Degradation in Convolutionally Coded V-BLAST System and A Performance Improvement Method

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Abstract—The performance degradation in convolutionally coded V-BLAST system is analyzed in this paper. Compared with coded linear filtering approach, it is shown that the coded V-BLAST suffers from serious degradation due to the effect of error propagation. Considering the constraint of the system, a performance improvement method is proposed. It adjusts the weighting of soft bits, where the weighting for one stream is found based on the equivalent error rate of that stream. Additional computation needed by the proposed method is negligible. Both theoretical analysis and simulation verify the effectiveness of the proposed method.

Index Terms—V-BLAST, Convolutional Code, Error Propagation

I. INTRODUCTION

In spatial division multiplexing system, Vertical Bell-lab Layered Space Time (V-BLAST) scheme [1][2] has been proposed as the receiver detection scheme. The ordered successive interference cancellation is employed to improve the raw performance by using the feedbacks from previous symbol detection stages. The uncoded V-BLAST has significant performance gain over the linear filtering approach. In [3][4], the convolutionally coded V-BLAST system is studied, and feedback method from decoded bit stage method is proposed, i.e. the decoded bit are re-encoded for cancellation. The method can provide good performance but has high computational complexity. There are other works addressing the coded V-BLAST system. In [5], it is shown that the iterative detection and decoding can significantly improve the coded V-BLAST performance. But the method exhibits high complexity. There are also some work in comparison between convolutionally coded V-BLAST and groupwise space time trellis coded MIMO-OFDM system [6].

However, the fundamental problems in the application of convolutionally coded V-BLAST system are not well addressed in the previous literatures. There lacks general performance analysis of Viterbi decoder under the interference caused by error propagation. Moreover, it is shown in simulations that, if the error propagation is not appropriately considered in receiver design, the coded performance degradation will be worse than the respective linear filtering approach. The problems include, what is the origin of this degradation, and is there any compensation method without significant complexity increase. These problems are the barriers in the practical application of V-BLAST in coded system. This paper will address the above problems.

The contributions of the paper include: The origin of the coded performance degradation is explained by an extracted system model with discrete distributed interference. The feasibility of using modified Viterbi decoder for performance improvement is discussed. A weighting method is proposed to compensate the degradation, and the weighting values are obtained by error rate mapping.

The paper is organized as follows. Problem description is given Section II. Improvement method is proposed in Section III. Simulation results are shown in Section IV, and the conclusions are drawn in Section V.

II. PROBLEM DESCRIPTION

In this section, the fundamental detection structure of spatial division multiplexing (SDM) is given firstly. Then the core characteristic of coded V-BLAST is modeled as an extracted system model with a discrete distributed interference. The feasibility of using modified Viterbi decoder metric is also given.

Assume $N_{SS}$ spatial streams, $N_{R}$ receiving antennas. The number of transmitting antenna is the same as the number of spatial streams. The channel on each subcarrier can be treated as flat fading, and the signal model is $Y = HX + N$, where $X$ is the $N_{SS} 	imes 1$ transmitting signal; $Y$ is the received $N_{R} 	imes 1$ signal, and $N$ is the $N_{R} 	imes 1$ AWGN noise. $H$ is the $N_{R} 	imes N_{SS}$ channel matrix. The transmitting symbol power is $P_X$. Assume MMSE detection is adopted in receiver, and the $N_{SS} 	imes 1$ detection output vector is $Z = GY$. By the principle of orthogonality, the filtering matrix is $G = [H^H H + \sigma_n^2 R_X^{-1}]^{-1} H^H$, or equivalently $G = R_X H^H H R_X H^H + \sigma_n^2 I_{N_{R}}^{-1}$, where the transmitting signal covariance matrix $R_X$ can be assumed as $P_X I_{N_{SS}}$. The post-detection SNR value $\gamma_i^{mmse}$ on the $i$th spatial stream is

$$\gamma_i^{mmse} = \frac{[P_X \cdot \text{Diag}(GH^H G^H)]_{ii}}{[\sigma_n^2 \cdot \text{Diag}(GG^H) + P_X \cdot \text{Diag}(B \cdot B^H)]_{ii}}$$

(1)

where the numerator $P_X \cdot \text{Diag}(GH^H G^H)$ is the signal power vector, and the denominator is the summation of noise power vector $P_X \cdot \text{Diag}(GH^H G^H)$ and interference power vector $P_X \cdot \text{Diag}(B \cdot B^H)$, where $B = GH - \text{Diag}(GH)$. 

