Controlling the ratchet effect through the symmetries of the systems: Application to molecular motors

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Abstract

We discuss a novel generic mechanism for controlling the ratchet effect through the breaking of relevant symmetries. We review previous works on ratchets where directed transport is induced by the breaking of standard temporal symmetries \( f(t) = -f(t + T/2) \) and \( f(t) = f(-t) \) (or \( f(t) = -f(-t) \)). We find that in seemingly unrelated systems the average velocity (or the current) of particles (or solitons) exhibits common features. We show that, as a consequence of Curie’s symmetry principle, the average velocity (or the current) is related to the breaking of the symmetries of the system. This relationship allows us to control the transport in a systematic way. The qualitative agreement between the present analytical predictions and previous experimental, numerical, and theoretical results leads us to suggest that for the given breaking of the temporal symmetries there is an optimal wave form for a given time-periodic force. Also, we comment on how this mechanism can be applied to the case where a ratchet effect is induced by breaking of spatial symmetries. Finally, we conjecture that the ratchet potential underlying biological motor proteins might be optimized according to the breaking of the relevant symmetries.

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1. Introduction

It is well known that the so-called ratchet potential refers to a certain kind of asymmetric potential, such as the sawtooth potential used by Feynman to discuss a possible way of violating the second law of thermodynamics Feynman et al. (1963) (see also Parrondo and Español (1996)). He showed that the ratchet and the pawl device can extract useful work only when the asymmetric system is out of equilibrium (the ratchet and the pawl are simultaneously coupled to thermal baths at different temperatures). Nowadays, the term ratchet is used to designate a system that is able to transport particles or energy through a periodic structure with nonzero macroscopic velocity when a zero average force is acting on it (Hänggi and Bartussek, 1996; Reimann, 2002). In general, the ratchet effect requires a departure from equilibrium as well as the breaking of temporal, spatial or field symmetries of the system considered Reimann (2002) (see Borromeo and Marchesoni (2005) and references therein for the ratchet effect in symmetric systems).

The control of directed energy transport (DET) by zero average forces in both point particles and spatially extended systems is a challenging problem which has attracted a great deal of attention in the last decade:
The dissipative quantum transport in extended peri-
tero, 2002a; Quintero et al., 2005; Braun and Kivshar,

While the dependence of the directed transport on
each of the above ratchet-controlling parameters has
been individually investigated experimentally, theoreti-
cally, and numerically, there is still no general criterion
to apply to the whole set of these parameters to optimally
control directed transport in general systems. We notice
that in all these systems, even when no explicit analytical
expression is derived, (Schiavoni et al., 2003; Gommers
et al., 2005), just to name a few.

In the simple case of a biharmonic force,

$$f_b(t; \omega, \epsilon_1, \epsilon_2, \varphi_1, \varphi_2) = \epsilon_1 \cos(\omega t + \varphi_1)$$

$$+ \epsilon_2 \cos(2\omega t + \varphi_2).$$

(7)

the shift-symmetry condition (1) is broken if and only
if $\epsilon_1 \neq 0$ and $\epsilon_2 \neq 0$, whereas the breaking of the time-
reversal symmetries (2) is controlled by the initial phases
$\varphi_1, \varphi_2$ together with the dissipation coefficient $\beta$. The
frequency $\omega$ defines the time-scale in which the force
acts on the system. We will assume that $\omega$ is in the range
of values for which the ac force is able to keep the sys-
tem out of equilibrium, such that the ratchet effect takes
place.

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cally, and numerically, there is still no general criterion
to apply to the whole set of these parameters to optimally
control directed transport in general systems. We notice
that in all these systems, even when no explicit analytical
expression is derived, (Schiavoni et al., 2003; Gommers
et al., 2005), the average velocity $\langle V \rangle$ presents the same
functional dependence on the amplitudes and the phases
of ac force when $\epsilon_1$ and $\epsilon_2$ are small enough:

$$\langle V \rangle \sim \epsilon_1^2 \epsilon_2 \text{har}(\varphi_2 - 2\varphi_1 + \varphi_0).$$

