. In our formulation we didn’t outline the dependence of $\beta$ on $M$. This fact can be well understood looking at the procedure which is commonly used to estimate $\beta$, [8]. In the following we assume that $\beta$ is constant for each $M$.

Using (11) the performance requirement in (7) can be re-written as:

$$ \text{Prob}\{ P_{e, block, AWGN}(\text{SINR}_{eff}) \geq P_0 \} = \text{Prob}(\text{SINR}_{eff} \leq \rho_0) \leq \zeta_0. \quad (13) $$

where $\rho_0$ is the performance target of the effective SINR ratio that can be obtained from [10]. As an example, for QPSK modulation with coding 1/2 from [10] we obtain $\beta = 1.57$ and $\rho_0 = 3.5$ dB for a block error probability of $10^{-2}$. To outline the dependence of (12) on the $(C/N)$ ratio assuming ideal power control$^3$ the $\text{SINR}_{eff}$ is:

$$ \text{SINR}_{eff} = \frac{C \alpha_m}{I_m + N}, \quad (14) $$

where $I_m$ is the interference on the $m$-th sub-carrier and $\alpha_m$ is the corresponding fast fading term. Using (14) equation (12) can be rewritten as:

$$ \text{SINR}_{eff} = -\beta \ln \left( \frac{1}{M} \sum_{m=1}^{M} e^{-(C/I_m)^{\alpha_m}} \right). \quad (15) $$

Under ideal power control $(C/I_m)$ doesn’t depend on $C$ and the $\text{SINR}_{eff}$ in (15) is an increasing function of $(C/N)$. Thus it is possible to search for the minimum $(C/N)$, e.g. $(C/N)_{\min}$, to be used in (6) for the calculation of $R$ such that (13) is verified.

In this paper for fixed $(C/N)$ the cumulative distribution function (CDF) of the $\text{SINR}_{eff}$ is numerically calculated for the selected fading and interference scenario. The value of $(C/N)$ satisfying (13) is used as $(C/N)_{\min}$ in (6).

Finally is should be noted that for AWGN channel (i.e. no fast fading and interference) the $\text{SINR}_{eff} = C/N$ so that $(C/N)_{\min} = \rho_0$.

A. Margins

It can be of interest to express the $(C/N)_{\min}$ obtained from (13) as the product of $\rho_0$ with fast fading and interference margins. To this purpose we (numerically) solve (13) to determine the following two values of $(C/N)_{\min}$:

1) $(C/N)_{\min}^{(FF)}$: the $(C/N)_{\min}$ in the absence of interference i.e. the only degrading cause is the fast fading (i.e. the $\{\alpha_m\}$ terms in (15));

2) $(C/N)_{\min}^{(FF+I)}$: the $(C/N)_{\min}$ when interference and fast fading are included.

Given $(C/N)_{\min}^{(FF)}$ and $(C/N)_{\min}^{(FF+I)}$ the fast fading margin can be defined as $M_a = (C/N)_{\min}^{(FF+I)}/\rho_0$ and the interference margin $M_I = (C/N)_{\min}^{(FF+I)}/(M_a \rho_0)$. The differences (in dB) between $M_a$ calculated for $M = 1$ with $M_a$ calculated for $M > 1$ provides the gain achieved by spreading the bits of one block over $M$ distinct sub-carriers.

IV. APPLICATIONS OF THE PROPOSED PROCEDURE

A. Interference scenario

The planning procedure outlined in the previous Section is applied to the calculation of the cellular parameters for the two cells scenario in Fig.1. The two cells are located at distance $D$ (referred to the center of the cells) and use the same OFDMA

$^3$We assume that power control can compensate for the average path loss as well as for shadowing (if present). The average power level $C$ is then a constant. The extension of the proposed procedure to the non-power controlled case is not difficult.
band. The downlink reference user is located at the cell border and it is interfered by uplink users within the interfering cell (see Fig. 1). Under ideal power control, the co-channel interference on the \( m \)-th sub-carrier of the reference user due to one terminal in the interfering cell can be approximated as:

\[
I_m = C \frac{L(d)}{L(R)} S_m \lambda_m, \tag{16}
\]

where \( L(\cdot) \) is the path loss, \( S_m \) is the shadowing term and \( \lambda_m \) represents fast fading which is assumed to be Rayleigh distributed with unit power. It approximates the effects of multipath propagation on the interferer-to-reference link. The attenuation model \( L(x) = MCL \cdot \min\{1, 1/x^{\gamma}\} \) has been considered; \( MCL \) is the minimum coupling loss and \( \gamma \) is the path loss exponent ranging from 2 to 5. The shadowing variables \( S_m \) are lognormal with identical standard deviation \( \sqrt{2} \sigma_S \) in dB.

