Impact of Wiener Phase Jitter Model on the Performance of Uplink OFDMA and Uplink MC-CDMA Systems

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Abstract

Performance degradation of uplink OFDMA and uplink MC-CDMA under Wiener phase jitter is pointed out in this paper using theoretical and simulation results. Beginning by characterizing the Wiener process, signal to noise degradation of presented waveforms is derived. Under the assumption of full load and small jitter, the degradation is shown to be varying with jitter variance and time. The main conclusion that can be achieved is the strong degradation that may occur to frequency locked multicarrier systems in the presence of Wiener phase jitter.

1. Introduction

The demand for high data rate services has been increasing very rapidly and there is no slowdown in sight. Often, both wired and wireless services require very reliable data transmission over very harsh environments. Most of these transmission systems experience many degradations, such as multipath, interference, noise and must meet many constraints, such as finite transmit power and most importantly finite cost. One technique that has recently gained much popularity due to its robustness in dealing with these impairments, is multicarrier modulation such as Orthogonal Frequency Division Multiple Acess (OFDMA) and Multi-Carrier Coded Division Multiple Access (MC-CDMA). The OFDMA technique has been proposed as an access technique for the return path in a Communication Area Television (CATV). Where, MC-CDMA system has been investigated in the context of mobile radio communication. However, these systems suffer from severe performance degradation in the presence of phase jitter. In fact, because the presence of interference, noise and other disturbances, synchronization errors: carrier phase errors and timing errors; may occur between receiver and transmitter. In most cases, the receiver uses a local oscillator to derive his own carrier phase by means of a feedback carrier synchronization mechanism. When the local oscillator output is phase locked, then error phase or jitter phase is considered to be stationary. Most papers dealing with effect of phase jitter on multicarrier systems have been focused on this assumption [3], [4] and [7].

In a previous work [8] and under the same assumption, we had compared two stationary models for the phase jitter (Gaussian and Rayleigh laws). We had pointed out that, although, the first seems to be more realistic, simulation results show that the second process performs better and gives low degradation for multicarrier systems.

To continue this comparison of jitter models, we suppose here that the local oscillator is being frequency locked instead of being phase locked. The phase noise resulting is then no more stationary and is modeled as a Wiener process.

The main objective of this paper is to examine the signal to noise degradation of OFDMA and MC-CDMA systems in the presence of Wiener phase jitter. We start by outlining the Wiener process and its characteristics. Then, we describe uplink OFDMA and uplink MC-CDMA systems. So general properties are derived and different equations are analyzed in section III. Performance degradation of the studied waveforms are examined in section IV. Simulation results are shown in section V while section VI concludes the paper.

2. Wiener Phase Noise

A wealth of information is available on the topic of phase noise. It’s well known that phase noise in oscillators becomes one of the limiting degradations in modern radio systems. However, most of the traditional analysis concentrate only on Orthogonal Frequency Division Multiplexing (OFDM) systems degradation ([12] and [9]). The effect of Wiener phase on multicarrier receiver systems OFDMA and MC-CDMA are rarely discussed.
The Wiener process serves as the most important model of Brownian motion. Brownian motion is the process of displacements of small particles suspended in a liquid [10]. In our context, the fluctuation or the random jitter caused by error synchronization, looks like the motion of these small particles. This is why, with no stationary case, the jitter is modeled by a Wiener process which is derived from the Gaussian process.

Let \( \phi(t) \) be this Wiener phase jitter, we can write:

\[
\phi(t) = \int_0^t \mu(u) du
\]  

(1)

where \( \mu(u) \) is a zero mean stationary white Gaussian random process with variance \( \sigma^2 \). The probability density function of \( \phi(t) \) can be written [6]:

\[
p_{\phi(t)}(\phi, t) = \frac{1}{\sqrt{2\pi\sigma^2 t}} \exp\left(-\frac{\phi^2(t)}{2\sigma^2 t}\right)
\]

(2)

At a fixed instant \( t \), the probability density function \( p_{\phi(t)}(\phi, t) \) mentioned above becomes a Gaussian process (see Figure 1). Where Figure 2 and Figure 3 show 3-D vari-

![Figure 1. Probability density function of Wiener phase jitter with different fixed values t](image1)

