A Frequency Error Resistant Blind CDMA Detector

Javier Romero-García, Riccardo De Gaudenzi, Senior Member, IEEE, Filippo Giannetti, and Marco Luise, Senior Member, IEEE

Abstract—This paper presents an enhanced version of the recently-proposed blind anchored interference-mitigating detector (BAID) [1] for code-division multiple-access transmission. Such a detector, named extended complex BAID (EC-BAID), features invariance to a phase error on the useful channel’s carrier and reveals resistant to a large frequency shift (e.g., Doppler shift) on interfering carriers. The EC-BAID is also shown to be insensitive to residual carrier frequency errors on the desired channel which is lower by three orders of magnitude with respect to the case of a data-aided minimum mean-square-error receiver. The performance of the EC-BAID is computed theoretically, and is validated by computer simulations under a variety of system configurations.

Index Terms—Adaptive equalization, CDMA, multiuser detection.

I. INTRODUCTION

The recently-proposed low-complexity linear blind anchored interference-mitigating detector (BAID) [1] minimizes the detrimental effect of multiple-access interference (MAI) on the bit-error rate (BER) performance of a code-division multiple access (CDMA) demodulator without requiring training sequences nor knowledge of interfering signals’ parameters. The BAID adaptively minimizes the detector mean output energy (MOE) and only requires side information about the useful channel signature sequence and chip timing, just like a conventional direct-sequence spread-spectrum (DS/SS) correlation receiver (CR). The blind linear MOE detector performance is affected by large replica code timing errors and/or frequency selective fading channels causing the so-called mismatch effect [1]. The issues of carrier phase offset recovery and sensitivity of the detector to a large frequency offset on the MAI were mentioned in [3]. We also note in margin that a promising extension of the blind MOE detector to the frequency selective channel was presented in [2], but with no mention to the issues above.

This letter deals with a further-enhanced version of the original BAID algorithm suited for (satellite) flat fading channels, named extended complex BAID (EC-BAID) that reveals robust in the presence of interfering signals affected by a residual carrier frequency shift. The new detector is also insensitive to a carrier phase offset on the useful signal. This allows the adoption of conventional carrier phase estimators operating at symbol rate on the output of the detector [3]. As a consequence of this property, it is also found that the sensitivity of the EC-BAID to a residual carrier frequency error on the desired signal is considerably smaller than that of the data-aided minimum mean-square-error (DA-MMSE) receiver [4]. The detector is analyzed for different signal formats, and it is shown that, in the absence of phase synchronization errors, QPSK modulation with real-valued spreading outperforms all of the others.

II. COMPLEX EXTENDED-BAID ADAPTIVE DETECTOR

A. CDMA Signal Format

The signal format we introduce here is quite general and corresponds to DS/SS with two-dimensional modulation. In the most general case, the incoming binary data stream at rate $R_b = 1/\tau_s$, for the $k$th user is split between the two phase-quadrature (P-Q) rails by means of a serial-to-parallel converter. The resulting symbols $a_k,p(u), a_k,q(u) \in \{-1, 1\}$, are independently spread by the P-Q signature sequences $c_k,p(l), c_k,q(l)$ (both with periodicity $L$) and filtered prior P-Q carrier modulation. The resulting complex $k$th signal is given by

$$c_k(t) = \sqrt{D P_k} \sum_{u=-\infty}^{\infty} [a_k,p(u) s_k,p(t - u T_s - \tau_k)$$

$$+ a_k,q(u) s_k,q(t - u T_s - \tau_k)] \times \exp\{j(2\pi f_k t + \phi_k)\}$$

$$s_k,h(t) = \sum_{l=1}^{L} c_k,h(l) g_T(t - l T_s), \quad h = p, q$$

(1)

where $D$ is an amplitude factor related to the signal modulation dimensionality (see Table I), $P_k$ is the $k$th signal power, $L$ is the period for both spreading sequences, $T_s$ is the chip time, $T_s = L T_c$ is the symbol time, $\Delta f_k$ is the $k$th carrier frequency offset with respect to the nominal frequency $f_0$, $\phi_k$ is the $k$th user carrier phase, $g_T(t)$ is the impulse response of the chip shaping filter, and $\tau_k$ represents the $k$th user signal delay. Without loss of generality, we also assume $0 \leq \tau_k < T_s$. 

