Decision Support

A general framework for multiresponse optimization problems based on goal programming

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Abstract

Setting of process variables to meet a required specification of quality characteristic (or response variable) in a process, is one of the common problems in the process quality control. But generally there are more than one quality characteristics in the process and the experimenter attempts to optimize all of them simultaneously. Since response variables are different in some properties such as scale, measurement unit, type of optimality and their preferences, there are different approaches in model building and optimization of MRS problems.

This study propose a general framework in MRS problems according to some existing works and some types of related decision makers and attempts to aggregate all of characteristics in one approach. The proposed framework contains four non-desirability parts of bias, response variation, errors in predictions and separation from responses’ specific region. We demonstrate the proposed framework with two examples of the literature and the results has been discussed with comparing of mentioned existing works.

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1. Introduction

Specifying appropriate operating conditions for an industrial process to achieve a quality character-

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surface (MRS). Three stages in these problems are: (1) data collection, (2) model building and (3) optimization.

Dual response surface (DRS) is one of MRS problems in which the mean and variance of a special quality characteristic is fitted by a polynomial surface and in the optimization stage, both variance and mean are optimized simultaneously.

The characteristics of MRS problems which it is necessary to attend to the model building and optimization stages are: different measuring units, different scales and magnitude, different types of optimality, different direction and different preferences of responses and also different types of decision makers.

In the next section, we will briefly review some existing approaches in MRS problems, and then we will provide a general framework and propose a unified approach based on goal programming. In Section 4, we will focus on decision making processes in MRS problems and give an approach to include consumers as another decision maker in the general framework.

Finally, we will illustrate the general framework with some examples and compare our method with those in the literature.

2. Review of existing works in MRS problems

Dual response surface (DRS) is a special case of MRS problems and researchers pays attention to one quality characteristic but they want to optimize mean and variance as robust parameter design (Onur and Doganaksoy (2001)). The mean and variance functions usually are as follows:

\[
\hat{\mu} = b_0 + X' b + X' B X,
\]

\[
\hat{\sigma} = c_0 + X' c + X' C X,
\]

which \(X\) denote a vector of controllable design variables for a selected quality characteristic and other parameters are coefficients.

When the number of quality characters increase, the MRS problems will be used. One of the existing approaches to MRS problems is the Priority based approach which the main objective (or response) will be the objective function and the other objectives will be treated as constraints with their desirability targets. Myers and Carter (1973) and Vining and Myers (1990), optimize the primary response with respect to the other responses by use of a Lagrange multiplier approach. Also Biles (1975), extended the idea of Myers and Carter by allowing more than two responses

Min (or Max) Primary response,
\[
s.t. \quad \text{Secondary response} = T_{sr},
\]

where \(T_{sr}\) is a specified target for secondary response and \(x\) is the experimental region.

One popular approach to multiple response optimizations is dimensionality reduction. In such approaches, the multiple response problem is converted to one single aggregated function. Lin and Tu (1995) propose the mean squared error instead of using a Lagrangian multipliers which is

\[
\min_x \text{MSE} = (\hat{y}_\mu - T_\mu)^2 + \hat{y}_\sigma^2,
\]

where \(\hat{y}_\mu\) is the estimated mean response, \(\hat{y}_\sigma\) is the estimated variance response and \(T_\mu\) is a target value for the mean of response. They also proposed a modified MSE approach in the situation where tight adherence to the target is essential, i.e.,

\[
\min_x \text{MSE'} = \lambda_1(\hat{y}_\mu - T_\mu)^2 + \lambda_2\hat{y}_\sigma^2,
\]

where \(\lambda_1\) and \(\lambda_2\) are pre-specified positive constants.

The single aggregated objective has often been defined as a desirability function or loss function. Each of them is discussed next. In the loss function approach, the deviation of responses from their targets is minimised. Pignatiello (1993) propose a quadratic loss function in multiresponse situations,

\[
L(y(x), \theta) = (y(x) - \theta)^T C (y(x) - \theta),
\]

where \(x = [x_1, x_2, \ldots, x_k]'\) denote a vector of controllable design variables, \(y = [y_1, y_2, \ldots, y_p]'\) is the vector of quality characteristics and depends on the controllable variables, \(C\) is a \(p \times p\) positive defined matrix of costs by DM representing the losses incurred when \(y\) deviates from its target, \(\theta\). Also the expected loss is

\[
\text{Min } E[L(y(x), \theta)] = (E[y(x)] - \theta)^T C (E[y(x)] - \theta) + \text{trace}(C \Sigma_j(x)),
\]

