Genetic Hybrid Predictive Controller for Optimized Dissolved-Oxygen Tracking at Lower Control Level

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Abstract—A hierarchical two-level controller for dissolved-oxygen reference trajectory tracking in activated sludge processes has been recently developed and successfully validated on a real wastewater treatment plant. The upper level control unit generates trajectories of the desired airflows to be delivered by the aeration system to the aerobic zones of the biological reactor. A nonlinear model predictive control algorithm is applied to design this controller. The aeration system itself is a complicated hybrid nonlinear dynamical system. The lower level controller (LLC) forces the aeration system to follow these set-point trajectories, minimizing a cost of energy due to pumping of the air and accounting for system operational limitations such as the limits on the allowed frequency of switching of the blowers and on their capacity. The predictive control is also applied to design the LLC based on a piecewise-linearized hybrid dynamics of the aeration system. Casting the mixed-integer nonlinear optimization problem under heterogeneous constraints due to the limits on the blower switching frequency into the approximated mixed-integer form is done at a cost of introducing large number of auxiliary variables into the lower level predictive controller optimization task. This paper derives another nonlinear hybrid predictive control algorithm for the LLC. It is directly based on the nonlinear hybrid dynamics and logical formulation of the switching constraint. A genetic algorithm is derived with dedicated operators allowing for efficient handling of the switching constraint and nonlinear hybrid system dynamics. The efficiency of the control algorithm is validated by simulation based on real data records.

Index Terms—Genetic algorithms, hierarchical control, hybrid predictive control, optimizing control, wastewater systems.

I. INTRODUCTION

AN ACTIVATED sludge wastewater treatment plant can be classified as a complex system due to its nonlinear dynamics, large uncertainty, multiple time scales in the internal process dynamics, and multivariable structure. In addition, rather limited measurements are available during plant operation.

Recent developments in control technologies and, particularly, in model predictive control (MPC), handling an uncertainty, estimation, trajectory tracking in nonlinear systems, and intelligent control triggered out new research and applications in this field. A hierarchical multilayer control structure that utilizes multiple time scales in the plant dynamics for robust optimized control of biological wastewater treatment plants was proposed in [1]. The control algorithms were synthesized at the supervisory control layer, slow control sublayer, and medium control sublayer (MCS) of the optimizing control layer. The hierarchical controller was validated by simulation based on real data records. In this paper, the fast control sublayer (FCS) is considered. A controller for an optimized tracking at the FCS of the dissolved-oxygen (DO) concentration trajectory prescribed by the MCS is synthesized. The DO control design was considered in, e.g., [2]–[6]. Under high variability of influent flow and pollutant concentrations, the plant operating point can vary considerably. An output from all these controllers is a desired airflow trajectory to be provided by the aeration system, i.e., the controller actuator. This is shown for the aeration system at the Kartuzy Wastewater Treatment Plant (WWTP), Northern Poland, with four aerobic biological reactor zones, which is shown in Fig. 1.

In this figure, \( S_{0,j}^{\text{ref}} \) and \( S_{0,j} \), \( j \in 1\ldots4 \), denote the prescribed and actual DO concentrations in the aerobic reactor zones, respectively, and \( Q_{\text{air},j} \) represents the airflows into these zones that control the DO concentrations. Typically, an aeration system itself is a complicated hybrid nonlinear dynamical system with faster dynamics compared to the internal dynamics of the DO at a biological reactor. It is a common approach, including the work described earlier, to neglect this dynamics and also the important operational limitations of this system imposed by the blower station. A blower that was once switched off cannot be switched on immediately but only after some period. If the period is long compared to the rate of change of the airflow, then this needs to be taken into account by a controller scheduling operation of the blowers. Otherwise, the desired airflow demand may not be met as the needed blower is disabled and cannot be activated when this is needed. The energy cost due to blowing of the air is taken into account at the MCS. However, due to the MCS time scale, only an average cost can be minimized when determining the optimized \( S_{0,j}^{\text{ref}} \) trajectories by this control layer [1]. An exact evaluation of this cost is possible only at the FCS that directly considers the aeration system. In a current industrial practice, the aeration system controllers are programmable logic controllers (PLCs) programmed PI algorithms that do not aim at all at combining the tracking functions with refining the cost optimization. A hierarchical two-level controller for DO reference trajectory tracking in activated sludge processes has been recently developed and successfully validated on a real wastewater treatment plant [7]. The controller structure is shown in Fig. 2.
Fig. 1. Configuration of the Kartuzy WWTP aeration system.

