On Manifold Structure of Cardiac MRI Data: Application to Segmentation

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Abstract

We develop theory and algorithms to incorporate image manifold constraints in a level set segmentation algorithm. This provides a framework to simultaneously segment every image of data sets that vary due to two degrees of freedom — such as cardiopulmonary MR images which deform due to patient breathing and heartbeats. We derive two formulations: a 4D level set which loosely couples the level set function between neighbors in the 2D image manifold and a multilayer level set function which uses different levels of the level set function to represent shapes that shrink or grow. We characterize the set of shape manifolds that the multilayer level set function can represent, and derive the evolution equations for both frameworks. We offer results of segmenting the left ventricle in cardiopulmonary MRI; by automatically discovering the 2D manifold structure of the image set then simultaneously segmenting every frame. Both extensions improve on frame-by-frame approaches, and a comparison of the results offers insight into their strengths and weaknesses.

1. Introduction

In cardiac imaging, the motion of the heart due to beating and the patient’s breathing is detrimental to most diagnostic methods. To correct for these two causes of motion, experimental protocols require that the patient holds his or her breath, and the images are only captured at the same phase of each heartbeat cycle. However, the breath-hold requirement is challenging for many of the sickest patients, and in some cases, the dynamics of the heart are of the greatest interest. Therefore, in this paper we develop pre-processing and segmentation tools for the analysis of free-breathing, ungated MR images.

Images which are taken with the same imaging geometry of the same patient have two principle degrees of freedom: the phase of the heartbeat and the phase of the breathing cycle. If pulse and respiratory data were available and synchronized with the images, then two images are likely to be similar if they have similar heart and breathing phases, even if they are temporally separated in the video sequence. When sufficiently many images are available, manifold learning algorithms, (typified by Isomap [16], Locally Linear Embedding (LLE) [14], and Semidefinite Embedding (SDE) [18]), are often effective at developing nonparametric representations of the 2D image manifold structure.

The main contributions of this paper are to describe and compare two natural extensions to Chan-Vese model [2] that exploit the structure of a 2D image manifold to improve image segmentation. We derive level set evolution equations for a 4D level set that loosely couples contours in neighboring manifold images [19] and a multilayer level set function that provides stronger constraints on the possible contour shapes. For ungated, free-breathing cardiac MRI data, we describe manifold learning methods to find a 2D embedding of the data. For many applications, the manifold structure may provide much more specific constraints than simply smoothness between neighbors. In particular, variations in the breathing phase lead to an approximately uniform translation of the heart, corresponding to a rigid translation of the (2D) shape segments. Alternatively, changes in the heartbeat phase lead to variations in the shape of the heart, but, largely, not its position.

For segmentation of MR imagery, several tools allow for the description of shapes within an image and support algorithms that automatically fit these shapes to image data. One such tool is level sets [4, 11], which represents 2D shapes as the zero-crossing of a surface. The standard evolution equations which drive the adaptation of the surface to the image data have very natural modifications which allow the level set to enforce manifold-based constraints. In experimental results, we find that these additional manifold constraints allow for segmentation of the left ventricle in images that are too low contrast to support single image segmentation.

The next section gives a brief background in manifold learning and its prior role in biomedical imaging, followed
by a description of the Chan-Vese framework for object segmentation. Section 3 derives new level set evolution equations that enforce manifold-based constraints on shape changes between images. Section 4 offers illustrative experiments on synthetic data and concludes with applications of segmenting low-contrast cine-MRI sequences of the left ventricle. These experiments illustrate the differences between our two proposed approaches, and that both improve on results using level sets on a frame-by-frame basis.

2. Background and Previous Work

This work integrates ideas from manifold learning and level set segmentation. To our knowledge, it is novel to combine these approaches. In order to ground our later presentation, we first introduce a very brief overview of manifold learning and recent research in the use of level sets in biomedical image analysis.

Manifold Learning, or nonlinear dimensionality reduction, offers tools to parameterize high-dimensional data that is drawn from a low-dimensional manifold. Principle Component Analysis (PCA) [7] and Independent Component Analysis (ICA) [6] are effective tools when the manifold is on or near a linear subspace, however many natural image sets, especially those of objects that move or deform do not lie near a low-dimensional linear subspace.

