Achieve the Degrees of Freedom of $K$-User MIMO Interference Channel with a MIMO Relay

Sujie Chen and Roger S. Cheng
Department of Electronic & Computer Engineering
Hong Kong University of Science & Technology
Clear Water Bay, Kowloon, Hong Kong
Email: {echsj, eecheng}@ust.hk

Abstract—This paper investigates the problem of achieving the degrees of freedom (DoF) of a $K$-user multiple-input-multiple-output (MIMO) interference channel with a MIMO relay, where each transmitter and receiver is equipped with $M$ antennas. We propose a two-hop relay aided transmission scheme based on the idea of interference alignment to achieve the full DoF $\frac{KM}{2}$. The scheme requires global channel state information (CSI) at the MIMO relay only while other nodes only need to have their local CSI. We also study the number of antennas needed at the MIMO relay to achieve interference alignment. In particular, we find that using just linear processing at the relay, the minimum number of antennas needed at the relay for each transmitter-receiver pair to achieve the maximum $\frac{M}{2}$ DoF per channel use is $(K - 1)M$, less than what is needed for decode-and-forward approach and all other reported approaches in existing literatures.

I. INTRODUCTION

Recent progress in multiuser information theory has led to many interesting results. In particular, DoF is used as a tool to characterize the asymptotic behavior of network capacity in high signal-to-noise ratio (SNR). For a $K$-user interference channel with every transmitter and receiver having $M$ antennas, [1] showed that if channel coefficients are time-varying/frequency-selective, the sum capacity can be characterized as

$$C(SNR) = \frac{KM}{2} \log (SNR) + o(\log (SNR)). \quad (1)$$

The pre-log factor $\left(\frac{KM}{2}\right)$ is hence defined as the DoF of the $K$-user MIMO interference channel. Here SNR is defined as the total transmit power of all nodes over the noise power at each node. The DoF achievable scheme in [1] is based on the idea of interference alignment. Each transmitter constructs its own pre-coding vector such that at every receiver, interference signals from unintended transmitters lie within a “wasted” space that does not intersect with the space spanned by the desired signals. The interference alignment scheme proposed in [1] requires global CSI at all nodes. However, obtaining global CSI could be quite impractical, especially when the channels are time-varying/frequency-selective.

Some works have been done to circumvent the global CSI requirement. In [5], a distributed algorithm for interference alignment to bypass the CSI requirement in a reciprocal network was proposed. In [4], the authors proved that cooperation through relays does not increase the DoF of a fully connected $K$-user interference channel. Therefore, the DoF of a $K$-user MIMO interference channel with a MIMO relay is still $\frac{KM}{2}$. However, the authors pointed out that relays might have potential benefits in terms of the CSI requirement. In [6], the authors considered the scenario where each transmitter and receiver is equipped with a single antenna, and a MIMO relay having more than $K$ antennas is designated to help all $K$ transmitters. By applying a number of hybrid encoding strategies and power allocation at MIMO relay, a transmission scheme which achieves the DoF is proposed [6]. The advantage of this scheme is that it requires global CSI at the MIMO relay only, while other nodes only need to have their local CSI. Thus, by utilizing a MIMO relay, the overhead of acquiring global CSI in all nodes can be greatly reduced.

In this paper, we consider a general scenario for $K$-user MIMO interference channel with a MIMO relay, where each transmitter and receiver is equipped with $M$ antennas. Based on the idea of interference alignment, we propose a DoF achievable scheme by making use of the MIMO relay. The proposed scheme relaxes the CSI requirement in [1][4] and assumes global CSI at the MIMO relay only, while other nodes only need to have their local CSI. Moreover, the MIMO relay in our proposed scheme applies linear transformation on its received signals instead of decode-and-forward. This represents a major reduction in processing complexity as compared to other approaches including [6]. Our results are interesting and rather surprising in that even with just linear processing at the MIMO relay, our scheme can achieve $\frac{KM}{2}$ DoF per channel use with less than $KM$ antennas at the MIMO relay. We prove that the minimum number of antennas needed is exactly $(K - 1)M$ in our scheme. It is important to note that even for the well known two-hop transmission scheme: Multiple-access transmission in the first hop followed by broadcast transmission in the second hop, the number of antennas needed at the MIMO relay is at least $KM$.