where the aim of the mentioned loss function is minimizing the bias from the targets and also variance of the responses. A quality loss function approach proposed by Ames et al. (1997) is based on the global quality loss function. In this approach, the correlation structure among the responses has been ignored.
\[
\min QLP = \sum_{j=1}^{m} w_j (\hat{y}_j - T_j^*)^2,
\]
where \( \hat{y}_j \) is the estimated response for quality criterion \( j \), with respective target value, \( T_j^* \), and \( w_j \) is the corresponding weight for each response. Vining (1998), proposed another definition of loss function by substituting of \( \hat{y}(x) \) for \( y(x) \) in (4), i.e.,
\[
L(\hat{y}(x), \theta) = (\hat{y}(x) - \theta)^T C(\hat{y}(x) - \theta),
\]
where \( \hat{y}(x) \) is the \( p \times 1 \) vector of predicted responses at \( x \). With this definition of loss, the expected loss will be,
\[
E[L(\hat{y}(x), \theta)] = (E[\hat{y}(x)] - \theta)^T C(E[\hat{y}(x)] - \theta) + \text{trace}[C \Sigma(x)],
\]
where \( \Sigma(x) \) is the \( p \times p \) variance–covariance matrix for \( \hat{y} \) at \( x \), and if \( \hat{y}(x) \) is an unbiased estimator of \( E[\hat{y}(x)] \) then \( E[\hat{y}(x)] \) equals \( E[y(x)] \). Expected loss proposed by Vining (1998) seeks to minimize both the deviation of the responses from their targets and the variance of predicted responses. Recently, Ko et al. (2005), propose a new loss function,
\[
L(\tilde{y}_{\text{new}}(x), \theta) = (\tilde{y}_{\text{new}}(x) - \theta)^T C(\tilde{y}_{\text{new}}(x) - \theta),
\]
\[
\tilde{y}_{\text{new}}(x) = \hat{y}(x) + e_{\text{new}}(x),
\]
where \( e_{\text{new}}(x) \) is a random error term having the same distribution as \( e(x) \). In their approach, the expected loss is obtained as
\[
E[L(\tilde{y}_{\text{new}}(x), \theta)] = (E[\hat{y}(x)] - \theta)^T C(E[\hat{y}(x)] - \theta) + \text{trace}[C \Sigma_i(x)] + \text{trace}[C \Sigma_j(x)],
\]
which contains bias, variance of responses and variance of predicted responses. The desirability function approach, transforms an estimated response \( \hat{y} \) to a scale free value, called the desirability \( d_i \) for the \( i \)th response. Overall desirability (\( D \)) is defined by combining the individual desirability values. Harrington (1965) proposed this approach; Derringer and Suich (1980) extended the approach by suggesting a systematic transformation of \( \hat{y}_i(x) \) to \( d_i \). Eq. (11) is an example of the transformation function for a the Larger The Better (LTB) type response variable,
\[
d_i = \begin{cases} 
0 & \hat{y}_i(x) \leq y_i^{\min} \\
\frac{(\hat{y}_i(x) - y_i^{\min})^T}{y_i^{\max} - y_i^{\min}} & y_i^{\min} \leq \hat{y}_i(x) \leq y_i^{\max} \\\n1 & \hat{y}_i(x) \geq y_i^{\max}
\end{cases}
\]
\[
D = (d_1 \times d_2 \times \ldots \times d_k)^{1/k},
\]
where \( t \) is a parameter reflecting the process economics for the response of \( i \).

Derringer (1994) proposed the weighted geometric mean for the overall desirability function. Kim and Lin (1998, 2000, 2002) suggested maximizing the lowest \( d_i \), which is equivalent to maximizing the overall desirability value of the responses.

Both of those mentioned approaches, loss function and desirability function approach, allow the user to consider the process economics, but the major advantage of the loss function approach is incorporating the variance–covariance structure of the responses and the major advantage of the desirability function approach is that the desirability values of each response are scale free and have the same magnitude. Chiao and Hamada (2001) used the specification region of the responses to find the optimal solution, i.e.,
\[
\max p(Y \in S), \quad x \in \Omega,
\]
where
\[
f(Y; \mu, \Sigma) = (2\pi)^{-m/2} |\Sigma|^{-1/2} \times \exp \left[ -\frac{1}{2} (Y - \mu)^T \Sigma^{-1} (Y - \mu) \right],
\]
where \( S \) is the specification region of all responses and \( \Sigma \) is the variance–covariance matrix for \( Y \) which has a normal distribution.

