Texture Based Segmentation: Automatic Selection of Co-Occurrence Matrices

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Abstract

Texture is one of the least understood areas in computer vision. One of the major shortcomings of texture segmentation approaches has been the ad-hoc selection of the set of feature vectors. We present an approach to qualitatively select a sub-set of a large (in principle infinite) set of co-occurrence matrices. A transportation measure is used to determine the difference between co-occurrence matrices resulting from various textures. This results in an ordered set of matrices, of which the resulting segmentation performance is directly related to the transportation measure. By combining segmentation results from various matrices the overall performance improves only when the matrices enhance different image areas. The most probable candidates for this can be obtained by using the same transportation measure applied to n dimensional co-occurrence data. Again, this results in an ordered set. Texture segmentation results indicate a monotone increase in performance when adding subsequent matrices results from the ordered set.

1. Introduction

Texture is one of the least understood areas in computer vision. Although no generic texture model has emerged so far a number of problem specific approaches have been developed successfully [3, 1, 7]. One of the major shortcomings of texture segmentation approaches has been the ad-hoc selection of the set of feature vectors. Recently, approaches have been investigated which aim to automatically determine a feature vector to be used for segmentation purposes [8]. The work described here can be seen to belong to the latter category. We present an approach to qualitatively select a sub-set of a large (in principle infinite) set of feature vectors. A transportation measure is used to determine the difference between feature vectors resulting from various textures. Transportation approaches have been used previously in computer vision applications [10, 5], but here it has been used in an automatic feature selection process.

We consider the segmentation of texture images. Here the main aim is to distinguish between two (or more) textures (see Fig. 1: www.outex.oulu.fi) and use the extracted information to get optimal segmentation results. Separate training, testing and ground truth information is provided, which enables both quantitative and qualitative evaluation.

1. Real texture examples.

Although we use co-occurrence matrices to illustrate the principles of the developed approach, the transportation measure and the resulting ordered set of feature vectors can also be used for other approaches such as Active Appearance Modelling [2], texture based image simulation [9] and other texture based segmentation approaches [3, 11].

2. Methods

The initial step is to obtain a transportation based measure to indicate the difference between co-occurrence matrices from different textures. Using a large set of translations to generate co-occurrence matrices, this results in an
ordered set of co-occurrence matrices. The highest transportation measure is used to select the co-occurrence matrix for the initial segmentation. Subsequently, additional co-occurrence matrices can be used to improve the segmentation results, where the nth additional co-occurrence matrix is selected based on m transportation measures, where m is the size of the set of co-occurrence matrices.

2.1. Co-Occurrence Matrices

The co-occurrence of grey-levels in an image I at points \((p, p + t)\) separated by a translation \(t\) is represented by

\[
\Psi^t = [\psi^t_{i,j}]_{i,j \in N_g}
\]

and

\[
\psi^t_{i,j} = \# \{p \in I, p + t \in I \mid I(p) = i, I(p + t) = j\} \tag{2}
\]

where \# denotes the number of elements obtained from \(I\) and \(N_g\) denotes the set of grey-level values.

2.2. Transportation

Transportation is a well known linear programming approach which has been described extensively in the literature \([4, 10, 5]\). In the classical case the optimal transportation cost is determined to move goods from a set of warehouses to a set of shops. The cost tends to be related to the distance between warehouses and shops.

Here we use the same transportation approach to determine the difference/similarity between co-occurrence information from different textures. The cost function is determined by the Manhattan distance which is taken between positions in the co-occurrence matrices. The cost, \(k\), is given by

\[
k(i_1, j_1, i_2, j_2) = |i_1 - i_2| + |j_1 - j_2| \tag{3}
\]

where \(i_1, i_2, j_1, j_2\) represent grey-level values in the co-occurrence matrices from two textures.

The total cost associated with matching two matrices requires the summation over all positions \((i, j)\) and movement of occurrences. This is given by

\[
T^t = \sum_{i_1,j_1} \sum_{i_2,j_2} k(\psi^t_{i_1,j_1}, \psi^t_{i_2,j_2}) \tag{4}
\]

where \(\delta()\) indicates the number of occurrences that are moved from \(\psi^t_{i_1,j_1}\) to \(\psi^t_{i_2,j_2}\) at a cost \(k\) per occurrence. It should be noted that, because we use a constant image size, for each texture

\[
\sum_{i_1,j_1} \psi^t_{i_1,j_1} = \sum_{i_2,j_2} \psi^t_{i_2,j_2} \tag{5}
\]

2.3. Level 1 Segmentation

At every point \(p\) within an image \(I\) an estimation of the likelihood of that point belonging to a texture that generated the co-occurrence matrix \(\Psi^t\) is given by

\[
L^t(p) = \sum_{i_1,j_2} \psi^t_{i_1,j_2} (k(i_1, j_1, i_2, j_2) + 1)^{-1} \tag{6}
\]

where \(I(p) = i_1\) and \(I(p + t) = j_1\). The value 1 has been added in \((k() + 1)^{-1}\) to avoid the singularity at \(k() = 0\).

A likelihood estimate can be obtained for every texture under consideration. Subsequently, the various likelihood estimates can be combined to provide a probability measure for texture \(\alpha\) by

\[
P^t_\alpha(p) = L^t_\alpha(p)/\sum_\beta L^t_\beta(p) \tag{7}
\]

where \(\beta\) is the set of all textures under consideration.

