A two-parameter model for crack growth simulation by combined FEM–DBEM approach

R. Citarella *, G. Cricri
University of Salerno, Department of Mechanical Engineering, Fisciano, SA, Italy

ABSTRACT

This paper describes the application of a two-parameters crack growth model, based on the usage of two threshold material parameters \( (\Delta K_{th} \text{ and } K_{max,th}) \) and on the allowance for residual stresses, introduced at the crack tip by a fatigue load spectrum or by material plastic deformations. The coupled usage of finite element method (FEM) and dual boundary element method (DBEM) is proposed in order to take advantage of the main capabilities of the two methods.

The procedure is validated by comparison with available experimental results, in order to assess its capability to predict the retardation phenomena, introduced by a variable load spectrum or by a plastic deformation introduced with a tool on the panel (indentation).

In particular two different tests are made: the first test involve a CT specimen undergoing a load spectrum and the second one involve a dented panel undergoing a constant amplitude fatigue load. In both cases a satisfactory numerical–experimental correlation will be proved.

The main advantages of the aforementioned procedure are: the simplicity of the crack growth law calibra-

1. Introduction

Damage tolerance, used in the design of many types of structures, such as bridges, military ships, commercial aircraft, space vehicle and merchant ships, requires accurate prediction of fatigue crack growth under service conditions, and this is typically accomplished with the aid of a numerical methodology. Many aspects of fracture mechanics are more complicated in practice than in two-dimensional laboratory tests, textbook examples, or overly simplified computer programs. Load spectrum, plastic deformations, threshold effects, microstructural effects, small crack effects, multiple site damage (MSD) conditions, material parameters scatter, all complicate the process of predicting fatigue crack growth in real world applications. This paper focuses on some of these complications: crack retardation phenomena, induced by spike load spectra [1] or by plastic deformations (indentation).

An original implementation of the two-parameters crack growth approach [2–5] is presented (see also [6]), together with a validation procedure. In particular the coupled usage of finite element method (FEM) and dual boundary element method (DBEM) is proposed [7,8] in order to take advantage of the main capabilities of the two methods: FEM is more efficient for elastic–plastic analysis (needed to assess the residual stress profile) whilst DBEM allows an efficient automatic crack propagation, especially for complex geometry or for mixed mode conditions.

The DBEM [9], as implemented in the commercial code BEASY [10], is adopted for the crack propagation simulation whereas the FEM code ANSYS is used to calculate the residual stresses by elastic–plastic analysis.

The first validation of the procedure is numerical: it is shown how the results obtained with a procedure based on FEM and analytical formulas are the same as those obtained by a fully numerical FEM–DBEM coupled approach.

The second level of validation comes from a series of laboratory tests, realised in order to evaluate the capabilities of the procedure in predicting the crack retardation phenomena induced by an overload or by material plastic deformations.

2. Theoretical model

The load spectrum effects, as well as other transient effects due to plastic deformations in an area surrounding the crack tip, arise due to perturbation of the stress distribution ahead of the crack tip with respect to the steady-state stress field: for example, in presence of an overload, the stress field at crack tip is altered by residual stresses generated by the enhanced plastic deformations.
The basic effect of these residual stresses is to change the effective values of the total stress intensity factor (SIF) at the crack tip, with both $K_{\text{min}}$ and $K_{\text{max}}$ generally affected in the same way, so as to leave unchanged the parameter $\Delta K$. Consequently, the primary effects of residual stresses on crack growth rates are related to the $K_{\text{max}}$ variations and not to the $\Delta K$ variations.

This is accounted for by the aforementioned two-parameters approach. According to this theory [2–5], fatigue crack growth can be viewed, fundamentally, as a two-parametric problem, where two driving forces, $K_{\text{max}}$ and $\Delta K$, drive the growth of a fatigue crack. Since it is assumed that $K_{\text{max}}$ also enters as the major driving force for fatigue crack growth (in addition to the classical parameter $\Delta K$), the residual stresses, introduced by plastic deformations, can affect crack growth rates even if they do not affect the parameter $\Delta K$.

In addition, the theory assumes that there are two fatigue thresholds, $K_{\text{max,th}}$ and $\Delta K_{\text{th}}$, corresponding to the two driving forces and depending on the alloy microstructure, mode of slip and environment. These are asymptotic values in the $\Delta K$–$K_{\text{max}}$ graphs of the fatigue curves: both the driving forces must be simultaneously larger than the relative thresholds for fatigue crack growth to occur.