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The signature sequences $c_{k,i}(l)$ may be composite sequences\(^1\) such as Walsh Hadamard (WH) functions overlaid by extended pseudo-noise (PN) scrambling sequences with the same period and start epoch\(^2\). We also assume short codes, i.e., $T_s = LT_c$ in order for the EC-BAID to be applicable. The code length $L$ is thus coincident with the spreading factor $T_s/T_c$.

Equation (1) represents a variety of modulation and spreading formats as summarized in Table I. On the one hand, d-BPSK and QPSK-RS double the available codebook size for orthogonal sequences with no bandwidth penalty; on the other, d-BPSK provides increased robustness with respect to QPSK-RS in case of carrier phase errors\(^3\).

**B. Receiver Front-End Analysis**

Assume for the moment that the carrier frequency error for the useful channel 1 is perfectly compensated for by means of an ideal automatic frequency control (AFC) subsystem\(^4\) (i.e., $\Delta f_1 = 0$). The received signal will go through a baseband filter $g_R(t)$ performing Nyquist square-root raised-cosine chip matched filtering (CMF), followed by chip-time sampling (or interpolation in a digital modem). We will also momentarily assume $\gamma_1 = 0$, that is, perfect chip timing recovery.

We denote with $\nu(t) = n_{p}(t) + n_{q}(t)$ the (baseband equivalent) AWGN process, whose $I/Q$ components $n_{p}(t)$ and $n_{q}(t)$ have two-sided power spectral density $N_0$. The signal samples at the output of the CMF $g_R(t)$ at time $t_m = m \cdot T_c$ are

$$y(m) = \left[ \sum_{k=1}^{K} c_{k}(l) + \nu(t) \right] \otimes g_R(t) \bigg|_{t_m = m \cdot T_c}$$

$$\simeq \sum_{k=1}^{K} \sum_{l=-L}^{L} \frac{1}{\sqrt{2P_k}} \cdot [a_{k,p}(i + [m/L]J + i_{k,p}(mT_c - iT_s - [m/L]T_s) + a_{k,q}(i + [m/L]J) + i_{k,q}(mT_c - iT_s - [m/L]T_s)] \cdot e^{j[2\pi f_k T_c + \phi_k]} + n(mT_c)$$

$$y^c(r) = \left[ y^c_T(r), y^c_0(r), y^c_1(r) \right]^T$$

$$\mathbf{y}_{\nu}(r) = [y((r+w)T_c), \ldots, y((r+w)T_c + L - 1)T_c]^T$$

where $[\cdot]$ denotes integer part and $f_{k,h}(t) = s_{k,h}(t - \tau_k) \otimes g_R(t - \tau_k), h = p, q$. The discrete-time complex-valued white Gaussian noise process $n(mT_c) \triangleq n_{p}(mT_c) + n_{q}(mT_c)$ has zero-mean white independent real/imaginary components with variance $\sigma_{n_c}^2 \triangleq E\{n_c^2(m)\} = N_0 T_c, h = p, q$. Now, the description of the detector is simplified by adopting a vector notation (boldface symbols). We will deal first for simplicity with BPSK-RS and we will extend our consideration to bidimensional signal formats in Section III. The EC-BAID uses a three-symbol observation window to detect one information-bearing symbol. The $3L$-dimensional array of CMF samples observed by the detector is

$$\mathbf{y}^c(r) = \sum_{k=1}^{K} \sum_{l=-2}^{1} a_{k,r}(i + \mathbf{p}_{k,r}(r)) + n^c(r)$$

$$\mathbf{p}_{k,i}(r) \triangleq \mathbf{z}_{k,i}(r) \exp(j2\pi \Delta f_k rT_c)$$