Fig. 2. Information structure of a two-level hierarchical controller for optimized DO tracking.

ries that are manipulated variables, forcing $S_{o,j}$ to follow $S_{o,j}^{\text{ref}}$, $j \in J_0$. The plant influent rate and pollutant concentration are unmeasured disturbance inputs at the upper control level. However, they can be aggregated in the DO dynamic model by respiration representing the DO consumption. It was noticed in [5] that respiration is slowly varying. Therefore, its one-step-delayed estimate carried out based on DO measurements was taken as the disturbance constant prediction over the controlled output prediction horizon in the NMPC optimization task. The lower level controller (LLC) acts as an actuating system and takes the ULC outputs that are the reference trajectories $Q_{\text{air},j}^{\text{ref}}$ of the airflows to be provided. It uses the aeration system control handles that are the blower structure on/off, blower speeds and angular positions of the throttling valves in order to produce the airflow trajectories $Q_{\text{air},j}$ that follow the trajectories $Q_{\text{air},j}^{\text{ref}}$ prescribed by the ULC and minimize the electrical energy cost due to blowing of the air. The blower switching constraints are catered for by the LLC. The magnitude and rate limits on $Q_{\text{air},j}$ are naturally obeyed by the LLC, and they are also taken into account at the upper control level so that the feasible trajectories of the manipulated variables can be prescribed by the ULC to be delivered by the LLC. The predictive control is also applied to design the LLC based on a piecewise-linearized hybrid dynamics of the aeration system. In this paper, another nonlinear hybrid predictive control algorithm for the LLC is derived. It is directly based on the nonlinear hybrid dynamics and logical formulation of the switching constraint. A genetic algorithm is derived with dedicated operators allowing for efficient handling of the switching constraint and nonlinear hybrid system dynamics. The efficiency of the control algorithm is validated by simulation based on real data records. This paper is organized as follows. Section II presents the aeration system model for control purposes. A general hybrid model predictive controller at the lower control level is described in Section III. The genetic hybrid controller at the lower level is derived in Section IV, and its implementation is described in Section V. The application of the
genetic controller to Kartuzy WWTP is described in Section VI, and simulation results based on real data records are presented. Section VII concludes this paper.

II. MODELING THE AERATION SYSTEM

A. Physical Modeling

The aeration system physical model consists of the following main element models: blower station composed of variable- and fixed-speed blowers operating in parallel, collector pipe, and aeration segment units (see Fig. 1). Each aeration segment unit $j \in J_a$ is composed of throttling valve, single equivalent diffuser, and collector–diffuser pipe. Model equations were derived and validated on a real WWTP in [8]. An electrical analogy of the overall aeration system model is shown in Fig. 3 (see [7]).

In Fig. 3, each blower $r \in I_b$ is represented by the current source. Each fixed- and variable-speed blower is described by a nonlinear function $Q_{b,r} = f_{b,r}(x_{b,r}, p_{b,r}, n_{b,r})$, where $Q_{b,r}$, $p_{b,r}$, and $n_{b,r}$ are the blower output airflow, pressure drop across the blower referred to atmospheric pressure $p_a$, and motor rotational speed, respectively, and the binary variable $x_{b,r} \in \{0, 1\}$ defines the on/off status of the rth blower. The speed variables $n_{b,r}$ are discrete valued for $r \in I_f$ and continuously valued for $r \in I_v$, and their ranges are $N_f$ and $N_v$, respectively. The collector pipe is modeled as a fluid-flow capacitance $C_{c}(p_c)$, and the fluid-flow resistance is negligible. Hence, $p_b = p_c$ for $r \in I_b$. Each aeration segment unit $j \in J_a$ is represented in Fig. 3 by the RC circuit, followed by a hydrostatic pressure source (hydrostatic pressure of sewer) $\Delta p_{h,j}$ in cascade. As the fluid-flow resistance of the collector–diffuser pipe is negligible, the unit system is modeled as a fluid-flow capacitance $C_{d,j}$ catering for the collector–diffuser pipe with two resistances $R_{v,j}$ and $R_{d,j}$ regarding the throttling valve and diffuser, respectively. The throttling valve resistance $R_{v,j}$ is a known nonlinear function of the valve angular position $\varphi_j$ so that

$$\Delta p_{v,j} = f_{v,j}(Q_j, \varphi_j) = R_{v,j}(\varphi_j)Q_j^3, \quad j \in J_a$$

where $c_4$ is a parameter.

If $\Delta p_{h,j} \geq \Delta p_{d,j}^{open}$, where $\Delta p_{d,j}^{open}$ is the diffuser opening pressure, the diffuser opens, and $Q_{air,j} = R_{d,j}Q_{air,j}$. The overall physical model reads

$$\frac{dQ_j}{dt} = \frac{1}{k_c V_p c} \sum_{r \in I_b} f_{b,r}(x_{b,r}, p_{b,r}, n_{b,r}) - \sum_{j \in J_a} Q_j$$

$$\frac{dQ_{air,j}}{dt} = \frac{1}{R_{d,j} C_{air,j}} (Q_j - Q_{air,j}), \quad j \in J_a$$

$$p_c = \Delta p_{d,j}^{open} + R_{d,j}Q_{air,j} + f_{v,j}(Q_j, \varphi_j) + \Delta p_{h,j} + p_a, \quad j \in J_a$$

where $n_{b,r}$, $x_{b,r}$, and $\varphi_j$ ($j \in J_a$ and $r \in I_b$) are control variables, and $p_c$ and $Q_{air,j}$ ($j \in J_a$) are state variables and, at the same time, output variables.