Since residual stress effects manifest primarily through a variation in $K_{\text{max}}$ levels, a crack growth rate variation is produced by an overload (intermingled in a baseline constant amplitude block cycle) or by a component plastic deformation (e.g., by indentation). An arrest in crack growth can occur if these stresses are sufficiently high, i.e., $K_{\text{max}}$ falls below $K_{\text{max,th}}$. As a matter of fact, the crack growth law is assumed to be of the form [4]:

$$\frac{da}{dN} = A(\Delta K - \Delta K_{\text{th}})^n(K_{\text{max}} - K_{\text{max,th}})^m$$

(1)

and is calibrated by best fitting the material parameters $(A, n, m, K_{\text{max,th}}, \Delta K_{\text{th}})$, based on constant amplitude fatigue experimental tests.

It is important to note that the threshold parameters are strongly related with the microstructure of the considered alloy, since they are related to crack growth rates, acting at the microstructure scale.

3. Test case No. 1: crack growth simulation under variable spectrum load by combined FEM/analitical approach

3.1. Introduction

The two-parameter model is initially tested using crack growth experimental data from an aluminum CT specimen, 2024 T351 and 2024 HP-T3 clad sheet, undergoing a load spectrum in which spike loads are intermingled in a baseline constant amplitude cycle block. The baseline cycle is ranging from $P_{\text{min}} = R * 800$ and $P_{\text{max}} = 800N$ ($R$ is the baseline stress ratio), and the spike load is characterised by an overload ratio $R_{\text{OL}} = P_{\text{max,OL}}/P_{\text{max}} = 2$ ($P_{\text{max,OL}}$ is the maximum overload).

The residual stress field due to a spike load is calculated by comparing two FEM elastic–plastic simulations. The impact of such residual stresses on the driving forces is calculated, then the crack growth model is applied, and its results compared with experimental crack growth rates after the spike load application.

It is remarkable that the two-parameter crack growth law, whose validity is expected to be extended to any overload ratio, is calibrated using only experimental data from constant amplitude test.

3.2. Determination of the threshold parameters $(\Delta K_{\text{th}}, K_{\text{max,th}})$

The experimental threshold data are provided by a sufficient number of tests on CT specimens (whose width and thickness are respectively equal to $w = 25.4\ mm$ and $t = 3.2\ mm$) undergoing constant amplitude fatigue traction load. The experimental crack growth rates near the threshold regime are determined and their interpolation provided for each stress ratio $R$ (Fig. 1). In Table 1 the corresponding threshold values, $\Delta K_{\text{th}}$ and $K_{\text{max,th}}$ are reported in numerical form.

![Fig. 1. Threshold crack growth rates vs. $\Delta K$: experimental data and numerical interpolation, for 2024 T351 (left) and 2024 HP-T3 (right).](image-url)
The following equation [4] is used in order to fit the experimental points, with coordinates \((D_{K\text{th}}, K_{\text{max,th}})\), obtained for different values of \(R\)-ratio:

\[
D_{K\text{th}} = D_{K\text{th}}/C_{3\text{th}} + B (K_{\text{max}}/C_{0\text{th}})^{K_{\text{max,th}}/C_{3\text{th}}}\]

with \(K_{\text{max}} = D_{K\text{th}}/(1/R)\) (2)

The regression coefficients \(B, l, D_{K\text{th}}, K_{\text{max,th}}\) are calculated, by minimization of the fitting error.

### 3.3. Determination of the material parameters \((A, n, m)\)

In order to obtain the material parameters \((A, n, m)\), Eq. (1) is fitted to data taken from constant amplitude experimental tests on CT specimens, at different \(R\)-ratios. The resulting values for \(A, n, m\), valid for every \(R\)-ratio in the considered range, are reported in Table 2.

In Fig. 2, plots of Eq. (1), with the above values of the material parameters \(A, n, m\), for variable \(R\) ratios, are superimposed to experimental results obtained from a fatigue load that is built with the addition of a spike load in a constant amplitude baseline sequence. \(K_{\text{max}}\) here is simply calculated as \(\Delta K/(1 - R)\), where \(R\) is the stress ratio of the baseline cycle. As expected, the correlation is accurate but in the transient part following the overload application.

We will show in the following that also this transient part is correctly simulated by Eq. (1) when allowance for residual stresses is introduced in the SIF calculation and, consequently, \(K_{\text{max}}\) and \(\Delta K\) are not anymore related by the fixed baseline stress ratio \(R\) (a varying stress ratio, based on the residual stresses influence, is implicitly taken).

### 3.4. Residual stress assessment

#### 3.4.1. Introduction

The overload retardation effect is evaluated by considering the crack growth law (Eq. (1)), in which the SIF is equal to the sum of the nominal SIF, corresponding to the remote load, plus the SIF corresponding to the contribution of the internal residual stresses, induced by the plastic flow at the crack tip.