$$\mathbf{z}_{k,i} \triangleq \left[ \mathbf{z}_{k,i+1}, \mathbf{z}_{k,i}^T, \mathbf{z}_{k,i-1}^T \right]^T$$

$$\mathbf{n}^c(r) = \left[ n^c_T, n^c_0, n^c_1 \right]^T$$

$$\mathbf{z}_{k,i} \triangleq \left\{ \begin{array}{ll} 0, & \text{for } i = 2; i = -3 \\
\lambda_{i,k}(0), & \text{for } i = -2, -1, 0, 1 \\
\lambda_{i,k}(L - 1), & \text{for } i = 2, -1, 0, 1 \\
\end{array} \right.$$

$$\lambda_{i,k}(l) \triangleq \sqrt{2P_k} s_{k,h}(lT_c - iT_s) \exp(j2\pi f_k T_c + \phi_k)$$

$$\mathbf{n}_{\nu}(r) = \left[ n_{\nu}^c_T, n_{\nu}^c_0, n_{\nu}^c_1 \right]^T$$

$$\mathbf{n}(r) = [n((r+w)T_c), \ldots, n((r+w)T_c + L - 1)T_c]^T$$

$$w = -1, 0, 1.$$

It is also handy to introduce the $3L \times 4K$ complex matrix $\mathbf{Z}^c$ that groups all of the user vectors

$$\mathbf{Z}^c = \left[ \mathbf{z}_{k,-2}, \mathbf{z}_{k,-1}, \mathbf{z}_{k,0}, \mathbf{z}_{k,1}, \ldots, \mathbf{z}_{K,-2}, \mathbf{z}_{K,-1}, \mathbf{Z}_{k,0}, \mathbf{Z}_{k,1} \right].$$

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\(^1\)Composite sequences are necessary in multibeam, multisatellite systems, or cellular terrestrial systems wherein the scrambling PN is beam/sector unique, and different WH signature sequences are assigned to each different user within the same beam/sector.

\(^2\)This limiting assumption will be removed later in Section III, when applying with numerical results.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Modulation</th>
<th>Spreading</th>
<th>$D$</th>
<th>$T_s/T_c$</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK-RS</td>
<td>BPSK</td>
<td>real (RS)</td>
<td>2</td>
<td>1</td>
<td>$a_{k,q} = 0$  \hspace{1cm} $c_{k,q} = 0$</td>
</tr>
<tr>
<td>BPSK-CS</td>
<td>BPSK</td>
<td>complex (CS)</td>
<td>1</td>
<td>1</td>
<td>$a_{k,q} = a_{k,q}$ \hspace{1cm} $c_{k,q} \neq c_{k,q}$</td>
</tr>
<tr>
<td>d-BPSK</td>
<td>2×BPSK</td>
<td>2×real</td>
<td>2</td>
<td>1</td>
<td>$a_{k,q} \neq a_{k,q}$ \hspace{1cm} $c_{k,q} = c_{k,q}$</td>
</tr>
<tr>
<td>QPSK-RS</td>
<td>QPSK</td>
<td>real (RS)</td>
<td>1</td>
<td>2</td>
<td>$a_{k,q} \neq a_{k,q}$ \hspace{1cm} $c_{k,q} \neq c_{k,q}$</td>
</tr>
</tbody>
</table>
The steady-state configuration of the linear detector and its performance are directly related to this matrix, as shown in Section III. The relevant system parameters (i.e., power, delay, distribution of frequency and phase offsets, and chip shaping) are embedded in $\bf{Z}$ as well. In particular, the samples of the desired user’s signature sequence are contained in $\bf{Z}_{\Delta t}^0$.