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For two spatial stream system adopting V-BLAST, assume the 1st spatial stream has larger channel gain and it is detected firstly. Given the fading channel matrix \( \mathbf{H} = [\mathbf{h}_1 \, \mathbf{h}_2] \), the first row of \( \mathbf{G} \) is used as the filtering vector for the 1st spatial stream. The signal on the 2nd spatial stream after cancellation is \( \mathbf{y} = \mathbf{h}_1(x_1 - \hat{x}_1) + \mathbf{h}_2 x_2 + \mathbf{n} \), where \( \hat{x}_1 \) is the re-modulated signal after decision. Assume there is no error propagation, the filtering vector on the 2nd spatial stream is \( \mathbf{w}_2 = P \mathbf{h}_2 \mathbf{h}_2^{H} + \sigma_n^2 I_{N_k}^{-1} \). The post-detection SNR on the 1st spatial stream is \( \gamma_1^{\text{mmse}} \), which is defined in (1).

If there is no detection error on the first spatial stream, the post-detection SNR on the 2nd spatial stream is

\[
\gamma_2^{\text{vblast}} = \frac{P_X |\mathbf{w}_2 \mathbf{h}_2|^2}{\sigma_n^2 |\mathbf{w}_2|^2} \tag{2}
\]

Once there is symbol detection error on the 1st stream, the post-detection SNR on the 2nd spatial stream will become

\[
\gamma_2^{\text{vbl}} = \frac{P_X |\mathbf{w}_2 \mathbf{h}_2|^2}{P_E |\mathbf{w}_2 \mathbf{h}_1|^2 + \sigma_n^2 |\mathbf{w}_2|^2} \tag{3}
\]

where \( P_E \) is the power of error symbol from the 1st spatial stream. For BPSK, \( P_E = 4P_X \), and the probability the 1st stream in symbol error can be approximated as \( P_{se} = c_r f_c (\sqrt{\gamma_1^{\text{mmse}}}/2) \).

Introducing a random variable \( I = x_1 - \hat{x}_1 \), for the two spatial stream and BPSK case, the distribution of the interference \( I \) is \( f(I) = \frac{P_{se}}{2\sqrt{P_X}} \cdot \delta(I - 2\sqrt{P_X} + \frac{P_{se}}{2}) + \frac{P_{se}}{2\sqrt{P_X}} \cdot \delta(I + 2\sqrt{P_X}) + (1 - P_{se}) \cdot \delta(I) \).

The demapped soft bits on the 2nd stream suffer from this discrete distributed interference.

If the interference is considered as Gaussian noise, the equivalent power of this interference is \( P_i = P_E \cdot P_{se} \), and the SNR on the 2nd spatial stream can be computed by

\[
\gamma_2^{\text{Pwr}} = \frac{P_X |\mathbf{w}_2 \mathbf{h}_2|^2}{P_i |\mathbf{w}_2 \mathbf{h}_1|^2 + \sigma_n^2 |\mathbf{w}_2|^2} \tag{4}
\]

Before analyzing the details of the influence of this interference, several aspects that determine the coded performance need to be clarified. There are three aspects: codec, transceiver and channel. The codec includes transfer function of the convolutional code and the decoding algorithm. The transceiver includes detection method (and its associated soft bit weighting), modulation and interleaving. The channel includes fading distribution, interference distribution and Gaussian noise power. In the following analysis, we focus on the analysis of relationship of soft bit weighting in detection and decoding algorithm, under the presence of interference. Other aspects as the transfer function of convolutional code, and the interleaver structure, are not included in the analysis and formulation. The following results will show the effectiveness of these assumptions, the drawbacks and how to compensate the drawbacks.