(8)

where har means indistinctly cos or sin and where the
phase $\varphi_0$ depends on the dissipation strength and fre-
quency (see the analysis below). Notice that when the
noise is considered in the system or the amplitudes of
the biharmonic force are not small enough, a more com-
plicated formula for the average velocity is expected,
(Borromeo and Marchesoni, 2006). These common fea-
tures of the ratchet effect in a great diversity of systems
motivate us to investigate the possibility of controlling
the transport in a systematic way, regardless of the sys-
tem (we only impose on the generic system the breaking
of the aforementioned temporal symmetries for the oc-
currence of the ratchet phenomena when the amplitudes
of the ac force are small enough). The key to solving this
problem is that the ratchet effect in all previous systems
takes place as a consequence of the same cause: indeed,
the breaking of the symmetries (1) and (2).
2. Relationship between cause and effect

Curie’s principle, (Reimann, 2002), establishes that the symmetry elements of causes must be found in the produced effects. In the following we apply this principle to the ratchet effect caused by the time-periodic ac force \( f_b(t; \omega, \epsilon_1, \epsilon_2, \varphi_1, \varphi_2) \) (7) and show that the average of the velocity is given by Eq. (8).

Since the ratchet effect is characterized by a non-zero average velocity \( \langle V \rangle \), our task is to determine the relationship between the properties of biharmonic ac force \( f_b(t; \omega, \epsilon_1, \epsilon_2, \varphi_1, \varphi_2) \) (7) and \( \langle V \rangle \). Notice that non-zero average velocity appears when the symmetries (1) and (2) are both broken, so without loss of generality we can assume that

\[
\langle V \rangle \sim s(\epsilon_1, \epsilon_2, \varphi_1, \varphi_2),
\]

(9)

where \( s(\epsilon_1, \epsilon_2, \varphi_1, \varphi_2) \) is an unknown function that depends on the independent parameters \( \epsilon_1, \epsilon_2 \) and \( \varphi_1, \varphi_2 \) that control the shift and the time-reversal symmetries, respectively. We will discuss separately the dependence of \( \langle V \rangle \) on the frequency \( \omega \) and damping coefficient \( \beta \).

2.1. Control of the shift symmetry by \( \epsilon_1 \) and \( \epsilon_2 \)

From Maclaurin’s series of the function \( s(\epsilon_1, \epsilon_2, \varphi_1, \varphi_2) \) we obtain

\[
s(\epsilon_1, \epsilon_2, \varphi_1, \varphi_2) = \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} c_{k,n}(\varphi_1, \varphi_2) \epsilon_1^k \epsilon_2^n,
\]

(10)

where \( c_{k,n} \) are the coefficients associated to the series. Let us analyze in details some properties of this function:

1.A If \( \epsilon_1 = 0 \) or \( \epsilon_2 = 0 \) the shift symmetry (1) is not broken and \( \langle V \rangle = 0 \), so \( s(\epsilon_1, 0, \varphi_1, \varphi_2) = 0 \) and \( s(0, \epsilon_2, \varphi_1, \varphi_2) = 0 \). This implies that all the coefficients \( c_{k,0} \) and \( c_{0,n} \) in the series vanish.

2.A The transformations \( \epsilon_i \to -\epsilon_i \) with \( i = 1, 2 \) change the tilt of the wave form in \( f_b \), and so the average velocity also must change its sign. This means that \( s(\epsilon_1, \epsilon_2, \varphi_1, \varphi_2) = -s(-\epsilon_1, -\epsilon_2, \varphi_1, \varphi_2) \) and then, \( k + n \) must be an odd number.

3.A Taking into account these two properties and since \( \epsilon_1 \) and \( \epsilon_2 \) are small enough, we obtain that

\[
s(\epsilon_1, \epsilon_2, \varphi_1, \varphi_2) = c_{1,2}(\varphi_1, \varphi_2) \epsilon_1^{2} + c_{2,1}(\varphi_1, \varphi_2) \epsilon_1^{2} \epsilon_2 + O(\epsilon_1^3).
\]

(11)

2.2. Control of the time-reversal symmetry by \( \varphi_1 \) and \( \varphi_2 \)

1.B The ratchet effect does not depend on the origin of time. So, \( \langle V \rangle \) must be invariant under any translation in time, \( t \to t + t_0, \forall t_0 \in \mathbb{R} \). This change of variable implies that

\[
f_b \to f_b = \epsilon_1 \cos(\omega t + \varphi_1) + \epsilon_2 \cos(2\omega t + \varphi_2),
\]