The rest of this paper is organized as follows. Section II describes the system model. The DoF achievable scheme based on interference alignment is proposed in Section III, we also discuss the requirement on the MIMO relay in terms of the number of antennas in this section. Finally, Section IV
concludes the paper.

Notations: $A^T$ and $A^{-1}$ stand for transpose and inverse of matrix $A$, respectively. $\text{vec}(A)$ denotes the vectorization of $A$ formed by stacking the columns of $A$. $A \otimes B$ denotes Kronecker product between $A$ and $B$, $I_m$ stands for identity matrix with size $m \times m$.

II. SYSTEM MODEL

Consider a $K$-user MIMO interference channel with a MIMO relay as shown in Fig. 1. Each transmitter and receiver has $M$ antennas while there are $R$ antennas at the MIMO relay. For transmitter $i$, $i \in \{1, 2, \ldots, K\}$, there is one message intended for its corresponding receiver, thus they form a transmitter-receiver pair. Assume that global CSI is available at neither transmitters nor receivers but is available at the MIMO relay. The relay is assumed to operate in half-duplex mode, i.e., it cannot receive and transmit at the same time.

We denote $H_{j,i}$ as the $M \times M$ channel matrix between transmitter $i$ and receiver $j$, $F_i$ denotes channel matrix between transmitter $i$ and MIMO relay with size $R \times M$, $G_j$ denotes channel matrix between MIMO relay and receiver $j$ with size $M \times R$. Furthermore, assume all channel coefficients are independently and identically drawn from a continuous distribution, thus, all channel matrices are non-degenerate (full rank) with probability one.

III. DOF ACHIEVABLE SCHEME USING MIMO RELAY

Note that it has been shown in [4] for a $K$-user MIMO interference channel with a MIMO relay where each transmitter and receiver is equipped with $M$ antennas, the upper bound on the DoF is $\frac{KM}{2}$. This bound is quite general meaning that it is valid regardless of global CSI knowledge being available and the number of antennas at the MIMO relay. However, how to achieve this bound under different assumptions remains an issue. In this section, we propose a two-hop relay aided interference alignment scheme which is able to achieve this bound. We also consider the feasibility conditions in terms of the number of antennas required at the MIMO relay.

A. Two-hop Interference Alignment Scheme

In the first hop, each transmitter sends information to its corresponding receiver and MIMO relay. The signals received at receiver $j \in \{1, 2, \ldots, K\}$ and MIMO relay can be respectively expressed as

$$y_j(1) = \sum_{i=1}^{K} H_{j,i} x_i + z_j(1), \quad (2)$$

$$y_R = \sum_{i=1}^{K} F_i x_i + z_R, \quad (3)$$

where $x_i$ is the $M \times 1$ data vector transmitted by transmitter $i$, $y_j(1)$ denotes the $M \times 1$ received signal at receiver $j$ in the first hop, and $y_R$ denotes the $R \times 1$ signal vector received at MIMO relay. $z_j(1)$ and $z_R$ denote noise vectors at receiver $j$ and MIMO relay in the first hop, respectively. All elements in noise vectors are independently and identically distributed (i.i.d.) zero-mean complex Gaussian with unit variance.