C. EC-BAID Detector Outline

The EC-BAID is just a linear detector operating on the chip-rate sampled received signal $\bf{y}(m)$ to yield the symbol rate signal $b_1(r)$ as follows:

$$b_1(r) = \frac{1}{L} \bf{x}_1^T(r) \cdot \bf{y}^e(r)$$  \hspace{1cm} (6)

where $\bf{x}_1^T(r)$ is the $3L$-dimensional array of the complex-valued detector coefficients. It is apparent that detection of each symbol calls for observation of three symbol periods (i.e., the current, the leading, and the trailing ones). This suggests the threefold pipelined parallel implementation of the detector sketched in Fig. 1, wherein the first unit processes the $(r-1)$th, the $r$th and the $(r+1)$th symbol periods for the detection of the $r$th symbol, the second unit processes the $r$th, the $(r+1)$th, and the $(r+2)$th periods for the detection of the $(r+1)$th symbol, and the third unit processes the $(r+1)$th, the $(r+2)$th, and the $(r+3)$th periods, for the detection of the $(r+2)$th symbol. The output data stream is obtained by sequentially selecting at rate $1/T_s$ one of the three EC-BAID detector outputs by means of a multiplexer unit. We need thus a further clock reference ticking at the so-called super-symbol rate $R_{ss} = 1/(3T_s)$, i.e., once every three symbols. Using the unique decomposition $r = 3s+n$ ($1 \leq n \leq 3$), the $n$th EC-BAID output sample ($n = 1, 2, 3$) can be computed as

$$b_1(3s+n-1) = \frac{1}{L} \bf{x}_1^T(s) \cdot \bf{y}^e(3s+n-1)$$  \hspace{1cm} (7)

with $s$ running at super-symbol rate. To achieve blind adaptation the complex detector coefficients are anchored to the user’s signature sequence [1], [3]

$$\bf{h}_1^{e+n}(s) = c_1^e + x_1^{e+n}(s)$$

$$\bf{c}_1^e = \begin{bmatrix} c_1 \\ 0 \\ \end{bmatrix}$$

$$\bf{x}_1^{e+n}(r) = \begin{bmatrix} x_1^{n-1}(r) \\ x_1^n(r) \\ x_1^{n+1}(r) \\ \end{bmatrix}$$  \hspace{1cm} (8)

with the “anchor” constraint $c_1^T \cdot \bf{x}_1^e = 0$. The error signal for detector $n$ is

$$\bf{e}_1^{e+n}(s) = b_1(3s+n-1) \left[ \bf{y}^e(3s+n-1) \right.$$

$$- \left. \frac{\bf{y}^e(3s+n-1)^T \cdot c_1^e}{L} \right]$$  \hspace{1cm} (9)

where the asterisk denotes complex conjugation. If the three detectors were running independently, the updating equation for each detector would be simply

$$\bf{x}_1^{e+n}(s+1) = \bf{x}_1^{e+n}(s) - \gamma \bf{e}_1^{e+n}(s)$$  \hspace{1cm} (10)

with $s$ ticking at super-symbol rate. In practice, it is expedient to combine the error signals generated by the different EC-BAID
III. EC-BAID PERFORMANCE

Let us now focus on the analytical derivation of the mean square error (MSE) at the detector output. We will not dwell on the different arrangements of the EC-BAID for the modulation/spreading schemes listed in Table I [3], [7]. Instead, we directly consider the complex-valued detector output (6) which can be reformulated as

\[ b_1(r) = \frac{1}{L} \textbf{h}_1^T \textbf{z}_{r,0} r_1(r) + [i_p(r) + i_q(r)] + [v_p(r) + v_q(r)] \]

(12)

where \( i_p(r) \) and \( i_q(r) \) represent the residual MAI terms at the output of the in-phase and quadrature detectors, respectively, while \( v_p(r) \) and \( v_q(r) \) are the in-phase and quadrature filtered units to speed up detector convergence and adaptation noise. This may be done for instance (see [6] for details)

\[ x_i(s)+1 = x_i(s)-\gamma \left[ e_i^{-1}(s-1) + e_i^{-2}(s-1) + e_i^{-3}(s-1) \right]. \]

(11)
Fig. 3. (a) BER versus $\Delta f_0 \cdot T_s$ for EC-BAID detector: BPSK-RS, $L = 64, K = 19, [C/f]_{sc} = -6$ dB, (b) BER versus $\Delta f_1 \cdot T_s$ for different detectors: BPSK-RS, $L = 64, K = 19, [C/f]_{sc} = -6$ dB.