When these data are drawn from low-dimensional, nonlinear manifolds, recently proposed methods (e.g., Isomap, LLE, and SDE) generate low-dimensional coordinates of data that preserve locally defined properties of the high-dimensional data set. These algorithms have been used in various applications, including classification, recognition, tracking, and to a limited extent, biomedical image analysis [9].

2.1. Embedding of Cardiopulmonary Manifolds

For data sets which are sampled from a low-dimensional manifold which is topologically planar, all of the methods mentioned in the previous section provide similar results. We choose to employ the Isomap algorithm for learning these embeddings. The Isomap procedure for dimensionality reduction starts by computing the distance between all pairs of images (using some distance function such as SSD pixel intensities). Then, a graph is defined with each image as a node and undirected edges connecting each image to its \(k\)-closest neighbors (usually choosing \(k\) between 5 and 10). A complete pair-wise distance matrix is calculated by solving for the all-pairs shortest paths in this sparse graph. Finally, this complete distance matrix is embedded into some low dimension by solving an Eigenvalue problem (Multidimensional Scaling (MDS) [1]). The dimensionality of the embedding can be chosen as desired, but ideally is the number of degrees of freedom in the image set, in our case two (the two intrinsic dimensions of variability are the heartbeat and breathing).

Previous work that applies manifold learning to biomedical image analysis suggests modifying Isomap to use image distance functions other than pixel intensity differences [12, 15]. For data sets with deformable motion, the suggested distance function is computed as the phase difference of local complex Gabor filters:

\[
||I_1 - I_2||_{\text{motion}} = \sum_{x,y} \Psi(G_{(\psi,V,\sigma)} \otimes I_1, G_{(\psi,V,\sigma)} \otimes I_2) + \Psi(G_{(\psi,H,\sigma)} \otimes I_1, G_{(\psi,H,\sigma)} \otimes I_2)
\]

where \(G_{(\omega,\{V,H\},\sigma)}\) is defined to be the 2D complex Gabor filter with frequency \(\psi\), oriented either vertically or horizontally, with \(\sigma\) as the variance of the modulating Gaussian, and \(\Psi\) returns the phase difference of the pair of complex Gabor responses above some threshold \(\tau\); we choose \(\tau\) to be the 50-th percentile filter magnitude. Figure 1 shows four example images of the heart, illustrating both the non-rigid and the rigid deformations. The Isomap embedding computes a 2D coordinate \((u,v)\) for each original image. These coordinates correspond to the phase of the breathing cycle and the magnitude of the breathing deformation respectively. Even though Isomap does not intrinsically provide guarantees about the meaning of its output coordinate, we have found the parametrization of cardiopulmonary imagery to be consistent because the deformation due to breathing is much larger.

An even sampling of this manifold simplifies the numerical implementation of the level set segmentation in the subsequent sections. However, the given image sequence may not be evenly distributed in the manifold space. Ide-
ally, it is desirable to have a continuous image function \( f(x, y, u, v) \) to describe all possible cardiopulmonary images. One may interpolate the image function \( f \) locally by fitting a thin-plate smoothing spline to the given images \( \{ I_i | i = 1 \cdots n \} \) and their associated manifold position \( \{(u_i, v_i) | i = 1 \cdots n\} \), such that, for each pixel position \((x_j, y_j)\), \( f \) minimizes the following weighted sum:

\[
(1 - \alpha) \sum_{i=1}^{n} |I_i(x_j, y_j) - f^j(u_i, v_i)|^2
+ \alpha \int \left| \frac{\partial^2 f^j}{\partial u^2} \right|^2 + 2 \left| \frac{\partial^2 f^j}{\partial u \partial v} \right|^2 + \left| \frac{\partial^2 f^j}{\partial v^2} \right|^2 dudv \]

where \( \alpha \in [0, 1] \) is a smoothing parameter, and \( f^j \) corresponds to the image function defined on the pixel location \((x_j, y_j)\). We use the Matlab toolbox function \( tpatch \), which chooses this smoothing parameter “in an ad hoc fashion in dependence on the [data]" [10].

In Section 3, we use this manifold parameterization to help extend standard level set segmentation algorithms. These are introduced next.