The blower states $x_{b,r}$ are, in this model, the input variables, and their dynamics will be derived in the next section. This model is described by a set of nonlinear differential and algebraic equations, where the algebraic equations (4) implicitly determine flows $Q_j, j \in J_a$, in terms of the states and control inputs.

B. Modeling the Blower Switching Constraints

The blower states can be immediately changed in a controlled manner every $k_l T_l$ time units, where $k_l$ is an integer variable representing discrete time at the lower control level, while $T_l$ is the sampling interval. The switching time is assumed zero so that, with $x_{b,r}(k_l) \equiv x_{b,r}(k_l T_l)$, $x_{b,r}(t) = x_{b,r}(k_l + 1)T_l = x_{b,r}(k_l + 1), t \in [(k_l T_l, (k_l + 1)T_l]]$.

Every blower can be switched on or off, implying changes of the blower state between 0 and 1. Let us define the switching control variables for the blower $r \in I_b$ as the binary and discrete-time variables $u_{b,r}^on$ and $u_{b,r}^off$.

An overall blower station dynamic can now be described for $r \in I_b$ as [7]

$$x_{b,r}(k_l + 1) = x_{b,r}(k_l) - u_{b,r}^off(k_l) + u_{b,r}^on(k_l), \quad r \in I_b$$

subject to the constraints

$$u_{b,r}^on(k_l) + u_{b,r}^off(k_l) \leq 1, \quad r \in I_b$$

$$x_{b,r}(k_l) - u_{b,r}^off(k_l) \geq 0$$

$$x_{b,r}(k_l) + u_{b,r}^on(k_l) \leq 1, \quad r \in I_b.$$  

As it was stated before, a blower that is switched off cannot be switched on immediately but only after a certain standstill period $N_s T_l$. In order to derive dynamical constraints on the values of the switching control variables $u_{b,r}^on$ and $u_{b,r}^off$ that meet this condition, a dedicated standstill state variable $S_{b,r}(k_l)$ is introduced to denote a number of steps over which the blower $r$ has remained switched off until time instant $k_l$. By using (5), a recursive equation driving evolution of $S_{b,r}$ over time can be written as

$$S_{b,r}(k_l + 1) = (1 - x_{b,r}(k_l)S_{b,r}(k_l) + u_{b,r}^off(k_l) - u_{b,r}^on(k_l)) \times S_{b,r}(k_l) + 1 - x_{b,r}(k_l) + u_{b,r}^off(k_l) - u_{b,r}^on(k_l), \quad r \in I_b.$$  

The standstill switching limit for the blower station can now be expressed as the variable constraints

$$S_{b,r}(k) - N_s x_{b,r}(k_l + 1) \geq 0, \quad r \in I_b.$$
III. LOWER LEVEL HNMPC

The control variables at the lower control level are as follows: \( \hat{u}_{\text{eff}}^{\text{bl}}, \hat{u}_{\text{eff}}^{\text{b1}}, n_0, \) and \( \varphi \) composed of \( u_{\text{eff}}^{\text{bl}}, u_{\text{eff}}^{\text{b1}}, n_{\hat{r}, r} \) (\( r \in I_b \)), and \( \varphi_j \) (\( j \in I_f \)), respectively. The switching control inputs are implemented in the system by simple PLCs. The speed control inputs are forced in the system by inverters. The throttling valve openings are forced by local PLC loops around the valves (see Fig. 1). Due to nonlinearity of the input–output model and constraints and as the variables involved are integers and also continuously valued, a hybrid nonlinear MPC (HNMPC) will be applied to derive the LLC algorithm.

A. Constraints

There are two types of constraints to be accounted for: the constraints implied by the physical aeration system model and the switching constraints.

1) Hard Model-Based Constraints: The speed and valve opening control variable trajectories are piecewise constant and determined by sequences of the vectors \{\( n(k_l), \varphi(k_l) \}\} for such that \( n_0(t) = n_0(k_l), t \in [k_l T_a, (k_l + 1) T_a] \), \( \varphi(t) = \varphi(k_l), t \in [k_l T_a, (k_l + 1) T_a] \). The aeration system model response over \( t \in [k_l T_a, (k_l + 1) T_a] \) is then determined by \( n(k_l), \varphi(k_l), x(k_l + 1) \) to produce the outputs \( p_c(t), Q_{\text{air}}(t), Q_{\text{f}}(t) \). The steady-state values of the outputs are very quickly achieved [7]. Hence, denoting \( p_c(k_l + 1) \stackrel{\Delta}{=} p_c(k_l + 1) T_a \), \( Q_{\text{air}}(k_l + 1) \stackrel{\Delta}{=} Q_{\text{air}}(k_l + 1) T_a \), \( Q_{\text{f}}(k_l + 1) \stackrel{\Delta}{=} Q_{\text{f}}(k_l + 1) T_a \), \( 2)–(4) \) yield the steady-state model of the aeration system in the form