### A. Extracted System Model

A system model extracting the core problem is depicted in Figure 1. The transmitted coded bits are assumed to be modulated by BPSK and demodulated coherently. The symbols after modulation is \( s_{j,m} = \sqrt{E_c} \cdot (2c_{j,m} - 1) \), where \( E_c \) is the symbol power in the extracted system model. Channel gain for each soft bit is \( h_{j,m} \). The additive Gaussian noise is \( n_{j,m} \), and follows \( \mathcal{N}(0, \sigma_n^2) \). The discrete distributed interference is \( I_{j,m} \), and follows PDF \( f(I) = \sum_{q=1}^{Q} p_q \delta(I - w_q) \), where \( Q \) is the number of discrete interference. The received signal is \( y_{j,m} \).

Soft-decision decoding is deployed and the soft bits after demodulation are \( r_{j,m} \). Viterbi algorithm is used in receiver convolutional code decoding. The signal received can be written as \( y_{j,m} = h_{j,m} \cdot s_{j,m} + n_{j,m} + I_{j,m} \). It is assumed that the demodulation block performs soft demapping and equalization. Thus the soft bits fed to the Viterbi decoder can be written as

\[
r_{j,m} = s_{j,m} + n_{j,m}/h_{j,m} + I_{j,m}/h_{j,m} \tag{5}
\]

The metric for the \( i \)th path of the \( B \) branches is \( M^{(i)} = \sum_{j=1}^{B} \mu_j^{(i)} \), where \( \mu_j^{(i)} = \log f(r_j | c_j^{(i)}) \). \( c_j^{(i)} \) is output bit vector of the \( j \)th branch on the \( i \)th path and \( r_j \) is the received soft bit vector. If the interference \( I_{j,m} \) is neglected, for the \( n \)th soft bit, the distribution of \( f(r_{j,m} | c_j^{(i)}) \) is \( \mathcal{N}(s_{j,m}^{(i)}, \sigma_n^2/h_{j,m}^2) \), and the maximization of \( M^{(i)} \) leads to the maximization of

\[
\mu_j^{(i)} = h_{j,m}^2 \cdot r_{j,m} \cdot s_{j,m}^{(i)} \tag{6}
\]

Thus if the adopted Viterbi decoder is for AWGN channel, the channel power \( h_{j,m}^2 \) of each soft bit needs to be multiplied on that soft bit. Or equivalently, the post-detection SNR of the soft bit needs to be multiplied.

If the interference term \( I_{j,m} \) is considered, and further assume the discrete distribution has three states

\[
f(I) = p_1 \delta(I - w_1) + p_2 \delta(I - w_2) + (1 - p_1 - p_2) \delta(I) \tag{7}
\]

The mean of distribution \( f(r_{j,m} | c_j^{(i)}) \) also has three states, \( s_{j,m}^{(i)} + w_1/h_{j,m} \), \( s_{j,m}^{(i)} + w_2/h_{j,m} \), and \( s_{j,m}^{(i)} \) with probability \( p_1, p_2 \) and \( (1 - p_1 - p_2) \) respectively. The maximization of
It becomes assumed.

This error will eventually cause large first-event error and propagation from the firstly-detected stream, the changed signal distribution that determines the Viterbi decoder path metric is 

\[ \mathcal{P}(x|y) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{(y - \mu)^2}{2\sigma^2}} \]

in the Gaussian channel. Given this constraint, metric (6) is used to analyze the origin of the degradation. In the practical system, the Viterbi decoder is generally only designed for Gaussian channel. Given this constraint, metric (6) is used in the following sub-section B to analyze the origin of the performance degradation.

**B. Origin of the Degradation**

Under the presence of the interference, the signal distribution that determines the Viterbi decoder path metric is changed. Once the soft bits on the secondly-detected stream are weighted an over-valued SNR, and there is error propagation from the firstly-detected stream, the changed signal distribution will cause large path metric computation error. This error will eventually cause large first-event error and coded bit error. In the following, the path metric pairwise error probability is given to explain the origin of the degradation. Without loss of generality, it is assumed that the all-zero coded bit sequence is transmitted and the probability of error is determined in favor of another sequence. Throughout the analysis, BPSK modulation and coherent demodulation are assumed.