Using (16) and accounting for the uncompensated fast fading the signal-to-noise plus interference ratio on the \( m \)-th sub-carrier \( (m = 0, 1, 2, ..., M - 1) \), \( SINR_m \) is:

\[
SINR_m = \frac{\alpha_m}{\frac{L(d)}{L(R)} S_m \lambda_m + \frac{N}{C}}. \tag{17}
\]

As expected, interference doesn’t depend on \( C \). In the following we assume that the random variables \( \alpha_m, \lambda_m \) and \( S_m \) are statistically independent. This assumption holds when the sub-carriers allocated to each block are randomly positioned over the entire signal band. In the WiMAX system this approach is well approximated by the partial usage sub-channel (PUSC) permutation mode, [10]. The values of \( SINR_m \) in (17) have been obtained by simulation randomly varying the position of the interferer inside its cell in accordance to a uniform spatial distribution. The distance \( d \) in Fig. 1 between the interferer and the reference user is obtained by a straightforward application of the Carnot’s theorem. The \( SINR_m \) in (17) are used for the calculation of the CDF of the \( SINR_{eff} \) in (15). It is not difficult to extend the procedure to other sub-carriers allocation modes and then to compare performance \( SINR_{eff} \).

\( ^4 \)Due to sectorization with a large number of sectors (6 or more) to be adopted in OFDMA systems, it seems to be reasonable to assume only one cell causing co-channel interference on the reference cell/sector. Only the first tier is considered but results can be easily extended to cells in the second tier and so on.

Figure 1. Two cells interference scenario for multi-carrier system.

Figure 2. Cumulative distribution function for \( SINR_{eff} \) - \( M = 4, 20 \) and \( (C/N)_{min} = 10, 15 \) dB.

Figure 3. Outage probability as a function of the \( (C/N)_{min} \) for variable \( M \) and \( D/R \).

B. Simulation Results

Results have been obtained using the following parameters: reference target for QPSK (cod. 1/2) \( \rho_0 = 3.5 \) dB and \( \beta = 1.57 \) assumed to be constant for each \( M \), BS antenna gain \( G_R = 10 \) dB, shadowing standard deviation \( \sigma_S = 8.5 \) dB. In Fig. 2 the CDF of the \( SINR_{eff} \) are plotted for different values of \( M \) and \( (C/N)_{min} \) and \( \gamma = 3 \). As expected, the increase of \( (C/N)_{min} \) allows to decrease outage since interferer becomes a constant value \( \rho_0 \). The drastic increase in the slope of the CDF passing from \( M = 4 \) to \( M = 20 \) can be explained looking at the \( SINR_{eff} \) formulation in (12). In fact, when \( M \) increases the \( SINR_{eff} \) tends to become a constant value approaching \( -\beta \ln(E(e^{-SINR/\beta})) \) for \( M \to \infty \).

In Fig. 3 outage probability is reported as a function of \( (C/N)_{min} \) for \( M = 1 \) (single carrier case) and for \( M = 12 \) and \( (D/R) \) equal to 2 or 6. As expected, the increase of the reuse ratio \( (D/R) \), allows to decrease outage since interference is reduced (on average). Again from Fig. 3, it can be noted...
the increase of the number of allocated sub-carriers per block, \( M \), allows to reduce the \( (C/N)_{min} \) required to achieve the outage target. This is due to the action of the decoder which allows block recovery even when one (or more) sub-carriers inside the block are in outage. As an example, from Fig.3 it can be observed that fixing the outage target to 5% gains of about 6 dB and 7 dB with respect to the single sub-carrier case are achieved for \( M = 12 \) and \( M = 20 \), respectively when \( D/R = 6 \).

In Fig.4 the outage probability is reported as a function of the \( (C/N)_{min} \) for variable \( M \) and \( D/R \). The performance improvement with the increase of \( \gamma \) is evidenced.

In Fig.5, the required \( (C/N)_{min} \) is reported as a function of the number of allocated sub-carriers \( M \), fixing the outage probability to 5%. Looking at the curve for \( (D/R) = 6 \) if the receiver can allocate \( M = 20 \) sub-carriers to each block instead of \( M = 4 \), the required \( (C/N)_{min} \) is 10 dB instead of 14 dB thus achieving 4 dB of extra margin. This performance improvement can be used for relaxing the requirements on the terminal and/or to increase the coverage radius. In fact, as indicated in Fig.5 this gain can be traded off in terms of the required \( (D/R) \) ratio to compensate for interference i.e. if the required \( (C/N)_{min} = 14 \) dB, the system can be designed setting \( M = 4 \) and \( D/R = 6 \) or, by selecting \( M = 8 \) with \( D/R = 3 \). In the latter, a tighter spatial reuse of the bands is achieved at the expense of a reduced granularity in the resource allocations (e.g. the blocks to be allocated to each terminal are larger). The results corresponding to \( M = 1 \) allows to put into evidence the margins required for single carrier planning that are higher than those required for multi-carrier link thus leading to wrong dimensioning of the cell network parameters. Finally it should be observed that the performance improvement which is achievable by increasing \( M \) rapidly saturates for \( M \geq 20 \).

V. CONCLUSIONS

The problem of cell planning for multi-carrier systems operating over frequency selective channels has been addressed. Cellular network parameters have been obtained using the effective SINR defined in (12) instead of the SINR for the single sub-carrier as used in classical planning. The adoption of \( SINR_{eff} \) allows to include in planning the number of sub-carriers \( M \) used to transmit bits belonging to the same block. It was observed that the selection of \( M \) can be traded off with the \( D/R \) reuse ratio. In particular, it was observed that \( D/R \) can be lowered at the expense of a reduced granularity in resource allocation over the frequency axis. Finally, system performance rapidly saturates with the increase of the number of sub-carriers \( M \);

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