\[
\sum_{r \in I_b} f_{\hat{r}, r}(x_{\hat{r}, r}(k_l + 1), p_c(k_l + 1), n_{\hat{r}, r}(k_l + 1)) +
- \sum_{j \in I_a} Q_{\text{air}, j}(k_l + 1) = 0
\]

with the simple bound constraints on the variables

\[
p_{\text{min}} \leq p_c(k_l + 1) \leq p_{\text{max}}, \quad Q_{\text{air}, j}(k_l + 1) \geq 0
\]

\[
x_{\hat{r}, r}(k_l + 1) \in \{0, 1\}, \quad r \in I_b
\]

\[
n_{\hat{r}, r}(k_l) \in N_r, \quad r \in I_b
\]

\[
n_{\hat{r}, r}(k_l + 1) \in N_r, \quad r \in I_b
\]

As the model is implicit, the decision variables in the HNMPC using this model are as follows: \( x_{\hat{r}, r}(k_l + 1), n_{\hat{r}, r}(k_l), p_c(k_l + 1), Q_{\text{air}, j}(k_l + 1), \) and \( \varphi(k_l) \). The model equations (10) and (11) and the inequalities (12) and (13) are the equality and inequality constraints, respectively, in the HNMPC optimization problem.

2) Switching Constraints: Equations (5) and (8) constitute the equality constraints, while the inequalities (6), (7), and (9) form the inequality constraints in the HNMPC optimization problem. Notice that the optimization problem is a nonlinear mixed integer. Its dynamics is entirely due to the switching frequency limit.

B. Performance Function

The electrical energy cost due to blowing of the air is proportional to the collector pressure \( p_c \). Hence, a performance function at time instant \( k_l \) over the prediction horizon \( H_P^l \) at the lower level is formulated as

\[
J^L = \sum_{i=1}^{H_P^l} \left\{ \eta(k_l + i - 1) p_c(k_l + i | k_l) T_a + \sum_{j \in I_a} \left( Q_{\text{air}, j}(k_l + i | k_l) - Q_{\text{air}, j}(k_l + i | k_l) \right)^2 + \sum_{r \in I_b} x_{\hat{r}, r}(k_l + i | k_l) n_{\hat{r}, r}(k_l + i - 1 | k_l) \right\}, \quad (14)
\]

The first term in (14) represents the electricity cost, and the coefficient includes the time-varying electricity tariff. The second term represents the airflow tracking error, where the reference airflow \( Q_{\text{air}, j}(k_l + i | k_l) \) is prescribed by the ULC. The third term encourages the controller to minimize a number of blower switchings and to adjust the speeds of the already switched variable-speed blowers rather than to switch another fixed-speed blower.

C. Control Algorithm

At time instant \( k_l T_a \), the HNMPC at the lower control level solves its optimization task by minimizing the performance function (14) with respect to the following decision variables:

\[
x_{\hat{r}, r}(k_l + i | k_l), \quad u_{\text{eff}, i}(k_l + i - 1 | k_l), \quad u_{\text{eff}, i}(k_l + i - 1 | k_l),
\]

\[
S_b(k_l + i | k_l), \quad n_{\hat{r}, r}(k_l + i - 1 | k_l), \quad p_c(k_l + i | k_l), \quad Q_{\text{air}, j}(k_l + i | k_l), \quad \varphi(k_l + i | k_l), \quad i \in \frac{1}{1}, \quad H_P^l
\]

which include both the control and output variables over the prediction horizon \( H_P^l \). Given the control inputs at \( k_l \) over \( H_P^l \), the outputs \( Q_{\text{air}, j}(k_l + l | k_l) \) and \( p_c(k_l + l | k_l) \) and states \( x_{\hat{r}, r}(k_l + l | k_l) \) and \( S_b(k_l + l | k_l) \) are predicted by solving the dynamic and implicit steady-state hybrid models (5), (8), and (10) and (11), respectively. The initial states \( x_{\hat{r}, r}(k_l) \) and \( S_b(k_l) \) are taken from the measurements. This is an information feedback from the system that allows one to correct the model-based states when the switching hardware errors occur. Only the first control step optimized values \( \hat{u}_{\text{eff}, i}^{\text{bl}}(k_l | k_l), \hat{u}_{\text{eff}, i}^{\text{b1}}(k_l | k_l), \hat{n}_b(k_l | k_l), \) and \( \hat{\varphi}(k_l | k_l) \) are applied to the aeration system, and at time instant \( (k_l + 1) \) \( T_a \), the aforementioned procedure repeats based on the updated airflow references \( Q_{\text{air}, j}^{\text{ref}}(k_l + 1 + i | k_l + 1) \).