![Figure 2. 3-D Probability density function of Wiener phase jitter](image2)

![Figure 3. 3-D Probability density function of Wiener phase jitter (phase zoomed)](image3)

3. Systems Description

3.1. Uplink OFDMA

the transmitters of different users are in different locations. This implies that the signals transmitted by different users exhibit different phase jitter \( \phi_l(t) \). The conceptual block diagram of the uplink OFDMA is shown in Figure 4. For each user, the data symbols to be transmitted to the basestation are organized into \( (N_F + \nu) \) blocks where \( N_F \) is the FFT point and \( \nu \) is the length of the cyclic prefix. Let \( a_{i,l} \) denotes the \( i^{th} \) data symbol transmitted by user \( l \). The transmitted sequence applied to the transmit filter \( p(t) \) yielding the signal \( s_l(t) \) given by:

\[
s_l(t) = \sum_{i=-\infty}^{+\infty} \sum_{n=-\nu}^{N_F-1} \frac{1}{\sqrt{N_F + \nu}} a_{i,l} e^{j2\pi \frac{nT}{T}} \times p(t - (i(N_F + \nu) + n)T - \tau_{c,l})
\]

(3)

where \( 1/T \) is the symbol rate per carrier and \( \tau_{c,l} \) is the time delay corresponding to the phase of the transmitter clock. The output of the ideal channel is disturbed by Wiener phase noise \( \phi_l(t) \) of user \( l \). The users signals are summed and
affected by the additive white Gaussian noise. So, we have:

\[ s(t) = \sum_{l=1}^{N_u} s_l(t) + w_{LP}(t) \]  

(4)

where, \( N_u \) is the number of users, which is equal to \( N_F \) under the assumption of a full load.

At the base station, the signal is applied to the receiver filter and sampled to give the \( v_{i,k} \) samples. Keeping the \( N_F \) samples outside the cyclic prefix, the output samples are fed to the FFT and then to an equalizer with coefficients \( g_{i,l} \). As a result, the samples \( z_{i,l} \) are given by:

\[ z_{i,l} = \sqrt{\frac{N_F}{N_F + \nu}} \sum_{l'=-\infty}^{\infty} \sum_{i',l'=-\infty}^{N_u-1} a_{i',l',l} I_{i',l',l'} + w_{i,l} \]  

(5)

where \( w_{i,l} \) is the term referred to the AWGN (Additional White Gaussian Noise) and \( I_{i',l',l'} \) is given by:

\[ I_{i',l',l'} = \frac{1}{N_F} g_{i',l'} \delta_{i',l'} \sum_{k=0}^{N_F-1} e^{-j 2 \pi k (l-l')/N_F} e^{-j \phi(t)} \]  

(6)

The quantity \( I_{i',l',l'} \) denotes the contribution of the symbol \( a_{i',l'} \) on the \( l^{th} \) FFT output during the \( i^{th} \) block. The samples \( z_{i,l} \) can be decomposed into four contributions: an average useful component \( I_{i,i,i,l} \), a zero mean useful component \( I_{i,i,l,l} - E(I_{i,i,l,l}) \) or self interference, an intersymbol interference \( (i' \neq i \text{ and } l' = l) \) and an intercarrier or interuser interference \((i' \neq l)\).

3.2. Uplink MC-CDMA

In the multicarrier CDMA technique, the original data stream is first multiplied with the spreading sequence and then modulated on the different carriers [4]. The conceptual block diagram of an uplink MC-CDMA system is shown in Fig. 5. The data symbols \( a_{i,l} \) denote the \( i^{th} \) symbol transmitted by user \( l \). Each data symbol is multiplied with the \( n^{th} \) chip during the \( i^{th} \) symbol interval of the sequence belonging to the user \( l \), \( c_i N_c + n_i l \). In this work, we consider orthogonal sequences consisting of user-dependent Walsh-Hadamard (WH) sequences of length \( N_c \). The maximum number of users, \( N_u \), equals \( N_c \). These orthogonal sequences make easy the separation of the different users at the base station. In fact in a multiuser scenario, each of the \( N_u \) users is assigned a unique spreading sequence. Each of the \( N_c \) components will be modulated on a different carrier using an IFFT of length \( N_F \). In order to avoid interference between successive MC-CDMA blocks, a cyclic prefix of \( \nu \) samples is used. The resulting transmitted samples \( s_{i,l} \) are given by:

\[ s_{i,l} = \frac{1}{\sqrt{N_c N_F}} \sum_{n=0}^{N_F-1} \sum_{k=0}^{N_F-1} a_{i,l} c_i N_c + n_i l e^{j 2 \pi k n / N_F} \]  