In the second hop, MIMO relay transmits to all receivers while the transmitted signal is generated by conducting a linear transformation on the received signal $y_R$ in the first hop. That is, the transmitted signal $x_R$ is given by

$$x_R = \Gamma y_R, \quad (4)$$

where $\Gamma$ is an $R \times R$ matrix. We refer to $\Gamma$ as beamforming matrix. The signal received by the $j$-th receiver in the second hop can be expressed as

$$y_j(2) = G_j x_R + z_j(2) = \sum_{i=1}^{K} G_j \Gamma F_i x_i + \tilde{z}_j(2), \quad (5)$$

where $\tilde{z}_j(2) = G_j \Gamma z_R + z_j(2)$ is the equivalent noise in the second hop. Thus, at the $j$-th receiver, the received signals in two hops form an $2M \times 1$ column vector:

$$y_j = \begin{bmatrix} y_j(1) \\ y_j(2) \end{bmatrix} = \left[ \begin{array}{c} H_{j,j} \end{array} \right] x_j + \sum_{i=1, i \neq j}^{K} \left[ \begin{array}{c} H_{j,i} \\ G_j \Gamma F_i \end{array} \right] x_i + \begin{bmatrix} z_j(1) \\ \tilde{z}_j(2) \end{bmatrix}. \quad (6)$$

Note that the first term is the desired signal and the second term is the interfering signals.

The available signal space at each receiver has $2M$ dimensions since two-hop transmission is applied, while the desired signal that a receiver needs to recover has $M$ dimensions. If interference signals from unintended transmitters can be concentrated in a $M$-dimension space that does not intersect with the space spanned by desired signal, simple zero-forcing suffices to null out interference at each receiver. We refer this “interference grouping” constraint as interference alignment constraint. On the other hand, at each receiver, its desired signal should fully span a $M$-dimension space after zero-forcing, and this can be referred as decodability constraint. A valid beamforming matrix $\Gamma$ should satisfy both constraints simultaneously at all receivers. Mathematically,
the following two rank constraints should be satisfied for all $j \in \{1, 2, \ldots, K\}$:

$$
\text{rank}\left[ \begin{array}{cccc}
H_{j,1} & \cdots & H_{j,j-1} & H_{j,j+1} & \cdots & H_{j,K} \\
G_j \Gamma F_1 & \cdots & G_j \Gamma F_{j-1} & G_j \Gamma F_{j+1} & \cdots & G_j \Gamma F_K
\end{array} \right] = M, \tag{7}
$$

$$
\text{rank}\left[ \begin{array}{cccc}
H_{j,1} & H_{j,2} & \cdots & H_{j,K} \\
G_j \Gamma F_1 & G_j \Gamma F_2 & \cdots & G_j \Gamma F_K
\end{array} \right] = 2M. \tag{8}
$$

Equations (7) and (8) represent interference alignment constraint and decodability constraint respectively. It is obvious that whether these two constraints are satisfied highly depends on the construction of beamforming matrix $\Gamma$. We find in the next subsection the minimum dimension requirement for the beamforming matrix $\Gamma$, or equivalently, the number of antennas needed at the MIMO relay in order to achieve the two constraints. Similar to [7], we view the construction problem of $\Gamma$ as solving a system of linear equations, once a proper solution is obtained, we can determine the number of antennas required at MIMO relay.

B. Requirements on MIMO Relay

The following theorem states the requirement on the number of antennas at MIMO relay.

**Theorem 1:** For a $K$-user MIMO interference channel with a MIMO relay where each transmitter and receiver has $M$ antennas, the two-hop relay aided transmission scheme with linear processing at the relay can achieve $\frac{K}{2}M$ DoF if and only if there are $R \geq (K-1)M$ antennas at MIMO relay.

1) *Proof of Necessary Condition:* We first prove necessary part by contradiction. Suppose when $R < (K-1)M$, there exists a valid beamforming matrix $\Gamma$ such that both (7) and (8) can be satisfied. Since the decodability constraint requires the matrix in (8) be full rank $2M$, matrix $G_j \Gamma$ which has size $M \times R$ should satisfy:

$$
\text{rank}[G_j \Gamma] = M, \ \forall j \in \{1, 2, \ldots, K\}. \tag{9}
$$

Otherwise, full rank constraint (8) cannot be achieved.