IV. GHNMP

There is no solver available at present that can solve the HNMPC optimization task in a robust manner meeting time constraints due to the online implementation requirements. In [7], the constraints were piecewise linearized by applying the routine differential linearization and propositional logic [10], and an absolute rather than the square of the tracking error was applied in (14), allowing for exact linearization of the performance function \( J^L \). Casting the mixed-integer nonlinear optimization problem into the approximated linear mixed-integer form was done at the cost of introducing a large number of auxiliary optimization (AO) variables. This makes solving of the optimization tasks still computationally demanding. We shall now reformulate the optimization problem so that
its viable suboptimal solution can be much more efficiently produced. The hard aeration system model constraints will be formulated in an explicit form assuming perfect reachability of the airflow trajectory prescribed by the ULC. Hence, their iterative solving will not be required. Moreover, the switching constraints will be formulated in a soft manner, and a dedicated genetic operator will be derived to handle them. Finally, adjusting at a low computing cost, only the determined blower speeds and valve opening angles will allow optimized tracking also of not exactly reachable reference airflow trajectories. These, when embedded into a state-of-the-art genetic algorithm, will produce a very efficient genetic hybrid nonlinear model predictive controller (GHNMPC).

A. Solving Hard Model-Based Constraints

We shall now assume that the airflow prescribed by the ULC can be exactly achieved by the aeration system over the prediction horizon. Thus, there exist feasible trajectories of the lower level control variables such that the following holds:

\[ Q_{\text{air},j}(k_t + i|k_t) = Q_{\text{air},j}^\text{ref}(k_t + i|k_t), \quad j \in J_r, \quad i = \frac{1}{H_p} \]

(16)

The unknown airflows, \( Q_{\text{air},j}(k_t + i|k_t), \quad j \in J_r \), in the aeration system steady-state model constraints (10) and (11) can now be eliminated to produce

\[
\sum_{r \in I_b,j \neq j} f_{b,r}(x_{b,r}(k_t + i|k_t), p_c(k_t + i|k_t), n_{b,r}(k_t)) + \sum_{j \in J_r} Q_{\text{air},j}(k_t + i|k_t) = 0 \tag{17}
\]


\[
P_c(k_t + i|k_t) = \Delta p_c^{\text{open}} + R_{d,j}Q_{\text{air},j}^\text{ref}(k_t + i|k_t) + \Delta p_{h,j} + p_a + f_{u,v}(Q_{\text{air},j}^\text{ref}(k_t + i|k_t), \varphi_j(k_t)), \quad j \in J_r. \tag{18}
\]

Given the status of the blowers, \( x_{b,r}(k_t + i|k_t), \quad r \in I_b, \quad i = \frac{1}{H_p} \), and the blower speed controls \( n_{b,r}(k_t + i|k_t), \quad r \in I_b, \quad i = \frac{1}{H_p} \), applying an affine approximation of \( f_{b,r} \) and using (1), the aforementioned constraint equations can now be explicitly solved with respect to \( p_c(k_t + i|k_t) \) and \( \varphi_j(k_t + i|k_t) \) for \( j \in J_r, \quad i = \frac{1}{H_p} \). Indeed, from (17), the collector pressure can be obtained. The valve opening controls \( \varphi_j(k_t + i|k_t) \) can now be calculated from (18) using (1). Let us notice that the HNMPC optimization problem is now significantly simplified. The variables \( Q_{\text{air},j}(k_t + i|k_t), \quad p_c(k_t + i|k_t), \) and \( \varphi_j(k_t + i|k_t) \), \( j \in J_r, \quad i = \frac{1}{H_p} \), are no longer the decision variables in the problem [see (15)]. Clearly, the second term in (14) disappears. Therefore, the predictive control optimization task at instant \( k_t \) can be written as

\[
\min \left\{ J_{\text{LG}} = \frac{H_p}{i=1} i f(k_t + i - 1|k_t) p_c(k_t + i|k_t) + \sum_{r \in I_b} x_{b,r}(k_t + i|k_t)n_{b,r}(k_t + i - 1|k_t) \right\} \tag{19}
\]

with respect to the decision variables

\[
x_{b,r}(k_t + i|k_t) \quad n_{b,r}(k_t + i - 1|k_t) \quad p_c(k_t + i|k_t) \quad \varphi_j(k_t + i - 1|k_t) \quad S_{b,r}(k_t + i|k_t), \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \ Quad}
C. Correction of Throttling Valve Openings and Blower Speeds

If (16) does not hold, \( p_c \) and \( \varphi_j \) become infeasible. Although this may not be the case, meaningful and feasible control actions may exist. The question on how to maintain appealing reformulation of the optimization problem and recover such control schedules at a low-cost modification of the best infeasible solution provided by a genetic solver arises.