(7)

In uplink MC-CDMA, the transmitter of each user derives a clock signal and a carrier oscillator signal from a network synchronization reference signal [4]. Hence, the transmitter of user \( l \) has a user dependent carrier phase \( \theta_{c,l}(t) \) and clock phase \( \tau(t) \). The signal at the output of the transmit filter \( P(f) \) can be written as:

\[ s_l(t) = \sum_{i=-\infty}^{+\infty} \sum_{k=-\nu}^{N_F-1} s_{i,(N_F+\nu)+k,l} \times p(t - (k + i(N_F + \nu))T - \tau(t(N_F + \nu) + k,l)) \]  

(8)

where \( T \) is the transmit clock signal period.

The output of the dispersive channel is affected by the carrier phase difference

\[ \phi_l(t) = \theta_{c,l}(t) - \theta_c \]  

(9)
where $\theta_t$ is the carrier phase of the oscillator used by the basestation and $\theta_{c,i}(t)$ is the transmitter carrier phase.

The received signal disturbed by the (Additive White Gaussian Noise) AWGN is applied to the receiver filter and sampled. After removing the cyclic prefix, the receiver de-modulates the samples using FFT (Fast Fourier Transform) procedure. The output of the FFT is multiplied with the $g_{i,n}$ coefficients of the equalizer and the spreading sequence of the considered user $l$, yield the samples $z_{i,l}$ given by:

$$z_{i,l} = \sqrt{\frac{N_F}{N_F + \nu}} a_{i,l} I_{i,i,i,l} + \sqrt{\frac{N_F}{N_F + \nu}} \sum_{i' = -\infty}^{+\infty} a_{i',l} I_{i,i',i,l} + \sqrt{\frac{N_F}{N_F + \nu}} \sum_{n=0}^{N_u-1} \sum_{n'=-\infty}^{+\infty} a_{i,i',l,l} I_{i,i',i,l} + w_{i,l} \quad (10)$$

Where

$$I_{i,i',l,l'} = \frac{1}{N_c N_F} \delta_{l,i'} \sum_{n,n'=0}^{N_c-1} G_{N_c+n,i}^* G_{N_c+n',l} g_{i,n}$$

The quantity $I_{i,i',l,l'}$ represents the contribution of the symbol $a_{i',l'}$ to the output of the receiver of the $l'$th user during the $i$th MC-CDMA block.

In (10), the first contribution is the useful component. This contribution can be further decomposed into an average useful component $E(I_{i,i,i,l})$ and a zero mean fluctuation $I_{i,i,i,l} - E(I_{i,i,i,l})$ or self interference (SI). The second contribution ($i' \neq i$ and $l' = l$) is the intersymbol interference (ISI), caused by other symbols from the same user. The third contribution ($l' \neq l$) denotes the multiuser interference (MUI). The last contribution is the additive noise term.

### 4. Performance Degradation

In this section, we compute the degradation of the signal to noise ratio (SNR) when carrier phase jitter is present for uplink OFDMA system and uplink MC-CDMA system. The performance degradation of these systems is studied with Wiener phase jitter model. If $\text{SNR}(0)$ is the signal to noise ratio in the absence of carrier phase jitter, then, the degradation is given by:

$$\text{deg}_\text{s} = -10 \log \frac{\text{SNR}(\phi_t)}{\text{SNR}(0)} \quad (12)$$

where

$$\text{SNR}(0) = \frac{N_F}{N_F + \nu} \frac{E_{s,l}}{E[w_{i,l}^2]} \quad (13)$$