AWGN samples. The MSE of the P component can be computed as follows:

$$c_p = E[|v_p(r)+\nu_p(r)|^2] = \frac{1}{2L^2}h_1^T F h_1 + \frac{1}{2L^2}R\{h_1^T G^* h_1\} - \frac{1}{L^2}R\{h_1^T z_{1,0}\}^2.$$ (13)

In a similar way for the Q rail, we obtain

$$c_q = \frac{1}{2L^2}h_2^T F h_2^* - \frac{1}{2L^2}R\{h_1^T G^* h_1\} - \frac{1}{L^2}R\{h_1^T z_{1,0}\}^2.$$ (14)

where $h_1 = x_1 + c_1 = \frac{I}{c_1^T F c_1} c_1$ and $F(\Delta r) = \Delta r^T F \Delta r + \sigma_r^2 I$.

$$G^* = \sum_{k=1}^{K} \sum_{l=-2}^{2} \delta_{\Delta w_k} \delta_{\Delta w_l} P_k P_l T.$$ (15)

whereby $\delta_{\Delta w_k} = 1$, for $\Delta w_k = 0$, 0 otherwise. For two-dimensional schemes like d-BPSK with different P-Q spreading
sequences, we have the two vectors $\mathbf{h}_P^0$ and $\mathbf{h}_Q^0$ for the two detectors operating on the P and the Q channels, respectively, whose output samples are $\theta_P^0$ and $\theta_Q^0$. Assuming perfect carrier phase offset compensation, the binary decisions on the P and Q rails are

$$\theta_P^0(r) = \text{sign} \{ \Re \{ \mathbf{b}_P^0(r) \} \} \quad \theta_Q^0(r) = \text{sign} \{ \Im \{ \mathbf{b}_Q^0(r) \} \}.$$  

Introducing the total signal-to-noise ratio at the EC-BAID output of the rail $l$ ($l = p, q$) as

$$\rho_l^0 \triangleq |\mathbf{h}_l^0|^2 \sigma^2_l/(2N_0),$$

assuming Gaussian statistics for the residual error on $\mathbf{b}_l^0$ [3], the resulting probability of error for the single rail $l$ is given by

$$P_e^l \approx Q(\sqrt{2\rho_l}), \quad l = p, q,$$  

where $\epsilon_P$ and $\epsilon_Q$ are computed by means of (13) and (14) on $\mathbf{h}_P^0$ and $\mathbf{h}_Q^0$, respectively, and $Q(x)$ represents the Gauss integral function. The resulting BER for mono- (1-D) and two-dimensional (2-D) modulation formats are, respectively

$$P_e^{1D} = P_e^p, \quad P_e^{2D} = \frac{1}{2} (P_e^p + P_e^q).$$  

### IV. NUMERICAL RESULTS

We will show in this section some numerical performance results obtained both by theory (Section III) and by computer simulation. The simulation model assumes $K$ independent DS-CDMA signals with square-root raised-cosine chip shaping and rolloff factor 0.2. The signature sequences are composed by an inner $L$ chip-long WH signature sequence overlaid by an extended PN sequence with the same period. When not otherwise specified, we assume 18 equally-powered interfering signals with a useful carrier-to-single interferer power ratio, $C/I_{\text{sc}} = -6$ dB, that corresponds to an overall $C/I = [C/I]_{\text{sc}}/K = -18.5$ dB.