(12)

with \( \varphi_1 = \varphi_1 + \omega t_0 \) and \( \varphi_2 = \varphi_2 + 2 \omega t_0 \). Consequently, \( 2 \varphi_1 - \varphi_2 = 2 \varphi_1 - \varphi_2 \) is an invariant quantity under translation in time. Taking into account \( \langle V \rangle \) (11),

\[
c_{1,2}(2 \varphi_1 - \varphi_2) = c_{1,2}(2 \varphi_1 - \varphi_2) \text{ and } c_{2,1}(\varphi_1, \varphi_2) = c_{2,1}(2 \varphi_1 - \varphi_2).
\]

2.B Furthermore, since \( f_b \to -f_b \) if \( \varphi_i \to \varphi_i + \pi \) with \( i = 1, 2 \), the average velocity also changes its sign, \( \langle V \rangle \to -\langle V \rangle \) and so,

\[
c_{1,2}(2 \varphi_1 - \varphi_2) = -c_{1,2}(2 \varphi_1 - \varphi_2 + \pi),
\]

(13)

\[
c_{2,1}(2 \varphi_1 - \varphi_2) = -c_{2,1}(2 \varphi_1 - \varphi_2 + \pi).
\]

(14)

Notice that the biharmonic force is invariant if we change simultaneously \( \epsilon_1 \to -\epsilon_1 \) and \( \varphi_1 \to \varphi_1 + \pi \). These two transformations also must leave invariant \( \langle V \rangle \), so from (11) we obtain that \( c_{2,1}(2 \varphi_1 - \varphi_2) = c_{2,1}(2 \varphi_1 - \varphi_2 + 2\pi) \) as expected from Eq. (14), but \( c_{1,2}(2 \varphi_1 - \varphi_2) = -c_{1,2}(2 \varphi_1 - \varphi_2 + 2\pi) \) which is in contradiction with Eq. (13), unless \( c_{1,2} = 0 \). Therefore, Eq. (11) reduces to

\[
s(\epsilon_1, \epsilon_2, \varphi_1, \varphi_2) = c(2\varphi_1 - \varphi_2)^2 \epsilon_1 \epsilon_2,
\]

(15)

where \( c(2\varphi_1 - \varphi_2) = c_{2,1}(2\varphi_1 - \varphi_2) \), being \( c(\theta) \) \( 2\pi \)-periodic in \( \theta = 2\varphi_1 - \varphi_2 \).

3.B From the two reversal symmetries given by (2), we will consider the case when the ratchet effect appears due to the breaking of the even-symmetry of the biharmonic force, i.e.

\[
f_b(t; \omega, \epsilon_1, \epsilon_2, \varphi_1 - \varphi_2) \text{ must be broken to get a nonzero average velocity. Using the property (1.B), it is equivalent to consider}
\]

\[
f_b(t; \omega, \epsilon_1, \epsilon_2, 2 \varphi_1 - \varphi_2)
\]

\[
= \epsilon_1 \cos(\omega t) + \epsilon_2 \cos(2\omega t + 2 \varphi_1 - \varphi_2),
\]

(17)

instead the biharmonic function \( f_b \) given by (7). This function fulfills the condition (16), giving rise to \( \langle V \rangle = 0 \), if and only if \( 2 \varphi_1 - \varphi_2 = k \pi \) being \( k \) an integer number. Consequently, \( c(k \pi) = 0 \). In addition, any value \( 2 \varphi_1 - \varphi_2 \neq k \pi \), allows \( \langle V \rangle \)
to be nonzero. In particular, if \( 2 \varphi_1 - \varphi_2 = \varphi_c = \pi/2 + k \pi \), the first harmonic in \( \langle V \rangle \) is an even function (i.e., \( \cos(\ldots) \)), whereas the second one is an odd function (\( \sin(\ldots) \)), so from this value we expect an extremum (maximum or minimum) of the average velocity. This implies that \( c(2 \varphi_1 - \varphi_2) \) exhibits an extremum at \( 2 \varphi_1 - \varphi_2 = \pi/2 + k \pi \). The simplest function satisfying all these requirements is

\[
c(2 \varphi_1 - \varphi_2) \sim \sin(2 \varphi_1 - \varphi_2).
\] (18)

Similar analysis can be carried out if the time-reversal symmetry necessary to break for the appearance of the ratchet effect is the odd one, i.e. \( f_b(t; \ldots) = -f_b(-t; \ldots) \).