The path metric is 

\[ M(i) = \sum_{j=1}^{B} \mu_{j,j}^{(i)} = \sum_{j=1}^{B} \sum_{m=1}^{P} \mu_{j,j,m}^{(i)} \]

where \( P \) is the number of soft bits on one branch. Assume \( M(0) \) represents the all-zero path, and \( M(1) \) represents the path that differs from the all-zero path in \( d \) bits, the probability of error in the pairwise comparison of \( M(0) \) and \( M(1) \) is 

\[ P_0(d) = P\{M^{(1)} \geq M^{(0)}\} = \frac{1}{B} \sum_{j=1}^{B} \left( \sum_{m=1}^{P} (\mu_{j,j}^{(1)} - \mu_{j,j,m}^{(0)}) \right) \geq 0 \]
spatial stream. Moreover, the sign of $\sum_{q=1}^{d} h_q$, the summation of channel realizations for the $d$ bits the two paths differ, varies for different paths. Since the channel realizations are random, term $\sum_{q=1}^{d} h_q$ should have equal opportunity to be positive or negative. And $\varepsilon$ is constant value for each spatial stream. Thus the term $-\varepsilon \cdot \sum_{q=1}^{d} h_q$ should also have equal positive and negative probabilities for each spatial stream. So for the overall performance, performance improvement can not be declared by adopting the modified Viterbi decoding metric (9).

III. IMPROVEMENT METHOD

Here we propose an approach computing the equivalent post-detection SNR. The basic idea is to find an equivalent SNR level with AWGN noise only, that has the same raw error performance compared with the system with both AWGN and discrete-distributed interference. It is called the method based on equivalent error rate mapping. For the two spatial stream case, Let $P\{E_1\}$ and $P\{E_2\}$ denotes symbol error probability on the 1st and 2nd stream respectively. $P\{E_1\}$ denotes probability of correct symbol detection on the 1st and $P\{E_2\}$ on the 2nd stream if the detection on the 1st stream is wrong. $P\{E_2|E_1\}$ denotes the 2nd stream’s probability of detection error if the 1st stream is correct. We thus have the following relationship

$$P\{E_2\} = P\{E_2|E_1\} P\{E_1\} + P\{E_2|\bar{E}_1\} P\{\bar{E}_1\}$$

(12)

In the aforementioned example in Section II, it is obvious that $P\{E_1\} = P_{se1}$ and $P\{\bar{E}_1\} = 1 - P_{se1}$. Define a mapping function $y = MF(x)$ which is the constellation error performance on AWGN channel, the error probabilities are $P\{E_2|E_1\} = MF(\gamma_{\text{vbllast}})$ and $P\{E_2|\bar{E}_1\} = MF(\gamma_{\text{vbllast}})$. The SNRs $\gamma_{\text{vbllast}}$ and $\gamma_{\text{vbllast}}$ are defined in (3) and (2). For BPSK, the mapping function is $y = erfc(\sqrt{x}/2)$. For spatial stream number larger than 2, the error probability on each stage can be obtained following the same methodology. The proposed method compute the equivalent SNR based on error rate mapping as follows

$$\gamma_{2}^{\text{ErrRate}} = MF^{-1}(P\{E_2\})$$

(13)

A concept graph can be drawn in Figure 3 to show the relationship of the SNRs aforementioned. The SNR on the 2nd SS versus time domain symbols are shown. The slow fading is assumed for all symbols under consideration. The MMSE detection SNR on the 2nd SS $\gamma_{2}^{\text{mmse}}$ are generally relative low, and are constant for all symbol time slots. After successive interference cancellation in V-BLAST, the discrete distributed interference by error propagation will occur. Thus the SNR will be as high as $\gamma_{2}^{\text{vbllast}}$ for symbols with no error propagation, and will be as low as $\gamma_{2}^{\text{vbllast}}$ for symbols with error propagation. It is inappropriate to apply the weighting of $\gamma_{2}^{\text{vbllast}}$ for all symbols. The SNR by equivalent error rate mapping $\gamma_{2}^{\text{ErrRate}}$ and equivalent power mapping $\gamma_{2}^{\text{Pwr}}$ are also plotted. The probability of error on the 1st stream is generally quite low, and the equivalent power of interference $P_i$ is generally quite small. So the SNR by equivalent power $\gamma_{2}^{\text{Pwr}}$ is generally quite close to the SNR assuming no error propagation $\gamma_{2}^{\text{vbllast}}$. By simulation experiment, it is found that $\gamma_{2}^{\text{ErrRate}}$ is smaller than $\gamma_{2}^{\text{Pwr}}$. These relationships are reflected in the graph.