Such an approach can help the experimenter to control the responses in their specification region; however it does not consider the deviation from the targets.

3. The proposed general framework

This study gives a general framework for existing works and combines them into an aggregated framework. We propose a formulation for the desirability and loss function approach. The constraints which are quadratic forms and objective function are as follows:
\[
\text{Min} \quad ND_{\text{overall}} = g_1 + g_2 + g_3 + g_4
\]
Subjected to:  
\[ \sum_{j=1}^{m} \gamma_j(1 - d_j)^2 - s_1 g_1 = T_1, \]
\[ \sum_j a_j \left( \frac{\sigma_j^2(x)}{\max(\sigma_j^2(x))} \right) - s_2 g_2 = T_2, \]
\[ \sum_j b_j \left( \frac{\sigma_j^2(x)}{\max(\sigma_j^2(x))} \right) - s_3 g_3 = T_3, \]
\[ c_{pr}(1 - d_{pr})^2 - s_4 g_4 = T_4, \]
and \( x \in \Omega, \)

where \( \gamma_j \) is the weight of the \( j \)th response and can be determined by the customer as illustrated in the following section, \( d_j \) is the desirability value for such responses \( s_k, T_k, (k = 1, 2, \ldots, 4) \) are user-defined weights and targets, respectively, \( \sigma_j(x) \) is the variance of observed values of the \( j \)th response at setting of \( x = (x_1, x_2, \ldots, x_r)^T \) as decision variable, \( \hat{\sigma}_j(x) \) is the predicted variance for such response at \( x, \Omega \) be a spherical or rectangular experimental region and \( ND_{overall} \) is the non-desirability of all aspects.

It is worth to mention that the proposed framework requires inputs (such as \( C \) matrix, shape of desirability function, etc.) from the decision maker and must be determined by DM before the formulation. Also this approach is based on the multivariate normality assumption and multivariate normality must be validated before solving the problem.

In addition \( d_{pr} \) is another desirability function for the probability of meeting the response specifications (or conformance with the specification region) and \( c_{pr} \) is the weight of related desirability. The first and last constraints can be added up in the constraint;

\[ \sum_{j=1}^{m} \gamma_j(1 - d_j)^2 - s_1 g_1 = T_1, \]

where \( \gamma_{m+1} = c_{pr} \).

This formula is a combination of the desirability function and loss function approaches. There are four terms \( (g_1, g_2, g_3, g_4) \) in this equation which are non-desirability of bias (\( ND_{bias} \)), non-desirability of response variation (\( ND_{variation} \)), non-desirability of errors in predictions (\( ND_{error} \)) and non-desirability of separation from responses’ specification region (\( ND_{S-region} \)). The optimal point minimizes all of these non-desirabilities. If mentioned assumptions cannot be validated or required inputs can not be determined by DM, related non-desirability cannot be computed and should be deleted from the formulation.

The above formulation provides the general framework and unifies the existing works. Comparison of our formula with the other techniques and parameter definitions is summarized in Table 1.

In Table 1, we can compare all existing works: it shows that the method of Lin and Tu (1995) does not contain \( ND_{variation} \) and \( ND_{S-region} \), the method of Ames et al. (1997) contains just \( ND_{bias} \), and Chiao and Hamada (2001) pay attention just to \( ND_{S-region} \).

Further Myers and Carter (1973) and Vining and Myers (1990) have paid attention to \( ND_{bias} \) and \( ND_{error} \), the method of Pignatiello (1993) does not contain \( ND_{error} \) and \( ND_{S-region} \). The method of Vining (1998) does not contain \( ND_{variation} \) and \( ND_{S-region} \) and finally the method of Ko et al. (2005) does not contain \( ND_{S-region} \). But the proposed unified approach contains all of them \( ND_{bias}, ND_{variation}, ND_{error} \) and \( ND_{S-region} \).

The problem is sensitive for setting variables \( (x) \) and \( T_k \) makes condition, so there is no effect in optimal solution.

### 4. Decision making in MRS problems

In this section, we would like to discuss about related decision makers in MRS Systems. Of course proposed approach needs lots of inputs must be defined by different type of decision makers and this section will discuss about different types of Decision Makers (DMs) for proposed approach.

It is clear that MRS problems contain decision making in all three stages of data collection, model building and optimization, so there can be more than one DM in the problem. But related DMs may be in different levels and the optimal solution for setting variables will occur when the solution can satisfy all of them. Different DMs in MRS problems are depicted in Fig. 1 and it shows that the outer layer relates to many decision makers (customers).