To begin with, let us notice that, due to SO operation, the blower state trajectories \( x_{th}(k_t + i|k_t) \), together with the blower speed trajectories, meet all the constraints except (21) and/or (22). We shall leave the former ones as they are and alter only the blower speeds and the throttling valve openings as follows. Under known \( x_{th}(k_t + 1|k_t) \), the time dependence between the airflow outputs, blower speeds, and throttling valve openings disappears. Hence, the predicted control actions need to be altered only over the time steps \( i \in \tilde{H}_p \) where the constraints are violated.

This is done by solving, with respect to \( Q_{air,j}(k_t + i|k_t) \), \( p_c(k_t + i|k_t) \), \( \varphi_j(k_t + i - 1|k_t) \), and \( n_{th,j}(k_t + i - 1|k_t) \), \( i \in \tilde{H}_p \), the following independent AO tasks [see (14)]:

\[
\min \left\{ \sum_{j \in a} \sum_{i \in \tilde{H}_p} v_{ij} \frac{Q_{air,j}^a(k_t + i|k_t) - Q_{air,j}(k_t + i|k_t)}{Q_{air,j}(k_t + i|k_t)}^2 + \gamma_i(k_t + i - 1) p_c(k_t + i|k_t) \right\} 
\]

subject to the constraints described by the equalities (10) and (11) with fixed \( x_{th}(k_t + i|k_t) \) and the inequalities (12) and (13).

The altered control trajectories minimize the weighted tracking errors, where \( v_{ij} \) denotes the volume of the \( j \)th aeration tank, and it is assumed that tank 1 has the smallest volume.

V. GENETIC SOLVER OF THE GHN MPC OPTIMIZATION TASK: IMPLEMENTATION

The genetic algorithm used in this paper is a simplified single-objective version of a state-of-the-art multiobjective algorithm NSGA-II [11]. It was chosen due to the used constraint handling scheme, namely, the constraint tournament. It has been proven to be very efficient in solving the optimization task.

An information structure of the employed GA is shown in Fig. 4. It operates, assuming perfect reachability of \( Q_{air,j}^{*}(k_t + i|k_t) \), in its main part, and this is followed by the AO correcting the obtained control variables so that their feasibility is regained if the reachability assumption is not satisfied. It has to be noticed that the mutation operator is used before crossover. This is motivated by supplying more diverse solutions to the crossover operator. Moreover, before every evaluation of the objective function (19), the SO defined in Section IV-B is used in order to assure the conformance of the switching frequency constraint. In this way, the resulting blower schedules are feasible with regard to the switching constraint (9), and also, (5), (6), and (7) are automatically met by the SO. The remaining constraints (13), (21), and (22) are handled mainly by the selection scheme. The resulting control actions forcing the operation of the aeration system over the entire prediction horizon are feasible and can be safely applied to the system.

A. Initialization

The used initialization scheme is a hybrid. A fixed number of initial solutions are obtained using a priori knowledge about the aeration system. The remaining solutions are initialized randomly. The algorithm using the proposed initialization scheme has an improved convergence because it starts with feasible or near-feasible solutions.

The procedure for generating the initial solutions is complex but yet very effective. First, the pressure drop values \( \Delta p_{c,j}(k_t + i|k_t) \) across the valves are randomly selected from their predefined ranges over the prediction horizon according to

\[
\Delta p_{c,j}(k_t + i|k_t) = U_{[0,1]} \Delta, \quad j \in J_a, \quad i = 1 : \tilde{H}_p
\]

where \( \Delta \in \{5, 10, 20\} \) and \( U_{[0,1]} \) is a random number of uniform distribution.

Next, for each trajectory of the valve pressure drops over the prediction horizon sequence, the collector pressures required to supply the reference airflow are computed from (8) at each prediction step for each aeration tank as

\[
p_{c,j}(k_t + i|k_t) = \Delta p_{c,j}(k_t + i|k_t) + \Delta p_{h,j} + p_{as}, \\
\]

\[
\quad j \in J_a, \quad i = 1 : \tilde{H}_p
\]

Clearly, the collector pressures differ as the valve pressure drops are randomly chosen. For the proceeding calculations, a mean value of the collector pressure for each prediction step is calculated as

\[
\bar{p}_{c}(k_t + i|k_t) = \frac{1}{J_a} \sum_{j \in J_a} p_{c,j}(k_t + i|k_t).
\]