It must be noted that, this weighting scheme may not be the optimal one which minimizes the coded performance. As described before, the coded performance depends on several aspects. The analysis presented in this paper focuses on some of them. The weighting values should also depend on the transfer function of the convolutional code and the interleaving scheme. To consider the influence of other aspects, we propose to multiply a scaling factor $\beta$ on the weighting values obtained by equivalent error rate. The justification of the scaling factor is to compensate for other aspects which are not considered in the theoretical formulation but exist in real system. Since these aspects are on codec and transceiver, the influence is deterministic for specific system. So they lead to a constant value of $\beta$ for specific system. The methodology of determining the scaling factor is left to trigger further study on this topic.

IV. SIMULATION RESULTS

The simulation is performed on the coded V-BLAST system following the transceiver structure of 802.11n EWC standard [8]. The modulation and coding scheme (MCS) No. 9 is chosen. This MCS has 2 spatial streams, QPSK modulation on both spatial streams, 1 encoded stream and 1/2 rate convolutional code. The convolutional code used in the standard has constraint length $K = 7$, generator 133 and 171 in octal and minimum free distance $d_{free} = 10$. For OFDM parameters, the parameters used include: 20 MHz bandwidth, 64 subcarriers, 56 used subcarriers and 800 ns cyclic prefix. Two transmitting antennas and two receiving antennas are used in the system. Perfect channel estimation is assumed in the
system. The receiver algorithm implemented is coded MMSE-VBLAST. The channel model used is the indoor channel model TGN model [9], TGN D channel with 390 ns maximum delay and 50 ns RMS delay is used. The Frame Error Rate (FER) performance is obtained as plotted in Figure 4. The x-axis parameter SNR in the two figures is defined as per receiving antenna, meaning that transmitting antennas have unit sum transmitting power, and the additive white Gaussian noise is added on each receiving antenna.

Two baseline methods are firstly given for comparison purpose. One is to use the SNR by treating the interference as equivalent power of a Gaussian source, as in (4). Another baseline method is simple equal weighting on all spatial streams. Several typical results are plotted for comparison in Figure 4. The curve of equivalent power suffers from serious degradation in FER performance. The reason is from the over-weighted soft bits on the secondly-detected stream. In simulation process, it is also observed that the weighting computed by equivalent power is sometimes quite large for the secondly-detected stream. For these cases, if there is an detection error on the firstly-detected stream, the error propagation basically causes current frame in error. This is the main source of high FER.

For the result of V-BLAST with equal weighting for all spatial streams, it can be observed that the performance of this approach is better than equivalent power weighting. The equal weighting has avoided the large 2nd stream weighting situations in equivalent power case, thus has better performance. It is necessary to note that, if people neglect the weighting problem when designing the V-BLAST system, the coded performance should be this result.

For the result of equivalent error rate, it has better performance than equal weighting and power equivalent weighting, which indicates that the method can find better weighting values for the V-BLAST system. But the performance improvement compared with equal weighting is not significant. This implies that there might be more room for further improvement. For the method of applying a scaling on weighting obtained by equivalent error rate, the performance has notable improvement. The reason, as mentioned before, is the weighting by equivalent error rate is not optimal in improving the performance. Several aspects in codec and transceiver is not considered in the formulation of equivalent error rate weighting, thus introduces these offset. In this simulation experiment, a scaling factor of 0.5 is applied. This number comes from experimental re-tries, not from theoretical analysis.

The simulation verifies that the degradation problem occur in 802.11n system adopting V-BLAST, and the method is effective for this system. For other system with different codec and transceiver, the performance degradation and the degree of method’s effectiveness might be different. More study is expected on the derivation of scaling factor, and other improving method. Further investigation can be performed to find the optimal weighting given a codec and transceiver structure.

V. CONCLUSIONS

The origin of degradation in convolutionally coded V-BLAST system is described by pair wise error probability of Viterbi decoder metric under the interference caused by error propagation. A method is proposed to improve the coded performance by adjusting the weighting of soft bits. The weighting value for a particular stream is obtained by mapping back from the error rate on that stream. Simulation has verified the analysis and method.

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