Then we consider interference alignment constraint (7). The matrix in (7) can be represented as

$$
\begin{bmatrix}
H_{j,1} & \cdots & H_{j,j-1} & H_{j,j+1} & \cdots & H_{j,K} \\
G_j \Gamma F_1 & \cdots & G_j \Gamma F_{j-1} & G_j \Gamma F_{j+1} & \cdots & G_j \Gamma F_K
\end{bmatrix}
= \begin{bmatrix}
I_M & 0 \\
0 & G_j \Gamma
\end{bmatrix}
\begin{bmatrix}
F_1 & \cdots & F_{j-1} & F_{j+1} & \cdots & F_K
\end{bmatrix}.
$$

According to the randomness and independence of channel coefficients, $D_j$ is full rank almost surely for all $j \in \{1, 2, \ldots, K\}$, i.e., the rank of $D_j$ only depends on its size with probability one. Applying Sylvester's rank inequality [9] on the equation above, we can obtain the following inequality:

$$
\text{rank}\left[ \begin{array}{cc}
I_M & 0 \\
0 & G_j \Gamma
\end{array} \right] D_j \geq \text{rank}\left[ \begin{array}{c}
I_M \\
0
\end{array} \right] + \text{rank}[D_j] - (M + R) \tag{11}
$$

where in (12) we have used the result obtained in (9). Since $R < (K-1)M$, we consider two cases:

1. $(K-1)M \leq R < (K-1)M$.

   In this case, we have $(M + R) \geq (K-1)M$. Thus, $\text{rank}[D_j] = (K-1)M$ almost surely. We can obtain the following inequality by substituting the rank of $D_j$ into (12):

$$
\text{rank}\left[ \begin{array}{cc}
I_M & 0 \\
0 & G_j \Gamma
\end{array} \right] D_j \geq KM - R. \tag{13}
$$

On the other hand, the constraint (7) requires

$$
\text{rank}\left[ \begin{array}{cc}
I_M & 0 \\
0 & G_j \Gamma
\end{array} \right] D_j = M.
$$

Thus, we have the following inequality:

$$
M \geq KM - R \Rightarrow R \geq (K-1)M, \tag{14}
$$

which contradicts the assumption $R < (K-1)M$. Hence, when $(K-2)M \leq R < (K-1)M$, a valid beamforming matrix $\Gamma$ does not exist.

2. $R < (K-2)M$.

   In this case, we have $(M + R) < (K-1)M$. Thus, $\text{rank}[D_j] = (M + R)$ almost surely. The following inequality can be obtained:

$$
\text{rank}\left[ \begin{array}{cc}
I_M & 0 \\
0 & G_j \Gamma
\end{array} \right] D_j \geq 2M,
$$

which is also an invalid result because interference alignment constraint (7) cannot be satisfied.

Thus, from the contradiction results, we have proved that if a beamforming matrix $\Gamma$ satisfies both (7) and (8), the number of antennas at MIMO relay has to be greater than or equal to $(K-1)M$.

2) *Proof of Sufficient Condition:* We prove the sufficient condition by constructing a valid beamforming matrix $\Gamma$. Note that for all values of $R \geq (K-1)M$, we only need to prove the extreme case when $R = (K-1)M$. Therefore, in the rest of sufficiency proof, $R = (K-1)M$ is assumed.

Consider interference alignment constraint (7) first. According to the non-degeneration of channel matrices, the first row block $[H_{j,1} \cdots H_{j,j-1} H_{j,j+1} \cdots H_{j,K}]$ has full rank $M$ almost surely for all $j \in \{1, 2, \ldots, K\}$. In order to satisfy the rank constraint (7), the second row block has to fall into the space spanned by the first block. Therefore, the interference alignment constraint (7) is equivalent to: For each $j \in \{1, 2, \ldots, K\}$, there exists an $M \times M$ matrix $A_j$, such that

$$
G_j \Gamma \begin{bmatrix}
F_1 & \cdots & F_{j-1} & F_{j+1} & \cdots & F_K
\end{bmatrix}
= \Lambda_j \begin{bmatrix}
H_{j,1} & \cdots & H_{j,j-1} & H_{j,j+1} & \cdots & H_{j,K}
\end{bmatrix}.
$$