2.2. Level Set Segmentation

In n-dimension space \( \Omega \), we define the evolving hypersurface \( C \) as the boundary \( \partial \Omega \) of regions of interest. We call \( \Omega^- \) the inside of \( C \) and \( \Omega^+ \) the outside of \( C \). For the cases of image segmentation, one approach is to define the contour as an energy minimization problem (the following presentation follows [2]):

\[
E(c^-, c^+, C) = \mu \cdot \text{Length}(C)
+ \lambda_1 \int_{\Omega^-} |I(x, y) - c^-|^2 dxdy
+ \lambda_2 \int_{\Omega^+} |I(x, y) - c^+|^2 dxdy \quad (2)
\]

where \( \mu, \lambda_1 \), and \( \lambda_2 \) are blending parameters. \( c^- \) and \( c^+ \) are constants depending on \( C \) and are usually the average of image intensity \( I(x, y) \) in the region \( \Omega^- \) and the outside \( \Omega^+ \), respectively. All parameter settings, such as \( \lambda_1 \), used in our experiments are listed in Section 4.2.

In problems of curve evolution, the level set method has been used extensively. Using the level set formulation, the boundary \( C \) is represented by the zero level set of a Lipschitz function \( \phi : \Omega \rightarrow \mathbb{R} \). Accordingly, the energy functional (2) can be written to evaluate the level set function \( \phi \) on the domain \( \Omega \):

\[
E(c^-, c^+, \phi) = \mu \int \delta_\varepsilon(\phi) |\nabla \phi| dxdy
+ \lambda_1 \int_{\Omega^-} |I - c^-|^2 (1 - H_\varepsilon(\phi)) dxdy
+ \lambda_2 \int_{\Omega^+} |I - c^+|^2 H_\varepsilon(\phi) dxdy \quad (3)
\]

where \( H_\varepsilon \) and \( \delta_\varepsilon \) are respectively the Heaviside and Dirac delta functions with a smoothed approximation of finite width \( \varepsilon \) [2].

The next section illustrates how to incorporate manifold constraints into the level set solution, including two modifications to the energy function \( E \) and the solutions for the corresponding evolution equations.

3. Level Set Segmentation on Image Manifolds

This section refines the standard Chan-Vese model for level set segmentation. Our goal is to segment all of the images in the data set simultaneously, and enforce additional constraints from the manifold structure. First we present a 4D level set formulation, that includes soft constraints on the contours on neighboring images in the manifold. Second, in order to provide stronger constraints on the segmentation, we derive a multilayer level set approach that defines the contour for every image using different level lines of a single level set function. In Section 4 we evaluate these approaches.

3.1. A 4D Level Set Approach

For cardiopulmonary image sequences, the images vary depending on their cardiac phase \( u \) and pulmonary phase \( v \) — the two degrees of freedom that parameterize the manifold shown in Figure 1. As described in Section 2.1, we automatically parameterize all the images and interpolate the result to generate evenly spaced samples of the image manifold \( f(x, y, u, v) \). Accordingly, the contour \( C \) that we seek is also a function of \( u \) and \( v \), and our goal is to describe \( C \) implicitly by the level set function \( \phi \) in 4-dimensional space \( \Omega \). Thus, a given cardiopulmonary image sequence specifies this contour by extending the energy functional (2) to four dimensions:

\[
\inf_{c^-, c^+, \phi} E_1(c^-, c^+, \phi), \quad \text{where} \quad \phi : \mathbb{R}^4 \rightarrow \mathbb{R} \quad (4)
\]

However, the manifold axes also correspond to specific kinds of deformation. The breathing of the patient results, approximately, in a translation of the heart. Therefore, we expect the variation of \( \phi \) in the \( v \) direction to be a uniform translation. That is, the shape deformation between neighboring images with respect to \( v \) should be consistent with a uniform translation. This induces a level set corollary to the classic optic flow constraint equation [5]:

\[
\frac{\partial \phi}{\partial x} \omega_x + \frac{\partial \phi}{\partial y} \omega_y + \frac{\partial \phi}{\partial v} = 0 \quad (5)
\]

where \((\omega_x, \omega_y)^T\) is the velocity vector that is constant over any given image, but may vary for different values of \( u \) and \( v \).
On the other hand, varying images along the other axis of the image manifold, deformations due to the cardiac cycle lead to image variation with minimal overall translation. For the special case of deformation caused by (non-uniform) heart expansion and contraction, we can express the constraint as:

$$\frac{\partial \phi}{\partial u} = \omega_u$$

(6)

where \(\omega_u\) is constant over the region of the heart for any given \(u\) and \(v\). This constraint enforces the condition that moving along the “heartbeat” axis simply adds or subtracts a constant value of the level set function \(\phi\) and therefore enforces that the shape either expands or shrinks.