The remaining steps of the initialization algorithm are discussed only for the case study aeration system composed of a single fixed-speed and a variable-speed blower, i.e., \( I_f = 1, \ I_v = 2 \), \( x_{b} = [x_{b,1}, x_{b,2}] \), and \( n_{b} = [n_{b,1}, n_{b,2}] \). This approach can be applied for different configurations of the blower station. However, its implementation becomes much more complex, and its explanation due to limited space would be rather tedious.
Using the previously obtained mean collector pressure at each prediction step \( \hat{p}_c(k_t + i|k_t) \) and prescribed by the ULC reference airflow into each aeration tank \( Q_{\text{air},j}^r(k_t + i|k_t) \), the blower control schedules are computed using an iterative algorithm. Given the states of the blowers and the speeds of the fixed-speed blower over the prediction horizon, the corresponding speeds of the variable-speed blower over the prediction horizon steps when \( x_{h,2}(k_t + i|k_t) = 1 \) can be calculated from (17) as

\[
n_{h,2}(k_t + i|k_t) = \frac{1}{b_2} \left\{ \sum_{j \in J_a} Q_{\text{air},j}^r(k_t + i|k_t) + a_2 p_a - c_2 + \left[ -a_2 \hat{p}_c(k_t + i|k_t) - a_3 n_1(k_t + i|k_t) \right] \times (a_1 \hat{p}_c(k_t + i|k_t) - a_3 p_a + b_1 n_{h,1}(k_t + i|k_t - 1|k_t) + c_2) \right\}. \tag{28}
\]

Starting from the highest speed of the fixed-speed blower and going down to complete shutdown, i.e., \( x_{h,2}(k_t + i|k_t) = 0 \), all combinations of speeds are checked for the fixed-speed blowers, and for each of them, the resulting variable-speed-blower speed is computed using (28). If the obtained speed is within the allowable range, the next prediction step is considered as well as the states over the prediction horizon. The SO operator is then applied to the obtained fixed-speed-blower state trajectory to assure its feasibility regarding the switching constraint (9). If changes are made, the variable-speed-blower speeds are recalculated using (28). When the recalculated speed lies outside the allowable range, it is modified to take the closest allowable value.

The described initialization scheme results in choosing the highest possible fixed-speed-blower schedule that allows one to supply the mean collector pressure \( \hat{p}_c(k_t + i|k_t) \). In order to further improve the initialization performance, the described scheme is performed again. However, this time, first, the switched-off fixed-speed blower is investigated, and its speed is checked when it is increasing. In this way, the lowest possible fixed-speed-blower speed schedule is determined.

### B. Selection, Crossover, and Mutation

The selection scheme is based on the constrained tournament [11]. This approach has been proven to be very efficient for solving constrained problems. It is based on comparing two random solutions from the population and choosing a better one with regard to both the objective function value and the overall constraint violation. The overall constraint violation is defined as a sum of all violations of the constraints over the entire prediction horizon.

Crossover is one of the most important genetic operators. Together with mutation, they are responsible for introducing new solutions to the populations. There are multiple crossover operators available. In this paper, the simulated binary crossover (SBX) operator is used [11]. As the name suggests, it simulates the working principle of a single-point binary crossover.

A polynomial mutation introduced by Deb [11] is applied. Similar to the SBX operator, it is based on a scalable polynomial probability distribution.

### C. Elitism

The implemented elitism operator is based on a concept of controlled elitism [11]. This implies that the number of elite solutions copied from the previous population is restricted. Therefore, a number of new solutions obtained by means of genetic operators in the current generation are preserved and copied to the next generation. This happens even if they are noticeably worse than the solution from the previous population. In this way, the diversity of the population is maintained, while the best solutions found so far are preserved and used in the search process.

### D. Stop Criterion

Exploiting \textit{a priori} knowledge of an aeration system is always worthwhile to design the stop criteria. Consider the case when \( Q_{\text{air},j}^r(k_t + i|k_t) \) is perfectly reachable by the aeration system and the third term in (19) is not restricting. Suppose that a feasible solution of the optimization task exists such that at least one throttle is fully open over the prediction horizon. Hence, the corresponding collector pressure \( \hat{p}_c(k_t + i|k_t) \) is minimal at every prediction step over \( H_{ij}^f \), and the performance function reaches its global minimum. Notice that such condition also becomes necessary if there is a sufficient controllability within the blower station, as it is possible to achieve the globally minimal collector pressure that is generated under a fully open throttle by feasible values of the blower station control handles. Such situation appears in our case study system. A nice and simple sufficient condition for optimality can then be set up. Clearly, it is not so in general. However, practical systems are typically designed in such a way that the aforementioned assumptions either hold or are close to hold. Hence, adding a certain tolerance to the throttle full-opening condition and a kind of safeguard condition yields a practically viable stop criterion. Namely, the algorithm stops if at least one of the following conditions holds:

1. The number of generations has reached the specified threshold.
2. At least one throttle meets \( \varphi_j^{\text{max}} - \varphi_j \leq \epsilon \) while having the overall constraint violation equal to zero, and the generation counter is larger than the specified number of generations.

Introducing this problem, a specific stop criterion allowed one to reduce the computation time by approximately 35% on the test problem while maintaining very high quality of the obtained results.