Applying vec($\cdot$) operation on both sides of (15), it can equivalently be represented as: For all $j \in \{1, 2, \ldots, K\}$,

$$
(F_j^T \otimes G_j) \text{vec}(\Gamma) = (H_j^T \otimes I_M) \text{vec}(A_j),
$$

where $I_M$ is an $M \times M$ identity matrix.
where we have used the property of Kronecker product that \( \text{vec}(AXB) = (B^T \otimes A)\text{vec}(X) \). Thus, for the \( j \)-th transmitter-receiver pair, if we treat \( \text{vec}(\Gamma) \) as unknown variables and \( \text{vec}(\Lambda_j) \) as auxiliary variables, they satisfy the following equation:

\[
\begin{bmatrix}
\vec{F}_j^T \otimes \mathbf{G}_j - \vec{H}_j^T \otimes \mathbf{I}_M
\end{bmatrix}
\begin{bmatrix}
\text{vec}(\Gamma)
\text{vec}(\Lambda_j)
\end{bmatrix}
= \mathbf{0}.
\]

We can aggregate the equivalent interference alignment conditions for all transmitter-receiver pairs into a unified system equation (16) which is shown at the bottom of the page. \( \mathbf{W} \) is the system matrix whose elements only depend on channel coefficients. By solving (16), we can obtain beamforming matrix \( \Gamma \) for interference alignment.

We want to emphasize that solving (16) only satisfies interference alignment constraint (7). Not all solutions satisfy decodability constraint (8). For example, \((K^2 - K + 1)M^2 \times 1\) all-zero vector is a valid solution to (16), however, with \( \Gamma = \mathbf{0} \), none of the receivers can decode their desired signals because those signals will be nulled \((\mathbf{G}_j^T \mathbf{F}_j = \mathbf{0})\) as well as interference. Therefore, we need to construct a solution to (16) properly, so that decodability constraint can also be satisfied.

We first show the following lemma which will be used in the construction of beamforming matrix \( \Gamma \).

**Lemma 1:** For system matrix \( \mathbf{W} \), if we remove one column block that contains \( \vec{F}_j^T \otimes \mathbf{I}_M, j \in \{1, 2, \ldots, K\} \), the resulting square matrix \( \mathbf{W}_j \) which is given below has full rank almost surely:

\[
\begin{bmatrix}
\vec{F}_1^T \otimes \mathbf{G}_1 - \vec{H}_1^T \otimes \mathbf{I}_M \\
\vec{F}_2^T \otimes \mathbf{G}_1 - \vec{H}_2^T \otimes \mathbf{I}_M \\
\vdots \\
\vec{F}_{K-1}^T \otimes \mathbf{G}_1 - \vec{H}_{K-1}^T \otimes \mathbf{I}_M \\
\vec{F}_1^T \otimes \mathbf{G}_2 \\
\vec{F}_2^T \otimes \mathbf{G}_2 - \vec{H}_2^T \otimes \mathbf{I}_M \\
\vdots \\
\vec{F}_{K-1}^T \otimes \mathbf{G}_2 - \vec{H}_{K-1}^T \otimes \mathbf{I}_M \\
\vdots \\
\vec{F}_1^T \otimes \mathbf{G}_K \\
\vec{F}_2^T \otimes \mathbf{G}_K - \vec{H}_2^T \otimes \mathbf{I}_M \\
\vdots \\
\vdots \\
\vec{F}_{K-1}^T \otimes \mathbf{G}_K - \vec{H}_{K-1}^T \otimes \mathbf{I}_M \\
\end{bmatrix}
\]

**Proof:** The proof of Lemma 1 is omitted here.

**Remark:** Since \( \mathbf{W}_j \) is shown to be full rank with probability one, all its columns span a full dimensional space with size \( K(K-1)M^2 \), and adding extra columns into it does not affect its rank. Therefore, system matrix \( \mathbf{W} \) is also full rank almost surely, i.e., \( \text{rank}[\mathbf{W}] = K(K-1)M^2 \) almost surely.