Enforcing these constraints is natural within the level set framework; both lead naturally to new terms in the evolution of the \(\phi\) function. Computationally, at each time step of the temporal evolution \(\phi\), we compute three parameters for each sample image at the manifold coordinate \((u, v)\). First, we compute the least-squares estimates of the vector \((\omega_x, \omega_y)^\top\) that corresponds to global motion as we move from one image to another in the \(v\) direction on the image manifold. Second, we compute the constant \(\omega_u\) that corresponds to the speed of the expansion or contraction of the shape as we move from one image to another on the manifold along the \(u\) direction.

That is, for a particular \(u, v\) value, we compute first \(\omega_x(u, v), \omega_y(u, v)\) by calculating \(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}\) over the entire image, and solving the resulting linear system (from Equation (5)). Second, we compute \(\omega_u(u, v)\) at a particular point \(u, v\) as the mean value (over all \(x, y\)) of \(\frac{\partial \phi}{\partial u}\).

Once we have computed \(\omega_x(u, v), \omega_y(u, v),\) and \(\omega_u(u, v)\), we can write the motion constraints as an energy functional:

$$E_2(\Gamma, \phi) = \eta_1 \int_{\Omega} \left( \frac{\partial \phi}{\partial x} \omega_x + \frac{\partial \phi}{\partial y} \omega_y + \frac{\partial \phi}{\partial u} \right)^2 d\Omega$$

$$+ \eta_2 \int_{\Omega} \left( \frac{\partial \phi}{\partial v} - \omega_u \right)^2 d\Omega$$

(7)

where \(\Gamma\) represents the collection of \(\omega_x, \omega_y\) and \(\omega_u\) from all input images, and \(\eta_1\) and \(\eta_2\) are blending parameters. The first term enforces rigid changes in shape by penalizing regions of \(\phi\) where the \(x, y, v\) derivatives are not consistent with the translation motion, and the second term penalizes the overall mean translational motion of the heart, which is minimal when motion is caused only by the heartbeat.

One can integrate this motion constraint module (7) into the previously defined energy term (4):

$$E(c^-, c^+, \Gamma, \phi) = E_1(c^-, c^+, \phi) + E_2(\Gamma, \phi)$$

(8)

Given an initial level set function \(\phi_0\), we minimize the above functional (8) by iterating two steps, first using the current estimate of \(\phi\) to estimate \(c^-, c^+\) and solving for \(\omega_x(u, v), \omega_y(u, v),\) and \(\omega_u(u, v)\), and then evolving \(\phi\).

### 3.2. A 2D Multilayer Level Set Approach

In this section, we consider the case when the contours on the manifold can be represented by level lines of the same level set function \(\phi\). There is a considerable limitation on the set of shapes the level set can define; formally it requires that the regions inside the contours have a total order under the subset relation. That is, if the contours are ordered from “smallest” to “largest”, each contour must fit completely inside all larger contours. When this property holds true for the shapes to be segmented, the multilayer level set provides much stronger constraints for segmentation than the 4D level set.

In [3], the set of boundaries of a single image segmentation is represented implicitly by a multilayer of level lines of a continuous function. In the case of a set of the images of a deformable object, if the object boundaries in the different images can be represented by level lines of a single continuous level set function as in Figure 2, we can extend the multilayer level set approach to segmenting the deforming object simultaneously for all images.

Given the image \(I_i\) at the manifold position \((u_i, v_i)\), we assume that the object boundary \(C_i\) corresponds to the \(l_i\) level of \(\phi\). Since the object of interest may also translate between images, there is a shift between the object boundary and its corresponding level line of \(\phi\). We denote by \(\Delta_i = (\Delta x_i, \Delta y_i)^\top\) this translation between the object boundary \(C_i\) and its associated level line \(l_i\) of \(\phi\).

Similar to [3], we extend the binary piecewise-constant level set segmentation model to the following energy functional \(E(c^-, c^+, \phi)\) as a level set form:

$$E(c^-, c^+, \{l_i, \Delta_i\}, \phi) = \mu \cdot \text{Area}(\phi)$$

$$+ \lambda_1 \sum i \int_{\Omega_i} |I_i(x + \Delta_i) - c^-|^2 (1 - H_\epsilon(\phi - l_i)) dx$$

$$+ \lambda_2 \sum i \int_{\Omega_i} |I_i(x + \Delta_i) - c^+|^2 H_\epsilon(\phi - l_i) dx$$

(9)

where \(x\) denotes the image coordinate \((x, y)^\top\). The first
term $\text{Area}(\phi)$ corresponds to the surface area of $\phi$, penalizing the perimeter of each level line and leads to a smooth function $\phi$ [8].