Each DM has special authority in the decision making process for the MRS problem. DM1, DM2 and DM3 have the same direction about the decision making but the direction of DM4 can be different.

If we delete some decision makers in the optimization stage, the optimal solution may not be the general optimization solution, because from the other decision maker point of view, optimal solution is different. Table 2, categorizes some of the existing works of MRS problems with respect to the type of decision maker.

Mentioned parameters in general framework can be defined by related decision makers. For example \( \gamma_j \) in Eq. (16) is defined by DM1 or DM4 (as...
illustrated in Fig. 2), type of responses, \( m \) in Eq. (16) \( \Omega \) and \( S \) in Eq. (13), are determined by DM2, cost matrix, shape of desirability function and \( s_k, T_k \), in Eq. (16) are defined by DM3.

<table>
<thead>
<tr>
<th>Existing technique</th>
<th>Parameters definition in the proposed method</th>
</tr>
</thead>
</table>
| Lin and Tu (1995)        | \( T_k = 0, s_k = 1, k = 1, \ldots, 4, \) \( j = 1, a_1 = c_{pe} = 0, \)
                         | \( b_1 = \max(\sigma_j^2(x)) \). |
| Lin and Tu (1995)        | \( T_k = 0, s_k = 1, k = 1, \ldots, 4, \) \( j = 1, a_1 = c_{pe} = 0, \)
                         | \( b_1 = \lambda_2 \max(\sigma_j^2(x)) \). |
| Lin and Tu (1995)        | \( T_k = 0, s_k = 1, k = 1, \ldots, 4, \) \( a_j = b_j = c_{pe} = 0, j = 1, \ldots, m, \)
                         | \( c_{pe} = 1 \) |
| Ames et al. (1997)       | \( T_k = 0, s_k = 1, k = 1, \ldots, 4, \) \( j = 1, a_1 = c_{pe} = 0, \)
                         | \( b_1 = \max(\sigma_j^2(x)) \). |
| Ames et al. (1997)       | \( T_k = 0, s_k = 1, k = 1, \ldots, 4, \) \( a_j = b_j = c_{pe} = 0, j = 1, \ldots, m, \)
                         | \( c_{pe} = 1 \) |
| Myers and Carter (1973)  | \( T_k = 0, s_k = 0, k = 1, k = 2, 3, 4, \) \( j = 1, a_1 = c_{pe} = 0, \)
                         | \( b_1 = \max(\sigma_j^2(x)) \). |
| Vining and Myers (1990)  | \( T_k = 0, s_k = 0, k = 1, k = 2, 3, 4, \) \( j = 1, a_1 = c_{pe} = 0, \)
                         | \( b_1 = \max(\sigma_j^2(x)) \). |
| Myers and Carter (1973)  | \( T_k = (T_j^*)^2, s_k = 0, k = 1, k = 2, 3, 4, \) \( j = 1, a_1 = c_{pe} = 0, \)
                         | \( b_1 = \max(\sigma_j^2(x)) \). |
| Vining and Myers (1990)  | \( T_k = (T_j^*)^2, s_k = 0, k = 1, k = 2, 3, 4, \) \( j = 1, a_1 = c_{pe} = 0, \)
                         | \( b_1 = \max(\sigma_j^2(x)) \). |
| Ko et al. (2005)         | \( T_k = 0, s_k = 1, k = 1, \ldots, 4, \) \( c_{pe} = 0, j = 1, \ldots, m, \)
                         | \( a_j = b_j = c_{j} \max(\sigma_j^2(x)) \). |
| Ko et al. (2005)         | \( T_k = 0, s_k = 1, k = 1, \ldots, 4, \) \( c_{pe} = 0, j = 1, \ldots, m, \)
                         | \( a_j = b_j = c_{j} \max(\sigma_j^2(x)) \). |

* This comparison can be made if \( C \) is a diagonal matrix, so each diagonal element of the matrix is the weight of the respective response.
Table 2

<table>
<thead>
<tr>
<th>Existing works</th>
<th>Related DMs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Derringer and Suich (1980)</td>
<td>DM2 and DM3</td>
</tr>
<tr>
<td>Pignatiello (1993)</td>
<td>DM3</td>
</tr>
<tr>
<td>Lin and Tu (1995)</td>
<td>DM3</td>
</tr>
<tr>
<td>Lee et al. (1996)</td>
<td>DM2 and DM3</td>
</tr>
<tr>
<td>Boyle and Shin (1996)</td>
<td>DM3 and DM4</td>
</tr>
<tr>
<td>Kim and Lin (1998)</td>
<td>DM2 and DM3</td>
</tr>
<tr>
<td>Proposed Method</td>
<td>DM1, DM2, DM3 and DM4</td>
</tr>
</tbody>
</table>

Fig. 1. Different types of DMs relating to MRS problems.