Then we construct a proper solution follow the steps below:

1. Note that when \( R = (K-1)M \), there are exactly \( M^2 \) degrees of freedom in solving system equation (16) because the number of unknowns is \( R^2 + KM^2 = (K^2 - K + 1)M^2 \) and \( \text{rank}[\mathbf{W}] = K(K-1)M^2 \). Set auxiliary matrix \( \mathbf{A}_j = \mathbf{I}_M \), then equation (16) can be transformed into the following form:

\[
\begin{bmatrix}
\text{vec}(\Gamma) \\
\text{vec}(\mathbf{A}_2) \\
\vdots \\
\text{vec}(\mathbf{A}_K)
\end{bmatrix}
= \begin{bmatrix}
\text{vec}(\mathbf{H}_1) \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

Since we have proved that \( \mathbf{W}_1 \) is full rank almost surely, column vector \( \mathbf{b}_1 \) must fall into the space spanned by the columns of \( \mathbf{W}_1 \), which guarantees the existence of solution to (16) under this setting. Hence, by solving (17), we can obtain a particular solution for (16):

\[
s = \alpha_1 s^1 + \alpha_2 s^2 + \cdots + \alpha_K s^K,
\]

where the superscript "1" means this particular solution is obtained by setting \( \mathbf{A}_1 = \mathbf{I}_M \). Similarly, we can obtain other \((K-1)\) particular solutions \( s^2, \ldots, s^K \) by setting \( \mathbf{A}_j = \mathbf{I}_M, j = 2, \ldots, K \).

2. Form a linear combination of \( s^j \)'s:

\[
s = \alpha_1 s^1 + \alpha_2 s^2 + \cdots + \alpha_K s^K,
\]

where the non-zero coefficients \( \alpha_j \), \( j \in \{1, 2, \ldots, K\} \) are randomly chosen and are independent of each other. Since each \( s^j \) is a solution to the homogeneous system equation (16), their linear combination result \( s \) is still a valid solution to (16). The linear combination in (18) is equivalent to

\[
\begin{align*}
\Gamma &= \alpha_1 \Gamma^1 + \cdots + \alpha_K \Gamma^K, \\
\mathbf{A}_j &= \alpha_1 \mathbf{A}_j^1 + \cdots + \alpha_j \mathbf{I}_M + \cdots + \alpha_K \mathbf{A}_j^K.
\end{align*}
\]

Note that from the expression of \( \mathbf{A}_j \) in (20), there is a full rank term \( \alpha_j \mathbf{I}_M \) for all \( j \in \{1, 2, \ldots, K\} \). The extra degrees of freedom to independently choose other \((K-1)\) coefficients \( \alpha_i \)'s \( i \neq j \) guarantees that the sum of matrices still be full rank almost surely, i.e., \( \mathbf{A}_j \) is full rank almost surely for all \( j \in \{1, 2, \ldots, K\} \).

Now we show \( \Gamma \) in (19) satisfies decodability constraint (8). Provided that interference alignment constraint (15) is
satisfied, the decodability constraint (8) can be simplified as
\[
\text{rank } [G_j F_j - \Lambda_j H_{j,j}] = M, \quad \forall j \in \{1, 2, \ldots, K\}. \quad (21)
\]
We need to prove $G_j F_j - \Lambda_j H_{j,j}$ is full rank almost surely for all $j \in \{1, 2, \ldots, K\}$. By solving $G_j F_j = \Lambda_j H_{j,j}$ (15), and substituting it into $G_j F_j - \Lambda_j H_{j,j}$, we obtain
\[
\text{rank } [G_j F_j - \Lambda_j H_{j,j}] = \text{rank } \left[ \Lambda_j \left( \tilde{H}_j F_j^{-1} F_j - H_{j,j} \right) \right] = \text{rank } \left[ \tilde{H}_j F_j^{-1} F_j - H_{j,j} \right], \quad (22)
\]
where (23) follows because square matrix $\Lambda_j$ is full rank with probability one. Now we only need to show $(\tilde{H}_j F_j^{-1} F_j - H_{j,j})$’s are full rank almost surely for all $j \in \{1, 2, \ldots, K\}$. Note that the second term $H_{i,j}$ is independent of $\tilde{H}_j F_j^{-1} F_j$ as well as the previous construction process of beamforming matrix $\Gamma$. Thus, conditioned on $G_j, F_j$, and $H_{i,j}$ for $i, j \in \{1, 2, \ldots, K\}, i \neq j$, the independence and randomness of $H_{i,j}$ assures that $(\tilde{H}_j F_j^{-1} F_j - H_{j,j})$ has full rank $M$ with probability one. Hence, decodability constraint (8) can be satisfied almost surely.