On the image manifold, object boundaries are similar for nearby images. Accordingly, for nearby images, their associated level lines should be also close to each other on the level set function. To approximate this property on the manifold, we model the level value $l$ as a function of $u$ by a cubic b-spline curve as the following matrix form:

$$l(u) = \frac{1}{6}[\bar{u}^3, \bar{u}^2, \bar{u}, 1] \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} p_{i-1} \\ p_i \\ p_{i+1} \\ p_{i+2} \end{bmatrix}$$

where $P = \{p_i, i = 1 \cdots m_1\}$ represents the b-spline control points which parameterize the level values, and $\bar{u} \in [0, 1]$ denotes the relative position of $u$ between $[p_i, p_{i+1}]$. For the sake of simplicity, we rewrite the above matrix form as:

$$l(u) = N_i(u)[p_1, p_2, \cdots, p_{m_1}]^\top$$

where $N_i(u)$ is a $m_1$-dimensional row vector in terms of $u$.

Similarly, we also model the shifts $\{\Delta_i\}$ as a function of $v$ by a cubic b-spline curve, which we define as follows:

$$\Delta(v) = N_\Delta(v)[q_1, q_2, \cdots, q_{m_2}]^\top$$

where $Q = \{q_i, i = 1 \cdots m_2\}$ represents a set of 2D control points which parameterize the shifts, and $N_\Delta(v)$ is a $m_2$-dimensional row vector in terms of $v$. The number of control points determines the degrees of freedom of the curve. Thus, selecting more control points allow for more local variance of level values $\{l_i\}$ or shifts $\{\Delta_i\}$, while a small number of control points imposes a stricter global constraint.

Finally, taking into account the neighborhood constraints on the image manifold, a given image set specifies the object contours by minimizing the following energy functional:

$$\inf_{c^-, c^+, \phi, P, Q} E(c^-, c^+, \phi, P, Q), \text{where } \phi : \mathbb{R}^2 \to \mathbb{R}$$

Minimizing the corresponding energy $E$ alternatively with respect to the unknowns, yields the associated Euler-Lagrange equations, parameterizing the descent direction by an artificial time $t$

$$\phi(0, x) = \phi_0(x)$$

$$\frac{\partial \phi}{\partial t} = \sum_i \delta_i \left( \lambda_1 |\hat{I}_i - c^-|^2 - \lambda_2 |\hat{I}_i - c^+|^2 \right) + \mu \text{div}(\nabla \phi / \sqrt{\text{div}^2 \phi + 1})$$

$$\frac{\partial p_j}{\partial t} = -\sum_i N^j_i(u_i) \int_{\Omega_i} \delta_i \left( \lambda_1 |\hat{I}_i - c^-|^2 - \lambda_2 |\hat{I}_i - c^+|^2 \right) \ dx + \lambda_2 |\hat{I}_i - c^+|^2 \nabla \hat{I}_i \ dx$$

where $H_i$ denotes $H_i(\phi - l_i)$, $\delta_i$ represents $\delta_i(\phi - l_i)$, $\hat{I}_i$ is corresponding to $I_i(x + \Delta_i)$, and $N^j$ denotes the $j$th element of the vector $N$. The parameter $c^-$ and $c^+$ can be obtained by simply taking intensity average of the inside regions $\{\Omega_i^-\}$ and the outside regions $\{\Omega_i^+\}$ respectively.

### 4. Experimental Results

This section describes results of our system which implements the constraints defined in the last section. We first consider a collection of artificially generated images in order to show the power of the multilayer level set segmentation in the presence of significant noise. We follow this with an application to finding the left ventricle wall shape in a noisy cardiac MRI sequence. In both cases, we set the blending parameter $\mu = 0.1$. The level set parameters $\lambda_1$ and $\lambda_2$ determine the importance of matching the intensity estimate for the inside and outside of the contour and are set to $\lambda_1 = \lambda_2 = 1$. We use $\epsilon = 1$ for approximating the Heaviside function $H_i$, and Dirac function $\delta_i$. For the multilayer level set method, we use five control points for parameterizing level values $\{l_i\}$ and four control points for modelling shifts $\{\Delta_i\}$. Deriving optimal choices for these parameters or other methods to automate the process for finding good parameters is the subject of continuing research.