2.4. Level n Segmentation

The initial (level 1) segmentation described in the previous section only uses a single co-occurrence matrix from each texture image. To extend this to the use of multiple co-occurrence matrices at the classification stage we use the following methodology.

At level 1 the highest transportation measure determines the first co-occurrence translation in an \(n\) sequence. The first translation was obtained from 2D co-occurrence data. The \(n^{th}\) translation can be obtained from \((n + 1)D\) co-occurrence data. There are several limitations on using a direct transportation approach as described in Sec. 2.2 for this higher dimensional case. The co-occurrence data becomes sparse and the size of the cost-function used in the transportation scales with the power of \((n + 1)\) which restricts the direct approach to low dimensional cases.

Instead of using \((n + 1)D\) co-occurrence data we obtain \(\sum_{i} (\sum_{i} i)\) 2D projection co-occurrence matrices from the \((n + 1)D\) co-occurrence data. The \(n^{th}\) co-occurrence matrices can be used to obtain \(\sum_{i} i\) transportation measures using the approach described in Sec. 2.2. An overall transportation measure is obtained by multiplying the \(\sum_{i} i\) transportation measures and normalising this by raising the result to the power \((1/\sum_{i} i)\). The resulting set of transportation measures can be ordered to provide the next co-occurrence translation.

This approach has the added advantage that only one additional set of transportation measures needs to be obtained for each next level in the sequence.

In this case, the likelihood is determined by an \((n + 1)\) dimensional version of Eq. 6.
3. Results

To evaluate the developed approach a number of aspects are considered. These range from a visual distribution of the transportation measure for different translations (see Sec. 3.1) and the relationship between the transportation measure and the classification performance (see Sec. 3.2) to the n level segmentation results (see Sec. 3.3). In all cases the set of co-occurrence matrices comprised all possible permutations within a $65 \times 65$ window. In addition, the co-occurrence matrices used a grey-level range reduced to 5 bits.

3.1. Transportation

As described in Sec. 2.2, the resulting total cost $T$ depends on the translation $t$ and the difference between the textures under investigation. Two examples of total cost distributions for the textures shown in Fig. 1 can be found in Fig. 2, where the result of a zero translation can be found in the centre of the distribution. Each other point in the distribution represents the total cost for the translation between that point and the centre. It should be clear that some of the repetitive structures within the texture images are represented in the total cost distributions. In addition, it is possible to use these distributions to select the translation which shows the largest difference between the co-occurrence matrices from two textures.

![Figure 2. Transportation total cost distribution for the texture examples shown in Fig. 1.](a) (b)

3.2. Level 1 Segmentation

Some example results of a level 1 segmentation can be found in Fig. 3, where white and black represent class and non-class, respectively. It should be made clear that these results are based on the co-occurrence distribution between two points in the image data and as such only a 2D feature vector is used at the classification stage.

![Figure 3. Level 1 segmentation (Eq. 7) of the real texture examples shown in Fig. 1.](a) (b)

Such level 1 segmentation results can be obtained for various co-occurrence translations. The segmentation results can be summarised by the area under the receiver operating characteristic curve [6]. For each such translation a transportation measure can also be obtained. Fig. 4 shows the correlation between these aspects for various texture combinations and a large number of translations.

![Figure 4. Transportation measure versus classification performance. Markers indicate different texture combinations: (△) Fig. 1a and (□) Fig. 1b.](a) (b)
The results in Fig. 4 show that for each texture combination there is a strong correlation (correlation coefficient equal to 0.92 and 0.98 for the examples shown) between the transportation measure and the segmentation based on co-occurrence data from each texture.

### 3.3. Level n Segmentation

The results of $n$ level segmentation is shown in Fig. 5, which again shows the area under the receiver operating characteristic curve as a function of the transportation measure. In this case an increase in the transportation measure represents subsequent levels in the segmentation process and here the graph represents levels 1 to 10. The level 1 results, the lowest transportation measure and classification results in Fig. 5, are associated with the highest transportation measure found in Fig. 4 for each texture combination.

**Figure 5.** Classification performance for cumulative, i.e. the number of co-occurrence matrices from an ordered set, segmentation results. Markers indicate different texture combinations: $(\triangle)$ Fig. 1a and $(\square)$ Fig. 1b.

The results in Fig. 5 indicate texture dependent behaviour. One of the texture combinations has a correlation coefficient close to unity. The second texture combination shows overall segmentation improvement as a function of the transportation measure, but this saturates for higher transportation values.

### 4. Discussion and Conclusions

The current approach can be used to discriminate between two textures. One of the aspects of further investigation will be to extend the presented framework to discriminate between more than two textures and colour space. In the first instance this will look at the possibility of decomposing the higher dimensional problem in the same way as was done in Sec. 2.4.

In summary, we have shown that a transportation measure based on a set of co-occurrence matrices from various textures can be used to select that co-occurrence matrix that provides an optimal segmentation for this 2D feature space. This can be extended to an $n$D co-occurrence space and a novel methodology is used to link this to $(\sum_i^n)\text{co-occurrence matrices}. At each level an optimal feature vector can be selected and is shown that an overall transportation measure is correlated to segmentation results.

### Acknowledgements

A research grant from *Prostate Cancer UK* is greatfully acknowledged.

### References