Since the calibration of Eq. (1) takes implicitly into account the residual stress contribution from the constant amplitude load, only the residual stresses generated by the overload effect need to be considered. Such residual stresses are calculated, for a given crack length, as a difference between the residual stresses arising from the load sequence that includes an overload, and those induced by the baseline load sequence (without overload), as detailed in the following steps:

1. Elastic–plastic FEM analysis of the CT specimen undergoing a baseline load cycle.
2. Elastic–plastic FEM analysis of the specimen undergoing a load cycle with a single spike load.
3. Calculation of the residual stress profile generated by the overload.
4. Calculation of the SIF due to the residual stresses (mode I) by means of Green’s functions.
5. Calculation of the total SIF due to both the remote and residual stresses, using the superposition principle.
6. Use of the crack growth law (Eq. (1)) to predict growth rates after a spike load.

---

### Table 1

<table>
<thead>
<tr>
<th>(R)</th>
<th>(\Delta K_{\text{th}}) (N/mm(^{1/2}))</th>
<th>(K_{\text{max,th}}) (N/mm(^{1/2}))</th>
<th>(\Delta K_{\text{th}}) (N/mm(^{1/2}))</th>
<th>(K_{\text{max}}) (N/mm(^{1/2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>94.9</td>
<td>99.9</td>
<td>107.2</td>
<td>112.8</td>
</tr>
<tr>
<td>0.15</td>
<td>82.2</td>
<td>96.7</td>
<td>90.3</td>
<td>106.2</td>
</tr>
<tr>
<td>0.3</td>
<td>69.3</td>
<td>98.9</td>
<td>78.9</td>
<td>112.6</td>
</tr>
<tr>
<td>0.5</td>
<td>56.7</td>
<td>113.4</td>
<td>65.0</td>
<td>130.0</td>
</tr>
<tr>
<td>0.8</td>
<td>50.6</td>
<td>253.1</td>
<td>58.2</td>
<td>291.0</td>
</tr>
</tbody>
</table>

\(\Delta K_{\text{th}} = \Delta K_{\text{th}}^* + B (K_{\text{max}}/K_{\text{max,th}})^{\lambda}\) with \(K_{\text{max}} = \Delta K_{\text{th}}/(1 - R)\)

### Table 2

| \(A, n, m\) values for the two alloys (\(\text{da}/\text{dN in mm/cycle and } \Delta K\text{ in N/mm}\(^{1/2}\)) |
|---|---|---|
| 2024-T351 | 2024 HP-T3 |
| \(A\) | 1.007\(e^{-10}\) | 6.745\(e^{-10}\) |
| \(n\) | 1.99 | 1.65 |
| \(m\) | 0.600 | 0.590 |

---

**Fig. 2.** Model calibration for 2024 T351 (left) and 2024 HP-T3 (right).
3.4.2. Materials constitutive law

A relationship is established between true stress and logarithmic strain based on the static tension tests data for the 2024HP-T3 and 2024HP-T351; such data are plotted in Fig. 3. The constitutive law in plastic flow conditions is assumed to be a power law (Ramberg–Osgood model):

\[ \frac{\varepsilon}{\varepsilon_0} = \left( \frac{\sigma}{\sigma_0} \right)^n \] (3)

and the following parameters can be calculated by fitting of experimental data:

- \( \sigma_0 = \sigma_y \) = 320 MPa
- \( \varepsilon_y = \sigma_y / E = 0.00457 \)
- \( \sigma_{true} = \sigma_u(1 + \varepsilon_f) = 537 \) MPa
- \( N = \ln(\varepsilon_{true}/\varepsilon_0) / \ln(\sigma_{true}/\sigma_0) = 7.2 \)

3.4.3. Determination of the overload residual stresses

The overload residual stress calculation requires two elastic–plastic FEM simulations (Fig. 4): first, the constant amplitude baseline cycle is imposed on the specimen model (points 1′–2′) so that, at the minimum load (point 2′), the material surrounding the crack tip is affected by a residual stress field as a consequence of the constant amplitude plastic zone; then, the spike load (overload ratio \( R_o = 2 \)) is applied and, after overload removal (point 2′′), the corresponding residual stress field, enhanced by the overload effect, is recorded.

Assuming that the principle of superposition holds true (such approximation is acceptable if the plastic zone dimension is sufficiently small), it is possible to assume that the stress field after the spike load removal (point 2′′ and 1′′) is the sum of the stress field produced by the current remote load plus the residual stress field inherited by the application of the previous spike load:

\[ \sigma_y;_{tot} = \sigma_y;_{appl} + \sigma_y;_{res} \]

With reference to Fig. 4, it is possible to write:

\[ \sigma_y(1) = \sigma_y(1) + \sigma_y(1);_{res} \]

and consequently \( \sigma_y(1);_{res} = \sigma_y(1) - \sigma_y(1) \) (41)
\[ \sigma_y^{(2)} = \sigma_y^{(2)} + \sigma_{y,\text{res}}^{(2)} \quad \text{and consequently} \quad \sigma_{y,\text{res}}^{(2)} = \sigma_y^{(2)} - \sigma_y^{(2)} \]

3.4.4. FEM elastic–plastic analysis

Only a quarter of the CT specimen is modelled because of the presence of two symmetry planes (Fig. 5): one coincident with the crack plane and the other perpendicular to the crack front at half thickness. Fig. 6 shows the three-dimensional finite element mesh adopted for the 3D analyses: 22,000 8-node linear elements are used and a model with about 75,000 degrees of freedom is obtained. With the adopted mesh, the linear elements give nearly the same accuracy of the 20-node quadratic elements in evaluating the residual stress profiles, so that the used mesh can be considered sufficiently accurate. The mesh is refined near the crack tip with 10 elements along the plastic radius, in order to properly capture the locally strong stress gradients.