Fig. 2(a) shows the probability density function (pdf) of $\rho$ for the CR, the C-BAID, and the EC-BAID, with random interferers’ carrier phases and code delays. The values of $\rho$ at the output of the CR exhibit a relatively large dispersion, due to the different delay and carrier phase assignments on the active channels. The effect of the interference-mitigating detectors is twofold. First, they obviously increase the mean value of $\rho$ so as to improve the mean quality of the link; second, they reduce the dispersion of $\rho$ around its mean value, and this has the effect of reducing the outage probability of the link as well. This attractive feature of the EC-BAID is further enhanced by the use of d-BPSK or QPSK-RS signal formats. Fig. 2(b) shows simulation results for the pdf of $\rho$ in the case of BPSK, d-BPSK, and QPSK. The occupied bandwidth being equal, d-BPSK/QPSK-RS allow the use of signature sequences having a repetition period which is twice that of the codes of the BPSK-RS or BPSK-CS cases. This doubles the codebook size (i.e., the maximum number of sequences available) and yields a two-fold capacity. Looking at Fig. 2(b), it can be also observed that, though BPSK and d-BPSK have similar average values of $\rho$, the variance is reduced for the latter. As for QPSK-RS, the average $\rho$ is higher and the deviation $\sigma_\rho$ is lower than for d-BPSK. In fact, in QPSK-RS each signal employs only one spreading sequence, while in d-BPSK two sequences are required for every user. This feature reduces the number of space dimensions occupied by the CDMA multiplex, thus enhancing the EC-BAID interference mitigation capability.

For the numerical results to follow, we assumed a worst case in which all the interferers are in-phase with the desired user signal [i.e., $\phi_k = 0$, $\forall k$ in (2)]. Also, a deterministic delay $\tau_k$, ...
(with \(0 \leq \tau_k < T_s, 1 \leq k \leq K\)) was assigned to every active user. The values of the delays were empirically selected, during preliminary trials, so as to provide a BER curve very similar to the average taken on all of the possible BER curves we would get by randomly assigning the delays to the \(K\) users. Furthermore, in order to reduce the computational effort, the following numerical results will be provided for the BPSK case only. The value of the adaptive gain \(\gamma\) used in the simulations was selected as a compromise between acquisition time and steady-state performance as follows: \(\gamma_{\text{C-BAID}} = 3 \times 10^{-4}\) and \(\gamma_{\text{EC-BAID}} = 1.2 \times 10^{-4}\).

One point in favor of the detector using complex-valued coefficients, is the invariance of its MSE performance in the presence of interfering signals with a frequency offset \(\Delta f\) with respect to the useful channel carrier [i.e., \(\Delta f_1 = 0\) and \(\Delta f_k = \Delta f, k = 2, \ldots, K\), in (2)]. Such offsets can be quite large compared to the symbol rate in a low- or medium-earth orbiting (LEO or MEO) multisatellite network. This important feature of the EC-BAID is testified by Fig. 3(a) for several values of \(E_b/N_0\).

The adaptation speed of the detector is limited by the step size \(\gamma\) in the updating equations. This is an issue when the desired user’s signal is affected by a small carrier frequency offset \(\Delta f_1\) in the receiver front-end. This problem is exemplified in Fig. 3(b) for EDA-MMSE algorithm,\(^3\) where the BER approaches 0.5 when the normalized frequency offset of the useful user is higher than \(10^{-6}\). The EC-BAID algorithm turns out to be more robust to frequency errors, presumably because it operates in a space “orthogonal” to the useful signal. The EC-BAID endures normalized frequency errors on the useful channel up to \(10^{-3}\), i.e., three orders of magnitude larger than those tolerated by the EDA-MMSE detector.

Finally, we evaluated the effect of a timing error on the useful signal [i.e., \(\tau_i \neq 0\) in (2)] and the relevant results are plotted in Fig. 4 for different detector types. As expected, the MMSE algorithms exhibit a better behavior than the blind detectors. In fact, the latter are forced by the anchoring condition to use \(c_i^T\) for signal detection, even if the presence of a timing error would suggest a change to this reference. On the contrary, the MMSE detectors have more freedom in synthesizing a set of coefficients which are matched to the condition of a received delayed signal with \(\tau_i \neq 0\). The standard deviation of the chip error required by the (E)C-BAID is about 0.05–0.1 chip intervals, which is anyway typical of conventional chip timing estimators, provided that the power unbalance between the CDMA signals is not so huge.

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**REFERENCES**


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\(^3\) Also in this case ‘E’ stands for ‘Extended’ and indicates that the observation window of the DA-MMSE algorithm encompasses three symbol intervals.