2.3. Control of the time-reversal symmetry through the dissipation

It is interesting to note the important role played by the dissipation in the breaking of the time-reversal symmetry. When this symmetry solely is violated by the damping one would expect a monotonic decreasing of the average velocity to zero as the damping is increased, but also in some cases an optimal value of the dissipation coefficient for the transport is observed. Therefore, due to the dissipation the initial phases \( 2 \varphi_1 - \varphi_2 \) given in Eq. (18) will be shifted by some value \( \varphi_0 \) and so, the function \( c \) also will depends on the damping coefficient.

From the above symmetry analysis it follows that the ratchet nonzero average velocity caused by the breaking of the shift-symmetry \( f(t) = -f(t + T/2) \) and time-reversal symmetry \( f(t) = f(-t) \) is given by

\[
\langle V \rangle \sim s(\epsilon_1, \epsilon_2, 2 \varphi_1 - \varphi_2) \sim \epsilon_1^2 \epsilon_2 \sin(2 \varphi_1 - \varphi_2 + \varphi_0),
\] (19)

where \( \varphi_0 \to 0 \) as dissipation vanishes. Notice that, in general \( \varphi_0 \) is not only a function of the damping coefficient, but also could function of the frequency \( \omega \).

We have found that the present theory confirms all previous experimental, theoretical and numerical results (Ajdari et al., 1994; Goychuk and Hänggi, 1998; Marchesoni, 1986; Salerno and Zolotaryuk, 2002b; Ustinov et al., 2004; Morales-Molina et al., 2003; Gorbach et al., 2006; Morales-Molina et al., 2006).

In particular, we can explain experimental results concerning the motion of atoms in a symmetric optical lattice without dissipation, (Schivoni et al., 2003), and with dissipation, (Gommers et al., 2005). Indeed, in (Schivoni et al., 2003), where a biharmonic force \( f_b(t; \ldots) = \text{(constant)} (1 - B) \cos(\omega t) + B \cos(2\omega t - \varphi) \) is used as a source of asymmetry to cause the transport of the atoms, the velocity of the center-of-mass of the atomic cloud is maximum at \( B \approx 0.33 \) and \( \varphi = \pi/2 \). Notice that the present theory predicts \( \langle V \rangle \sim (1 - B)^2 B \sin(\varphi) \). Thus \( \langle V \rangle \) presents a maximum at \( B = 0.33 \) and \( \varphi = \pi/2 \). In the presence of dissipation our theory also confirms the recent experiments in optical lattice (Gommers et al., 2005).

The ac force \( f_b(t; \ldots) \) given by (7) is a particular case of a more general biharmonic force,

\[
f(t; \omega_1, \omega_2, \epsilon_1, \epsilon_2, \varphi_1, \varphi_2)
= \epsilon_1 \cos(\omega_1 t + \varphi_1) + \epsilon_2 \cos(\omega_2 t + \varphi_2),
\]

where \( \omega_1 = q \omega_2 \) being \( p, q \) coprime integers.

3. Geometric properties of the optimal waveform

Let us illustrate with two examples how the ratchet effect can be controlled by the breaking of the relevant symmetries of the systems. We will consider the case in which the time-reversal symmetry is solely controlled by the damping. For the sake of concreteness, these findings will be discussed using the following working model for the driving force,

\[
f_{\text{ellip}}(t) = \epsilon f(t; T, m, \theta)
= \epsilon \text{sn}(\Omega t + \Theta; m) \text{cn}(\Omega t + \Theta; m),
\] (20)

where \( \text{cn}(\cdot; m) \) and \( \text{sn}(\cdot; m) \) are Jacobian elliptic functions of parameter \( m \), \( \Omega = 2K(m)/T \), \( \Theta = K(m) \theta/\pi \), \( K(m) \) is the complete elliptic integral of the first kind, \( T \) is the period of the force, and \( \theta \) is the (normalized) initial phase \( \theta \in [0, 2\pi] \). Note that fixing \( \epsilon, T, \) and \( \theta \), the force waveform changes as the shape parameter \( m \) varies from 0 to 1 (see Fig. 1). Physically, the choice (20) is motivated by the properties \( f_{\text{ellip}}(t; T, m = 0, \theta) = (1/2) \sin(2\pi t/T + \theta) \) and that \( f_{\text{ellip}}(t; T, m = 1, \theta) \) vanishes except on a set of instants that has Lebesgue measure zero, i.e., in such limits kink-mediated DET is not possible while it is expected for \( 0 < m < 1 \). Thus, one may expect that the average velocity exhibits (at least) an extremum as the shape parameter \( m \) is varied, the remaining parameters being held constant (as in Fig. 1). To demonstrate the above prediction we use the Fourier series of the force:

\[
f_{\text{ellip}}(t) = \epsilon \sum_{n=1}^{\infty} a_n(m) \sin \left[ n \left( \frac{2\pi t}{T} + \theta \right) \right],
\]

\[
a_n(m) = \frac{\pi^2 n}{m K^2(m)} \text{sech} \left[ \frac{n \pi K(1 - m)}{K(m)} \right].
\] (21)
Fig. 1. Function $f(t; T, m, \theta)$ (cf. Eq. (20)) vs. $t/T$ for $\theta = 0$ and 3 shape parameter values, $m = 0, 0.96, 1 - 10^{-6}$, showing an increasing symmetry-breaking sequence as the pulse narrows, i.e., as $m \to 1$.

Now, keeping only the first two terms of the series, we deal with the case of biharmonic force. For this biharmonic approximation, one straightforwardly obtains the following estimate for the average velocity $\langle V \rangle \sim \epsilon^2$}

$$\langle V \rangle \sim \epsilon^2 \epsilon_2 = \epsilon^3 a_1^2(m) a_2(m) = \epsilon^3 S(m)$$

$$= \epsilon^3 \frac{\text{sech}^2 \left[ \frac{\pi K(1-m)}{K(m)} \right] \text{sech} \left[ \frac{2\pi K(1-m)}{K(m)} \right]}{m^3 K^6(m)}.$$  \hspace{1cm} (22)

The absolute value of the average velocity presents a maximum as a function of the shape parameter at $m = m_{op} \simeq 0.960057$, as is shown in Fig. 2. Now, two competing mechanisms allow one to understand the appearance of the velocity extremum. Namely, the breaking-degree increase of the temporal shift symmetry, which yields an increase of the absolute value of the average velocity, and the (effective) narrowing of the force pulse as the shape parameter increases from 0, which in its turn yields a decrease of the absolute value of the average velocity. The former mechanism can be quantitatively characterized by

$$\langle D(t; T, m, \theta) \rangle = \frac{1}{T} \int_0^T dt D(t; T, m, \theta),$$

$$D(t; T, m, \theta) = -\frac{f(t + T/2; T, m, \theta)}{f(t; T, m, \theta)} = \frac{\sqrt{1-m}}{dn^2(\Omega t + \Theta; m)}, \hspace{1cm} (23)$$

where $dn(\cdot; m)$ is the Jacobian elliptic function. Eq. (23) indicates that the degree of deviation from the shift symmetry condition $\langle D(t; T, m, \theta) \rangle \equiv 1$ increases as $m \to 1$, irrespective of the period and initial phase values (see Fig. 3). Thus, an increase of the shape parameter (from 0) improves the degree of symmetry breaking (DSB) characterized by $\langle D(t; T, m, \theta) \rangle$ yielding a higher transport velocity, but it simultaneously narrows the pulse force yielding a lowering of the driving effectiveness of the force (the impulse of the force in half of the period decreases as $m \to 1$, see Fig. 3), which becomes the dominant effect for sufficiently narrow pulses. This explains the existence of the aforementioned extremum.

Let us consider this elliptic force acting on the damped sine-Gordon kink, which dynamic is described by

$$\phi_{tt} - \phi_{xx} + \sin(\phi) = -\beta \phi_t + f_{\text{ellip}}(t), \hspace{1cm} (24)$$

where $\beta$ represents the damping coefficient. Approximating the elliptic function by the first two terms of the Fourier series, a collective coordinate approach with two degrees of freedom, (Morales-Molina et al., 2003), $X(t)$ and $l(t)$ (respectively, position and width of the kink), can be directly applied to obtain the dynamics of these two CC in the presence of the elliptic force:

$$\dot{P} = -\beta P - q f_{\text{ellip}}(t), \hspace{1cm} (25)$$

Fig. 2. Left panel: It is shown that the function $S(m)$ yield a maximum at $m_{op} = 0.960057$. Right panel: For the optimal value of $m$ and $\theta = 0$, the corresponding optimal wave form is shown.
\[ l = \frac{l^2}{2l} + 1/(2\alpha l) - \beta l - (\Omega_R^2 l/2)(1 + M_0^{-2} p^2), \]