The elastic–plastic analyses are performed assuming an isotropic-hardening behaviour for the material, with Von Mises yield criterion and associative flow rule. In order to use the 3D FEM results for a 2D crack growth simulation, the residual stress profile is averaged along the thickness: after evaluating the residual stress profiles on different planes, equally spaced along the thickness \( B \), the average values are obtained by imposing an equilibrium condition Eq. (5),

---

**Fig. 6.** Mesh used for the 3D FEM analyses with close up of the area surrounding the crack tip.

**Fig. 7.** \( \sigma_y \) in the area surrounding the crack front, before (left) and after (right) the application of the spike load.

**Fig. 8.** Plastic zone extension, before (left) and after (right) the application of the spike load.
\[ \sigma_{\text{res}}(x) = \frac{1}{B} \int_{0}^{B} \sigma_{\text{res}}(x, z) \, dz \]  

(5)

Results from the elastic–plastic FEM analysis, for the 2024-T351, with \( R = 0.05 \) are shown in Figs. 7 and 8, before (point 1* in Fig. 4) and after (point 1 in Fig. 4) the overload application, for a load applied in y direction: in Fig. 7 the stress field \( \sigma_y \) corresponding to the maximum load (applied in y direction); in Fig. 8 the highlight of plastic zone extension.

In Fig. 9, a summary of all the calculated residual stress profiles, for the different materials and R-ratios, is shown. Due to the similarity of the stress–strain curves of the two materials (Fig. 3), for a given R-ratio the residual stress profile does not change between the two materials, so that the differences in the overload effect are due essentially to the different threshold values.

3.5. Residual stress effect on SIFs

3.5.1. Analytical approach

Using the superposition principle it is possible to evaluate the impact of the residual stress field induced by the spike load on SIF values. This is possible by considering the stress profiles \( \sigma_y \) along the crack path as previously calculated with the FEM analyses.

The Green’s Function \( G(x, \alpha) \), that provides the SIF value when two concentrated symmetric forces are applied on the crack faces (with a direction orthogonal to the crack edges), can be integrated to obtain the SIF corresponding to the residual stress profile \( \sigma_y \) (Eq. (6)):

\[ K_{\text{I, res}}(\alpha) = \int_{a_{OL}}^{\alpha} \sigma_{\text{res}}(x) G(x, \alpha) \, dx \]  

(6)

where \( \alpha_{OL} \) is the crack length when the overload is applied, \( \alpha (> \alpha_{OL}) \) is the current crack length, \( \sigma_{\text{res}}(x) \) is calculated from Eq. (5), and \( G(x, \alpha) \) is equal to

\[ G(x, \alpha) = \frac{1}{\sqrt{a}} \frac{1}{\sqrt{1 - \frac{\alpha}{a}}} f_1 \left( \frac{\alpha}{a} \right) f_2 \left( \frac{\alpha}{W} \right) \]  

(7)

The SIFs corresponding to residual stresses are calculated for both materials (Fig. 10) and used for the subsequent retardation effect evaluation (they are negative because of compressive residual stresses). Being the residual stresses \( \sigma_y(x) \) (\( x \) is the crack axis) very similar for the two materials, the corresponding SIF curves are very similar too.

Fig. 9. Residual stress profiles along the crack path.

Fig. 10. SIFs due to residual stresses.

Fig. 11. \( K_{\text{I, max, tot}} \) values for 2024-T351 (\( R_{OL} = 2 \)).

Fig. 12. \( DK_{\text{tot}} \) values for 2024-T351 (\( R_{OL} = 2 \)).
Fig. 13. Numerical and experimental crack growth rates for 2024T351 (left) and 2024HP-T3 (right).

Fig. 14. Crack length vs. cycle number for 2024-T351 (left) and 2024 T3 (right).

Fig. 15. Crack length vs. number of cycles: experimental results (left) and numerical results (right).
Following linear elastic fracture mechanics, in which the principle of superposition is valid, in presence of residual stresses the parameter $K$ can be expressed as

$$K_{\text{tot}} = K_{\text{appl load}} + K_{\text{residual stresses}}$$

Since $K_{\text{res}}$, the SIF corresponding to residual stresses, has the same values for both maximum and minimum applied load, it is possible to write:

$$K_{\text{max,tot}} = K_{\text{max,appl}} + K_{\text{res}}$$
$$K_{\text{min,tot}} = K_{\text{min,appl}} + K_{\text{res}}$$

and it follows that $\Delta K_{\text{tot}} = \Delta K_{\text{appl}}$.