Fig. 2. Customer (DM4) role in MRS decision making.

5. Examples for general framework

The proposed framework is illustrated with two experiments from the literature. In each example we have compared the results of proposed framework with the existing works.

Example 1. In an experiment on the quality of a printing process, described in Box and Draper (1987), and the experimenter tries to determine the

effects of speed \((x_1)\), pressure \((x_2)\) and distance \((x_3)\) on the quality of printing. The experiment is a \(2^3\) factorial design with three replicates at each point. The experimental data is given in Table 3.

The fitted response function for the mean and standard deviation of \(y\) together with the probability of meeting the specification region are;

\[
\hat{y}_\mu = 327.6 + 177.1 x_1 + 109.4 x_2 + 131.5 x_3 + 32 x_1^2 - 22.4 x_2^2 - 29.1 x_3^2 + 66 x_1 x_2 + 75.5 x_1 x_3 + 43.6 x_2 x_3 \\
\hat{y}_\sigma = 34.9 + 11.5 x_1 + 15.3 x_2 + 29.2 x_3 + 4.2 x_1^2 - 1.3 x_2^2 + 16.8 x_3^2 + 7.7 x_1 x_2 + 5.1 x_1 x_3 + 14.1 x_2 x_3. \\
\]

After calculation of the probability of being in the specification region of \(y_\mu \in (498, 502)\) by Eq. (14), the fitted response function will be;

\[
\hat{y}_\text{pr} = 0.1197 + 0.04157 x_1 + 0.0608 x_2 - 0.0465 x_1^2 - 0.03 x_1 x_2 + 0.035 x_1 x_3. \\
\]

As mentioned before, it is assumed that \(Y\) has a normal distribution. where \(y_\text{pr}\) is the probability of being in the specific region with specific mean and variance.

Finally by minimization of the objective function Eq. (15) the optimal setting variables will be
Table 4
Comparison of the results of general framework and some other existing works for printing process example

<table>
<thead>
<tr>
<th>Method</th>
<th>Optimal setting points</th>
<th>( ND_{\text{bias}} )</th>
<th>( ND_{\text{error}} )</th>
<th>( ND_{\text{S-region}} )</th>
<th>( ND_{\text{overall}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lin and Tu (1995)</td>
<td>(1, 0.07, 0.25)</td>
<td>0.30952</td>
<td>0.10442</td>
<td>0.8304</td>
<td>1.244</td>
</tr>
<tr>
<td>Vining and Myers (1990)</td>
<td>(0.614, 0.228, 0.1)</td>
<td>0.00023</td>
<td>0.1417</td>
<td>0.75</td>
<td>0.8958</td>
</tr>
<tr>
<td>Chiao and Hamada (2001)</td>
<td>(1, −1, 1)</td>
<td>7.23</td>
<td>0.212</td>
<td>0.577</td>
<td>8.025</td>
</tr>
<tr>
<td>Proposed ND based general framework</td>
<td>(1, −0.7673, 0.7295)</td>
<td>3.01e−5</td>
<td>0.1795</td>
<td>0.6277</td>
<td>0.80725</td>
</tr>
</tbody>
</table>

\* ND variation has not compared in this example, because there is not enough data for selected example and also it has not computed in other methods.

Table 5
Study data of example 2

<table>
<thead>
<tr>
<th>ID</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( y_1 )</th>
<th>( y_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>1</td>
<td>1</td>
<td>104.454</td>
<td>104.515</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>1</td>
<td>−1</td>
<td>104.12</td>
<td>103.93</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>1</td>
<td>−1</td>
<td>98.732</td>
<td>98.748</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>1</td>
<td>−1</td>
<td>100.192</td>
<td>100.38</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>−1</td>
<td>1</td>
<td>103.145</td>
<td>103.62</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>−1</td>
<td>1</td>
<td>106.078</td>
<td>106.74</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>−1</td>
<td>−1</td>
<td>113.515</td>
<td>113.62</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>−1</td>
<td>−1</td>
<td>109.895</td>
<td>109.97</td>
</tr>
</tbody>
</table>

\[ x = (1, −0.7673, 0.7295) \] and \( ND_{\text{overall}} = 0.80725. \) The comparison of our framework and some existing methods for printing process data is illustrated in Table 4. In this table, all non-desirability values have been computed by Eq. (16).