where the momentum \( P(t) \) is given by \( P(t) = M_0 l_0 \dot{X}/l(t), \Omega_R = \sqrt{2/(\pi l_0)} \) is the so-called Rice’s frequency, \( \alpha = \pi^2/12 \), and \( M_0 = 8 \), \( q = 2\pi \), and \( l_0 = 1 \) are, respectively, the dimensionless kink mass, topological charge, and unperturbed width. For this biharmonic approximation, one straightforwardly obtains the following expression for the average velocity of the kink,

\[ \langle V \rangle \equiv \langle \dot{X}(t) \rangle \equiv \frac{1}{T} \int_0^T \frac{P(t)(t)}{M_0 l_0} \, dt \]

\[ = \langle \dot{X}_0(t) \rangle + \varepsilon \langle \dot{X}_1(t) \rangle + \ldots = F(\epsilon, \beta, T) S(m), \]

(27)

with

\[ F(\epsilon, \beta, T) \]

\[ \equiv \frac{2q^2 \Omega_R^4}{M_0^4 (\beta^2 + 4\pi^2 / T^2)(\beta^2 + 16\pi^2 / T^2)^{1/2}} \times \sum_{i=1}^{2} \frac{2 \cos(\chi_2 - 2\chi_1) + (-1)^i \tilde{\beta}_l (-1)^{i-1}}{i \sqrt{(\Omega_R^2 - 4i^2 \pi^2 / T^2)^2 + 4i^2 \pi^2 \beta^2 / T^2}}, \]

(28)

where \( \chi_2 = \text{atan}(4\pi / \beta T), \chi_1 = \text{atan}(2\pi / \beta T), \tilde{\beta}_l = \text{atan}((\Omega_R^2 - 4\pi^2 / T^2) / 2\pi \beta), \) and \( \tilde{\beta}_2 = \text{atan}((\Omega_R^2 - 16\pi^2 / T^2) / 4\pi \beta) \), and where, as expected, \( \langle \dot{X}_0(t) \rangle = 0 \). From Eq. (22), we see that the average velocity is independent of the initial phase \( \theta \) while nonzero velocity exists for \( 0 < m < 1 \). From the approximated expression (27), the average velocity has a minimum at \( m_{\text{op}} = 0.960057 \). From the numerical solutions of the CC Eqs. (25) and (26) an extremum appears at \( m_{\text{op}} = 0.98 \) and from the numerical integration of the full problem Eq. (24), \( m_{\text{op}} = 0.98 \). Fig. 4 shows the average velocity of the center of the kink, obtained from the numerical integration of Eq. (24) and from the numerical solutions of the CC equations, versus \( m \).

Finally, another example we would like to discuss here is related to the ratchet effect induced by spatial asymmetry. In this case the particles, subjected to the external forces, move in the spatial asymmetric potential (Reimann, 2002). At a first stage, these ratchets models were developed to understand the transport of the molecular motors inside the cell (Maddox, 1994). Nowadays, these models have been improved, some of them taking into account the experimental results on the molecular motors (see (Buonocore et al., 2005) and references...
This mechanism allows us to estimate the average velocity as a function of the parameters of the ac force. The qualitative agreement between the present analytical predictions and previous experimental, numerical, and theoretical findings in several seemingly unrelated systems (Salerno and Zolotaryuk, 2002b; Ustinov et al., 2004; Morales-Molina et al., 2003; Marchesoni, 1986; Goychuk and Hänggi, 1998; Gorbach et al., 2006; Morales-Molina et al., 2006; Gommers et al., 2005; Schiavoni et al., 2003), leads us to suggest the existence of a universal waveform for optimal control of the ratchet effect for a given time-periodic ac force (in the case of breaking of temporal symmetries  \( f(t) = -f(t + T/2) \) and \( f(t) = f(-t) \) or \( f(t) = -f(-t) \)).

The main conclusion of this work is that Curie’s principle can be quantitatively applied to uncover universality in directed transport phenomena where a ratchet effect is induced by breaking of system’s symmetries. We also have studied two examples, where this mechanism can be characterized by the degree of symmetry breaking (DSB) and transmitted impulse of a more general biharmonic forces. The control of transport through the relevant symmetries of the systems also can be applied to other biharmonic forces and other ratchet systems where the asymmetry is created by a ratchet potential, although in the latter case a richer behavior is expected due to the interplay of parameters of the spatial potential and the ac force. Finally, we conjecture that a similar version of the aforementioned mechanism discussed in this paper could explain how the molecular motors optimize directed transport once the symmetries to be broken are identified.

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