For small $R$-ratios the minimum total SIF can be negative. Since this fact indicates that the crack tip is closed, the corresponding SIF should be set to zero. Taking into account the above consideration, the SIF, and the driving parameters used in the following, are adjusted as in the following:

$$K_{\text{max,tot}} = K_{\text{max,appl}} + K_{\text{res}}$$ (8.1)
$$\Delta K_{\text{tot}} = \Delta K_{\text{appl}} \text{ if } K_{\text{min,tot}} \geq 0$$ (8.2)
$$\Delta K_{\text{tot}} = K_{\text{max,tot}} \text{ if } K_{\text{min,tot}} < 0$$ (8.3)

The resulting $K_{\text{max,tot}}$ and $\Delta K_{\text{tot}}$ values are shown in Figs. 11 and 12 for the 2024 T351, but they are completely similar for 2024 HP-T3. In Fig. 12, in particular, it can be noted the effect of the position $S_p$ that increases with the decreasing $R$-ratio; such effect is particularly evident because the overload is very high and consequently the overload plastic zone is large.

3.6. Crack growth rates prediction

Crack growth rates can be predicted by using Eq. (1) where $\Delta K_{\text{tot}}$ and $K_{\text{max,tot}}$ include the contributions from remote applied stress and local internal stresses. Results are shown in Figs. 13–15 for the different $R$-ratios and materials (Al 2024-T351 and Al 2024HP-T3). In Fig. 13, the crack growth rates are plotted for comparison between experimental and simulation results: the agreement between the numerical and experimental minimum crack growth rates is comparable with the inherent scatter except for the case $R = 0.5$, where there seems to be an overestimation of the retardation phenomena. This can be explained observing that the two thresholds on $K_{\text{max}}$ and $\Delta K$ where approximated using a limit crack growth rate $\frac{\text{da}}{\text{dN}} = 1 \times 10^{-7} \text{ mm/cycle}$, whereas a slight reduction of such thresholds could be expected considering a lower value of $\frac{\text{da}}{\text{dN}}$: such slight variation would not affect significantly the cases with $R = 0.05$, $R = 0.15$ and $R = 0.3$ but, with $R = 0.5$ a strong impact is expected, due to the closeness between the $K_{\text{max,tot,min}}$ and the $K_{\text{max,tot,max}}$ as visible from Fig. 11. In Fig. 14 the total crack length vs. number of cycles is plotted: confirming, as expected, the same kind of discrepancy at $R = 0.5$.

In Fig. 15, a comparison between the behaviour of the two materials is represented, for experimental and simulation results: at various $R$-ratios, the model always gives higher retardation effects for 2024HP-T3 than for 2024 T351, and this effect increases with increasing $R$: the reason is to be found in the different thresholds values for the two materials.

4. Test case No. 2: crack growth simulation under variable load spectrum by combined FEM–DBEM approach

4.1. Introduction

The FEM procedure, after the calculation of residual stresses, would proceed through the crack propagation by complete domain remeshing at each step or by disconnecting some elements in a mesh that, since the beginning, is arranged keeping into account the crack propagation path. Such approach is not much efficient for such kind of problems, with strong difficulties to establish an automatic process, especially if the crack experience a mixed mode condition with a curved crack path.

Such drawbacks, in the crack propagation phase, can be circumvented adopting the DBEM. The DBEM incorporates two independent boundary integral equations: the displacement equation applied at the collocation point on one of the crack surfaces and the traction equation on the other surface. In this way it becomes possible to solve the crack problem in a single region formulation. This is implemented in a commercial code, where an automatic crack propagation analysis is possible, with allowance for
residual stresses, introduced as body loads (concentrated along lines of load). Such residual stresses are calculated along the prede
defined crack propagation path by the previously mentioned elastic–
plastic FEM analysis (see Section 3) and then introduced in the
DBEM model, on the propagating crack edges.

Even in case the crack path is not known a priori, it would be
possible to determine the (curved) crack propagation path by a
purely elastic DBEM analysis, where only the remote load is ap-
plied (without residual stresses), then calculate by FEM the resid-
ual stresses along such virtual line and import them in the DBEM
model. The underlying hypothesis is that the crack path is not
appreciably affected by few cycles with increased maximum load,
terminated in a constant amplitude fatigue load sequence. This is
acceptable in many real applications where the crack path is not
significantly affected by load spectrum residual stresses that, on
the contrary, heavily affects the crack growth rates.