#### 4.1. Simulation Data

We constructed an artificial data set by defining a shape and deforming it through a composition of a non-rigid deformation and a rigid translation along diagonal direction. Thus, this data set has a 2D manifold structure indexed by the magnitude of each deformation. One hundred images were created and each was then corrupted by additive white Gaussian noise and the introduction of small random white patches. Figure 3 depicts the shape deformation (left) and eight selected frames among the 100 generated images (top right).
Figure 3. Artificial data set generated by composing a non-rigid shape deformation (left) and a diagonal translation. On the right shows eight examples from 100 generated images with SNR 10dB. Random white patches are added to the images to make the segmentation more challenging.

Figure 4. The plot on the left shows the iso-contours of the estimated level set function $\phi$. The right shows example segmentation results from our multilayer level set approach. Without being informed by the image manifold, conventional level set methods may fail in this data set.

The noise in the image and the random patches make this a challenging data set for traditional level set approaches to converge to the correct boundary. Introducing a shape prior could possibly improve these results for this data set [13, 17]. However, for this work, it is our goal to illustrate the advantages of using the manifold structure of these images.

The first row of Figure 4 shows the iso-contours of the final estimate of $\phi$. The second row gives the contours which are the results of applying our algorithm, which exploits the manifold structure of these images. Note our proposed method is very robust to the added noise. Conventional level set methods fail to detect the correct object boundaries.

4.2. Left Ventricle Segmentation

We applied the 2D multilayer level set approach to segment the left ventricle from the data set shown in Figure 1. Since the cardiac phase $u$ on the manifold is periodic in the range $[0, 2\pi]$, we enforced this periodicity by forcing the two end points of b-spline curve to have the same value and derivative. Figure 5 show examples of the segmentation result for eight consecutive frames.

We also tested the 4D level set method on the this data set. We constructed the image function $f(x, y, u, v)$ of size $40 \times 40 \times 10 \times 10$, as shown in Figure 1, to regularly sample the manifold. The blending parameters $\eta_1$ and $\eta_2$ determine the importance of manifold constraints, and are set to $\eta_1 = \eta_2 = 0.1$. The initial level set $\phi_0(x, y, u, v)$, for each image, is defined by the signed distance function to the circle of radius 6 centered at the image center, with points inside the circle having a negative value. The segmentation result looks qualitatively similar to the one obtained from the multilayer level set.

We found that the manifold structures are most important for images that are especially low contrast or noisy. We compared the results from single-image level set segmentation with our proposed methods of a multilayer level set and a 4D level set. Figure 6 gives examples of images where the manifold based solutions differ significantly from the single image solution. In the first two cases, the manifold constraints show a significant improvement — the single image solutions are incorrect because of insufficient contrast. The last two cases show segmented shape boundaries that are different than the single image segmentation, and which may reflect more accurately the correct boundary, although it is difficult to quantify the improvement (see details on the figure).

The results for both methods look qualitatively similar, but a more quantitative comparison is possible by considering properties of the segmentation of the entire data set.
Figure 7. This figure plots the area inside the detected contour versus image frame number for the 2D multilayer level set method (red “+”), and the 4D level set method (blue “o”). The multilayer level set method gives results that are more consistent from beat to beat, because it provides a stronger constraint between images at the same part of the heartbeat cycle.

Figure 7 shows the area inside the detected contour for each frame plotted versus the image frame number (which corresponds to the temporal order). The plot from the multilayer level set method is more regular, while the contours obtained by the 4D level set tend to have more variations. One possible explanation is that the multilayer level set imposes a strong constraint on the contour shapes. The deformations in our data set are very well approximated by translations and (perhaps non-uniform) shrinking and expansion of the contour; so the segmentation is more robust with low-contrast, noisy images. Additionally, our implementation of the 4D level set requires a regular sampling of the image manifold, which is difficult or impossible to obtain if there is significant deformation or translation between neighboring samples on the manifold.

Summary. This work presents efforts towards integrating manifold learning and the simultaneous segmentation of large image sets. The natural applications of this may lie primarily in medical imaging, particularly cardiopulmonary images. However, variations of this approach may be applicable to any application domain for which there is a manifold structure to the data and may be extended to other computational shape representation tools.

References