$E_{s,l}$ is the energy per symbol on the $l^{th}$ carrier; $E(|w_{i,l}|^2)$ is the variance of the AWGN and $\text{SNR}(\phi)$, is the signal to noise ratio in the presence of the carrier phase jitter, defined as the ratio of the power of the average useful component $P_{IUI,l}$ to the sum of the powers of self interference $P_{SI,l}$, intersymbol interference $P_{ISI,l}$, interuser interference $P_{IUI,l}$ and the additive noise AWGN. This yields:

$$\text{deg}_\text{s} = -10 \log \frac{P_{IUI,l}}{1 + \text{SNR}(0)(P_{SI,l} + P_{ISI,l} + P_{IUI,l})} \quad (14)$$

In the other hand the output samples $z_{i,l}$, given for both systems OFDMA and MC-CDMA in equation (5) and (10), are similar. So same reasoning procedures will be presented for these systems.

In order to investigate SNR degradation of uplink OFDMA and uplink MC-CDMA systems when Wiener phase jitter is present, we define different powers of useful component, self interference, intersymbol interference and interuser interference, as follows:

$$P_{u,l} = |E(I_{i,i,i,l})|^2$$

$$P_{SI,l} = E(|I_{i,i,i,l} - E(I_{i,i,i,l})|^2)$$

$$P_{ISI,l} = \sum_{j'= -\infty}^{+\infty} E(|I_{i,j',i,l}|^2)$$

$$P_{IUI,l} = \sum_{k=0}^{N_u-1} \sum_{l'=0}^{+\infty} \sum_{l = 0}^{+\infty} \sum_{k=0}^{+\infty} \sum_{l' = 0}^{+\infty} E_{s,l'} E_{s,l} E(|I_{i,i',l,l}|^2)$$

If now we concentrate on the degradation expression with different power terms to define, then we shall introduce the term $E(e^{j\phi})$ as the average of the carrier phase jitter modeled by Wiener distribution.

This average is given by:

$$E(e^{j\phi}) = \int_0^{2\pi} e^{j\phi} \rho_\phi(t) d\phi = \frac{1}{\sigma_\phi \sqrt{2\pi t}} \int_0^{2\pi} e^{j\phi} \frac{e^{-\frac{\phi^2}{2\sigma_\phi^2}}}{\sqrt{2\pi}} d\phi$$

(15)

Thank to the last expression and after simplification, the degradation expression can be written:

$$\text{deg}_\text{s} = -10 \log \frac{A(t)}{1 + \text{SNR}(0)(2 - A(t))} \quad (16)$$

where

$$A(t) = \frac{1}{\sigma_\phi \sqrt{2\pi t}} \int_0^{2\pi} e^{j\phi} \frac{e^{-\frac{\phi^2}{2\sigma_\phi^2}}}{\sqrt{2\pi}} d\phi \quad (17)$$

Mathematical studies show that expression (16) takes sense, if and only if, the temporal size ‘t’ do not exceed certain
value $t_0$. After this value, $A(t)$ is annulled and the degradation expression can’t be defined. Our simulation will, then, be done under the assumption that $t < t_0$.

5. Results and Interpretations

As we can deduce from equation (16), the performance degradation depends on jitter variance, SNR(0) and time. The first result is obtained by considering this degradation at a given time. For different time values, Figure 6 shows performance degradation of OFDMA and MC-CDMA systems as a function of Wiener jitter variance. This figure indicates that the degradation increases with increasing time $t$ and increasing jitter variance. In fact, for $\sigma^2_{\phi} = 0.01$, the degradation has increased more than 10 times. All these remarks are confirmed by Figure 7. In this figure, it is shown that SNR(0) affects the performance degradation of different waveforms studied. In deed, these performances degrade when SNR(0) increases. From Figure 6 and Figure 7, OFDMA and MC-CDMA performance degrade considerably with Wiener phase jitter model. With this stochastic process, we’d better not spend a long period of time in correcting the phase offset by a phase locked loop (PLL) which is frequency locked, in order not to have a visible and obvious degradation.

6. Conclusion

In this paper, we have considered a non stationary Wiener phase jitter model while discussing performance degradation of uplink OFDMA and uplink MC-CDMA systems. We pointed out strong time varying degradation, which depends also on jitter variance. In addition, It is shown that the non stationary system suffers from obvious performance degradation, not only, when jitter variance increases, but also, when the system is frequency locked for a long time. These ideas can be used by designers when developing behavioral models for PLL correction systems.

References