4.2. Procedure description

If the superposition principle holds true, the residual stress ef-
fect on SIFs can be equivalently modelled by a distribution of trac-
tions on the advancing crack edges (Figs. 16 and 17): such trac-
tions are corresponding to the residual stresses existing on the virtual
line traced by the advancing crack (the material at this stage is
not cracked yet).

The following step is to realise a DBEM crack propagation anal-
ysis, with SIFs and crack growth rates automatically calculated by
the DBEM code, where the chosen crack propagation law (Eq.(1))i s
implemented.

The CT specimen, with the crack configuration at the moment in
which the overload is applied, is shown in Fig. 16: it undergoes a
remote traction load of 800 N, applied by means of two pins
(explicitly modelled in the DBEM model) and acting in combina-
tion with the residual stresses previously calculated by an FEM
elastic–plastic analysis. Such residual stresses are initially applied
directly on the crack edges (Fig. 17) and the related SIFs calculated.

In Fig. 17 it is possible to see the deformed plot of the crack tip,
comparing the case with and without residual stresses superim-
posed to the remote load: it is clear the beneficial effect of the com-
pressive residual stresses generated at the crack tip by the overload.

Applying the residual stresses directly on the crack edges, even
if more precise than using the body loads, has some drawbacks
with the DBEM code adopted: the automatic crack propagation is
prevented and manual intervention is needed to change the ap-
plied load on crack edges after each crack propagation step. Conse-
quently, a procedure is set up based on the simulation of the
aforementioned crack edge tractions by a couple of parallel body
load lines, with a small offset from the crack edges (Fig. 18).

A convergence analysis is made for this specific example with
$R = 0.15$, in order to compare the differences in the calculated SIFs,
as provided by the application of the tractions directly on crack
edges or, alternatively, on the aforementioned load lines with a
variable distance from the crack axis (Table 3). This is aimed to
understand how much the load lines must be close to the crack
axis in order to effectively simulate a load on the crack edges with-
in a given approximation. On the other hand, in order to avoid
numerical round off errors, it is necessary to keep the ratio be-
tween the load line distance and the crack tip element length
above a given threshold. Moreover, the use of double precision
for the numerical resolution becomes mandatory.

In Fig. 19 a deformed contour plot of Von Mises stresses in the
final cracked configuration is shown.

Considering the residual stresses obtained from a 3D FEM anal-
ysis, it is possible to compare the total SIFs ($K_I$) as calculated in an
FEM/analytical procedure (Fig. 11) with those calculated by a

<table>
<thead>
<tr>
<th>Load line distance from crack axis (mm)</th>
<th>$K_I$ (MPa $\text{mm}^{0.5}$)</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (load directly applied on crack edge)</td>
<td>186</td>
<td>0</td>
</tr>
<tr>
<td>0.01</td>
<td>203</td>
<td>9.1</td>
</tr>
<tr>
<td>0.005</td>
<td>196</td>
<td>5</td>
</tr>
<tr>
<td>0.0025</td>
<td>182</td>
<td>-2.1</td>
</tr>
</tbody>
</table>

Fig. 19. Von Mises stresses (deformed scale = 15).

Fig. 18. Crack opening due to remote load with superimposed residual stresses (acting as two body load lines).
two-dimensional DBEM analysis (Fig. 20), where the aforementioned residual stresses are provided as input: the results for a baseline stress ratio $R = 0.15$ and an overload stress ratio $R_{ol} = \sigma_{maxol}/\sigma_{maxbl} = 2$ are in a satisfactory agreement.

Again a convergence study is performed based on a variable distance of the body load lines from the crack axis and on a variable growth increment: the convergence is judged satisfactory (SIFs difference error below 2%) with a growth increment $\Delta a = 0.014$ mm and a load line distance from the crack axis $d = 0.005$ mm.

In Fig. 21 the crack model with highlight of applied body loads is shown: it is evident the beneficial effect of the superimposed residual stresses, that reduce the crack opening in that part of the crack that during the propagation is crossing the compressive residual stress field.

4.3. Crack growth rates prediction

Crack growth rates can be automatically provided by the DBEM code, by means of Eq. (1) where $\Delta K_{th}$ and $K_{max,th}$ include the contributions from both applied and residual stresses.

5. Analysis of results provided by test cases No. 1 and 2

The overload effect depends on the following material parameters:

1. The stress–strain curve (that affects the elastic–plastic calculations of the residual stress).
2. The thresholds $\Delta K_{th}$ and $K_{max,th}$ of the driving forces.
3. The constant amplitude crack growth curves.

For the two considered alloys, only the thresholds have a non-negligible effect on the model response differences, being the other parameters very close each other.

Appreciable differences between the numerical and experimental results exists only at $R = 0.5$, but the errors displayed can be considered acceptable, considering that the chosen overload ratio produces a $K_{max,th}$ that with its minimum value (with variable crack tip distance) approaches very closely the corresponding threshold, becoming extremely sensitive to the correct assessment of it.