Combining necessity and sufficiency proofs, we have proved that our two-hop relay aided interference alignment scheme can achieve $\frac{K M}{2}$ DoF if and only if there are at least $(K-1)M$ antennas at the MIMO relay.

C. Discussion

Note that in [1], the DoF achievable scheme for a $K$-user interference channel without MIMO relay requires the channel coefficients be time-varying/feature-selective, and each transmitter and receiver is assumed to have global CSI. The benefits of adding a MIMO relay in our proposed scheme is that the channel only need to be constant during the transmission of two hops. Furthermore, the signaling overhead in acquiring global CSI can be greatly reduced on both transmitter and receiver sides, which is important for practical applications. In the proposed scheme, global CSI is only assumed at the MIMO relay. There is no CSI requirement on transmitters, and the signaling overhead for receiver side is that the $j$-th receiver, $j \in \{1, 2, \ldots, K\}$, is required to obtain its corresponding auxiliary matrix $\Lambda_j$ from MIMO relay in order to decode its signal by zero-forcing the interference.

We can also draw a comparison between our proposed scheme and the well-known transmission scheme: A multiple-access transmission in which $K$ transmitters and the MIMO relay form a multiple-access channel (MAC) followed by a broadcast transmission in which the MIMO relay and $K$ receivers form a broadcast channel (BC). We refer to this scheme as MAC-BC scheme. The MAC-BC scheme can also achieve $\frac{K M}{2}$ DoF per channel use while it requires at least $K M$ antennas at the MIMO relay. Our proposed scheme is able to achieve the same DoF with $(K-1)M$ antennas. Thus, the proposed scheme saves $M$ antennas at the MIMO relay. For MIMO relay in the MAC-BC scheme, it has to decode all transmitters’ messages in multiple-access phase then re-encode and transmit them to all receivers in broadcast phase. However, in our proposed scheme, the MIMO relay only has to process its received signal with a linear transformation, which is similar to amplify-and-forward method in classic relay channels. Thus, the processing complexity of our scheme is lower than that of MAC-BC scheme. For performance comparison, although MAC-BC scheme may achieve better performance in terms of raw numbers of the capacity in finite SNR (due to dirty paper coding etc.), both schemes have the same performance in the sense that the sum capacity is characterized as $\frac{K M}{2} \log(\text{SNR}) + o(\log(\text{SNR}))$, i.e., both schemes achieve the full DoF. In short, the advantage of our proposed scheme is that it has low processing complexity and it requires less number of antennas at the MIMO relay if only DoF is taken as comparison criteria.

IV. CONCLUSION

We have proposed a MIMO relay aided interference alignment transmission scheme for $K$-user MIMO interference channel with a MIMO relay, where each transmitter and receiver is equipped with $M$ antennas. It can be shown that if and only if there are $R \geq (K-1)M$ antennas at the MIMO relay, our scheme is able to achieve the full DoF $\frac{K M}{2}$. The proposed scheme only assumes global CSI at MIMO relay, which greatly reduces the overhead in acquiring global CSI for other nodes. We have also shown that our proposed scheme has low processing complexity and requires less number of antennas at the MIMO relay as compared with MAC-BC scheme.

REFERENCES