The table shows that the proposed approach contains the least overall non-desirability in comparison to other methods.

Table 6
Variance–covariance matrix and probability value of meeting the specification region of example 2

<table>
<thead>
<tr>
<th>ID</th>
<th>Variance–covariance matrix</th>
<th>Probability value</th>
</tr>
</thead>
</table>
| 1  | \[
\begin{pmatrix}
0.204 & 0.168 \\
0.168 & 0.214
\end{pmatrix}\] | 0                 |
| 2  | \[
\begin{pmatrix}
0.161 & 0.269 \\
0.269 & 0.576
\end{pmatrix}\] | 2.43e−5           |
| 3  | \[
\begin{pmatrix}
0.08 & 0.061 \\
0.061 & 0.066
\end{pmatrix}\] | 0.9956            |
| 4  | \[
\begin{pmatrix}
0.093 & 0.129 \\
0.129 & 0.336
\end{pmatrix}\] | 0.9996            |
| 5  | \[
\begin{pmatrix}
9.518 & 5.637 \\
5.637 & 3.822
\end{pmatrix}\] | 3.29e−9           |
| 6  | \[
\begin{pmatrix}
12.49 & 8.54 \\
8.54 & 8.499
\end{pmatrix}\] | 0.0571            |
| 7  | \[
\begin{pmatrix}
5.41 & 4.167 \\
4.167 & 6.63
\end{pmatrix}\] | 0.5642            |
| 8  | \[
\begin{pmatrix}
6.352 & 10.72 \\
10.72 & 18.718
\end{pmatrix}\] | 0.489             |

It is worth to mention that in proposed approach \( T_k \) has no effect on optimal solution.

Example 2. In this example (Pignatiello, 1993), there are two response variables \((y_1, y_2)\) and again three setting variables \((x_1, x_2, x_3)\). The experimental data is shown in Table 5. In this example it is assumed that the targets of the responses are 103 and 73 and that the specification regions are \((97, 109), (70, 76)\) for \( y_1, y_2 \), respectively.

The variance–covariance matrix and computed probability value of being in the specification region for each point (by Eq. (14)) is reported in Table 6.

By use of these data, the fitted response functions for the mean, standard deviation and probability of meeting the specification region are:

\[
y_{1\mu} = 104.867 - 3.148x_1 + 2.379x_1x_2 - 0.35x_1x_3,
\]

\[
y_{2\mu} = 70.4514 - 0.3488x_1 + 3.592x_2 - 0.449x_1x_3 + 0.6144x_2x_3,
\]

\[
y_{1\sigma} = 1.613 - 0.254x_2 + 1.25x_3,
\]

\[
y_{2\sigma} = 1.7288 + 0.417x_1 + 1.214x_3 + 0.2625x_1x_3,
\]

\[
y_{pr} = 0.388 + 0.374x_2 - 0.1106x_3,
\]

where \( y_{1\mu}, y_{2\mu} \) are fitted response surfaces for the mean of \( y_1, y_2 \) and \( y_{1\sigma}, y_{2\sigma}, y_{pr} \) are fitted response surfaces for the \( y_{1\sigma}, y_{2\sigma} \) and \( y_{pr} \), respectively.

The comparison of general framework and some other existing approaches is in Table 7. It is worth
to mention that the proposed approach is just sensitive to $x$ as a decision variable and $T_k$ is used for aggregating of other objectives.

Table 7 shows that new General Framework can aggregate all approaches and generate optimal setting points for optimizing all approaches. In the mentioned example there were not experiment data (replicate) for some optimal solution settings, so comparing of methods can be done by the last column. The results show that proposed method contains the least non-desirability ($0.4525$) and it seems that the general proposed approach can find better solution according to whole aspects.

6. Conclusion

The proposed general framework yields a non-desirability approach which contains existing approaches and, as shown in the previous section, determines optimal setting points, so we can use the new approach and find optimal design. Mentioned examples show that the proposed approach has better solution with respect to all aspects. This approach has enough flexibility because of defining inputs by decision makers and also the other existing methods are reachable by some changes on the general framework.

Since the proposed approach contains different non-desirabilities, the related values can be compared and computed.

By using of proposed method, we can find the optimal solution on MRS problems with respect to all decision makers and the results can be calculated according the existing approaches.

References