### VI. CASE STUDY SIMULATION RESULTS

This section presents results of application of the derived controller to case study WWTP in Cartuzy, Northern Poland. The plant diagram is shown in Fig. 5. Two blowers supply the overall aeration system. The first one can run with two fixed speeds, while the second is of variable speed, so it is controlled by an inverter.

The control system operation was simulated over 48 h under influent scenarios showing large variations of the wastewater inflow into the plant and its pollution. Biological processes were modeled by applying ASM2d [12], and the overall model was calibrated based on plant data records. The model was then
implemented in the simulation package SIMBA [13] in order to get data from the controlled plant. Several sampling intervals were investigated, and the following values were selected: $T_i = T_f = 5 \text{ min}$. The blower standstill period was $N_{\text{b}} = 3$. The prediction horizons $H_P^U$ and $H_P^L$ at the upper and lower levels were ten steps each; $H_P^L$ needs to be long enough in order to accommodate the blower holding time.

The parameters of GHNMPC are as follows: population size of 100, polynomial mutation parameter of 1, mutation probability of 0.01, SBX parameter of 1, crossover probability of 0.9, maximum number of generations of 400, and minimum number of generations of 200. The stop condition parameter $\epsilon = 0.0001$. Computations were carried out on an Intel Core 2 Duo class computer in the Matlab environment. The time required to compute a single prediction step was approximately 3 s.

In Fig. 6, the $S_{\text{-ref,2}}$ prescribed by the MCS is presented together with the DO concentration resulting from GHNMPC operation. The resulting DO tracking is good. The airflow reference trajectory $Q_{\text{air,2-ref}}$ prescribed by the ULC is shown in Fig. 7 together with the trajectory provided by the derived GHNMPC. The operation of the blowers is shown in Fig. 8. In Fig. 8, the blower operation schedule is presented, which was computed using the third term in (14). Switching the fixed-speed blower will now be used only if the variable-speed blower has not been able to meet the airflow demand. The third term in (14) was then removed. The resulting operation of both blowers became very demanding, and it would probably result in increased maintenance requirements for both blowers. However, the resulting tracking performances were very similar. At least one throttle
is fully open. This suggests that the resulting operation of the aeration system is nearly optimal.

Let us now screen the operation of the overall DO controller at time 0.66 and short after. At this time instant, the LLC decides to switch off the fixed-speed blower because, according to the airflow reference prescribed by the ULC, engaging the fixed-speed blower over the prediction horizon is not needed, and therefore, optimizing the third term in (14) forces the blower to remain off. However, the disturbance prediction that is the prediction of the respiration (see [5]) used by the ULC has noticeably changed in the next time step. Hence, the airflow reference trajectory is updated such that the airflow demand cannot be met by the variable-speed blower alone, and the support of the fixed-speed blower becomes necessary. Unfortunately, the SO does not allow one to turn on the fixed-speed blower because the required number of standstill steps is not met yet.

This is shown in Fig. 9 for three different values of standstill. As the assumption (16) is not met, this results in infeasible solutions generated by the genetic solver. Therefore, the AO described in Section IV-C had to be engaged in order to regain feasibility and to choose the angular positions of the throttles minimizing the airflow tracking error. This results in additional error in the prediction of DO concentration by the ULC. As a perfect actuator is assumed in the DO concentration prediction model used by the ULC, this error is seen by the ULC as the model prediction error. Hence, it is detected by the controller through its output feedback mechanism, and consequently, the action is undertaken at the upper control level in order to compensate the error. The airflow trajectories prescribed by the ULC in the subsequent time steps gradually become perfectly reachable by the LLC, and finally, the AO actions are not needed any longer. This is shown in Fig. 9(a)–(c). Clearly, the time needed to regain the perfect tracking of $Q_{r_{ref}}$, $f \in J_d$, depends on the standstill length, as shown in Fig. 9(d). It is achieved after a short period that is not even seen in Fig. 7.

In summary, although we do not present in this paper a rigorous analysis of stability, for example, it is demonstrated by simulation that the approach to the LLC synthesis that is based on the airflow perfect reachability assumption followed by the AO, if this assumption is not temporarily met, achieves an excellent tracking performance in a robust and extremely efficient manner.

VII. Conclusion

This paper has considered a hierarchical two-level controller for optimized DO reference trajectory tracking in activated sludge processes that has been recently developed and successfully validated on a real wastewater treatment plant. A novel genetic hybrid nonlinear LLC has been synthesized. The genetic algorithm has been derived with dedicated operators allowing for efficient handling of the switching constraint and nonlinear hybrid system dynamics to efficiently solve the model predictive optimization task. The efficiency of the control algorithm has been validated by simulation based on real data records to show good tracking performance and an excellent computing efficiency that is crucial for online applications. Stability of the overall hybrid nonlinear dynamics has been verified by simulation, and it is still under research.
REFERENCES