Moreover, the results have been obtained without any calibration on the overload experimental data whereas the calibration experimental data derives from very few specimens undergoing constant amplitude fatigue load.

The coupled FEM–DBEM approach can guarantee a high level of versatility and accuracy to the procedure, so that it is recommended its adoption instead of a purely FEM based approach.

6. Test case No. 3: assessment of allowable dents on aeronautical panels based on the coupled usage of FEM–DBEM methodology

6.1. Introduction

In this test case a panel, made of aluminum alloy 6056 T78, is dented in an area in which crack propagation under constant amplitude fatigue load is to be analyzed: the denting process pro-
duce residual stress in the plastically deformed area, and such stresses combines with stresses coming from the remote fatigue load, influencing the crack propagation phase.

The DBEM, as implemented in a commercial code [10], is adopted for the crack propagation simulation, whereas an elastic–plastic analysis made with the FEM code ANSYS is used to calculate the residual stresses induced by the denting process.

The proposed application of the two-parameter approach (Eq. (1)) is verified against a series of laboratory tests, in order to evaluate its capability to predict the way in which the crack growth phenomena is affected by the plastic deformations in the cracked area.

6.2. Determination of the threshold parameters ($D_{Kth}/C_3$, $K_{max,th}$)

Due to the lack of experimental propagation data for the aluminum alloy 6056 T78 in the threshold range, the threshold data are...
assumed equal to those of the previously mentioned aluminum alloy 2024 HPT3, $\frac{K}{C3 \text{th}} = 58 \text{ N/mm}^3 = 2$ and $\frac{K}{C3 \text{max; th}} = 106 \text{ N/mm}^3 = 2$ (judging the two materials sufficiently similar from this point of view). Such decision imply another approximation, related to the fact that the 2024 HPT3 CT specimens, used for the threshold assessment, are 3.2 mm thick, whilst the dented coupon under investigation is 1.2 mm thick (this could have a non-negligible impact on the threshold values).

6.3. Determination of material parameters ($A, n, m$)

The material parameters ($A, n, m$) in Eq. (1) are chosen as best fitting coefficient to interpolate propagation data, generated by the previously calibrated Elber law (Eq. (9)), valid for 6056 T78 unclad weldable alloys and for a thickness in the range 0.8–3.2 mm:

$$\frac{da}{dN} = C_{\text{eff}}(A + B + R)^n \Delta K^n$$

where

$$C_{\text{eff}} = 1.608 \times 10^7; A = 0.61; B = 0.39; n = 3.5; \frac{da}{dN} = \text{[mm/cycle]; } \Delta K = \text{[MPa m}^{0.5}]$$

The crack growth rates (mm/cycle) are calculated at different $R$-ratios (from 0.1 to 0.9) in the range of $\Delta K = 200–700 \text{ MPa mm}^{0.5}$ (Fig. 22). The final result is

$$A = 2.69 \times 10^{-11}, \quad n = 2.08, \quad m = 0.696.$$  

6.4. Experimental test

The analyzed coupons were tested during the European project "IARCAS" (VI framework). They were dented (Fig. 23) and, afterwards, fatigue cycled up to rupture (Figs. 24 and 25), with a traction load applied along the longitudinal specimen direction. The tested coupon, made of aluminum alloy 6056 T78, have the following geometric dimensions and mechanical properties:

- Thickness: 1.2 mm; specimen length: 750 mm; specimen width: 220 mm; dent residual depth: 5 mm; indenter radius: 50 mm; Young modulus $E = 72 \text{ GPa}$, Poisson coefficient $\nu = 0.3$.

6.5. Coupled FEM–DBEM numerical procedure

6.5.1. Introduction

The numerical FEM–DBEM procedure is based on the following steps:

1. Calculate the residual stress scenario after denting, by means of a static elastic–plastic FEM analysis that simulate the skin–indenter impact. The 6056 T78 behaviour is defined by the following yielding and rupture values: $F_y = 315 \text{ MPa}$ and $F_u = 325 \text{ MPa}$. The base hypothesis are isotropic-hardening and Von Mises plasticity with associate flow rule.

2. From the overall residual stress scenario only the residual stresses along the experimentally observed crack path are extracted.

3. Such residual stresses are exported to the DBEM code adopted [10] and applied on the initial cracked configuration shown in Fig. 24 (the experimental crack growth monitoring starts with...
this configuration). Such stresses are modelled by line forces (a kind of body loads) along two parallel “load lines” that have a very low offset from the crack axis equal to 0.04 mm (the two load lines should ideally have a zero distance from the crack edges but, from a convergence study it is possible to assess that at a distance inferior to 0.04 mm there are no appreciable differences in the results).

4. A DBEM crack propagation analysis is performed in order to obtain the crack growth rates, with a load case defined by the superposition of residual stresses plus the remote fatigue load.

6.5.2. FEM elastic–plastic analysis

The analysis is performed through three load steps with the following boundary conditions:

- The dent is modelled as infinitely rigid and an aluminum strip is introduced between the coupon and the constraints. The z displacement is null out of the black line, where the plate sliding occurs with zero friction (Fig. 26).
- After displacing the denting tool in contact with the plate, an initial dent displacement equal to 14 mm is imposed (Fig. 27).
- The dent displacement goes back to 0 while the other boundary conditions are retained (Fig. 28).
- All the constraints are removed but the symmetry condition and the rigid motion constraints (Fig. 29).
- Calculation of $\sigma_Y$ residual stress near the denting area with $\sigma_{Y_{\text{max}}} = 206$ MPa (right).

In conclusion the residual stress field due to the plastic deformation induced by the denting process is obtained by sequentially removing the indenter and all the constraints. The final results

![Fig. 28. Load step 2 (the indenter is removed): z displacement with maximum residual dent depth equal to 5.3 mm (left) and Von Mises stresses (right) with $\sigma_{\text{eq, max}} = 378$ MPa.](image)

![Fig. 29. Load step 3 (all the constraints are removed): Z displacement (left); Von Mises stresses with $\sigma_{\text{eq, max}} = 361$ MPa (middle); $\sigma_Y$ residual stress near the denting area with $\sigma_{Y_{\text{max}}} = 206$ MPa (right).](image)

![Fig. 30. Graph plot of $\sigma_Y$ residual stress (N/m²) at the plane $y = 0$, calculated for top, bottom and middle shell surface.](image)

![Fig. 31. Load lines applied along the crack path for residual stress allowance.](image)
used in the two-dimensional DBEM crack propagation analysis, are
the $\sigma_Y$ residual stress at middle shell surface (this corresponds to
consider only the membrane stresses) and in correspondence of
the plane $y = 0$.

6.5.3. DBEM crack propagation

Having calculated the residual stresses, left by the denting pro-
cess on the coupon, it is possible to start the crack propagation
simulation aimed at reproducing the experimental crack propaga-
tion (Fig. 25). The initial scenario detected in the experimental test
is depicted in Fig. 24: up to this point a maximum remote fatigue
load $t_{\text{max}} = 140$ MPa with $R = 0.1$ is applied, whilst for the remain-
ing part of the crack propagation the maximum load is set to
$t_{\text{max}} = 70$ MPa with $R = 0.1$. The residual stresses are applied as
body loads along lines parallel and very close to the crack propaga-
tion path (they approximate stresses directly applied on the crack
edges) and usually they are both tangential and normal (Fig. 31)
but in this case only the normal (to the crack edges) components
are non-negligible (Fig. 32).

The DBEM model is shown in Fig. 32. In Fig. 33 the Von Mises
stresses and the crack deformed plot are shown at an intermediate
step of crack propagation: it is possible to see that the crack open-
ing is higher on the left hand side, where opening residual stresses
are acting, whilst the opposite occurs on the right hand side, where
compressive residual stresses are predominant.

The numerical and experimental graphs of crack advance vs. cy-
cles are shown in Fig. 34 for the two crack tips $a_1$ (external) and $a_2$
(internal), showing a satisfactory correlation. The effect of residual
stresses on SIFs is shown in Fig. 35.

![Fig. 32. DBEM coupon with residual stresses applied on crack edges and superimposed to the remote load.](image)

![Fig. 33. DBEM contour plot of stress results at the final stage of the propagation phase: the residual stress effect causes an unsymmetric crack opening.](image)

![Fig. 34. Numerical and experimental crack growth (right).](image)
6.6. Analysis of results

The procedure described takes advantage of the best capabilities of the two numerical methods (FEM and DBEM) and can be easily automated, but most of all it does not require extensive calibration tests because based on a physical description of the crack propagation phenomena across plastically deformed material. The differences between the calculated and experimental delay cycles are comparable with the inherent scatter but further tests are needed to obtain the precise threshold values for the 6056 T78 and to better calibrate the $A$, $n$, $m$ values.

Moreover a three-dimensional approach for both FEM and DBEM analysis could assess the importance of the bending stresses on the crack propagation, taking also into account the real stress distribution along the thickness, where a strong variation of residual stresses is evident (Fig. 30).

7. Conclusions

We have examined the overload effects and cold working effects on fatigue crack growth, and related contributing factors. Residual stresses due to respectively overload plastic zone and large plastic deformations, are shown to be a major factor that contributes to retardation (or acceleration).

The test cases presented confirm: (a) the validity of the two-parametric requirements, $\Delta K$ and $K_{\text{max}}$, (b) the role of residual stresses in understanding the fatigue crack growth behaviour in a component (they provide additional crack tip forces mostly through $K_{\text{max}}$).

Consideration of internal stresses, provide a self-consistency in terms of load spectrum effects and cold working effects.

References