A Precoding Scheme for DFT-Based OFDM to Suppress Sidelobes
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Abstract—In spectrum pooling scenario, the spectral leakage of DFT-based OFDM signal can be divided into in-band-out-of-subband (IBOSB) radiation and out-of-band (OOB) radiation. A precoding scheme is proposed to suppress the IBOSB sidelobes. The precoding design is based on the generalized eigenvalue problem. Simulation results demonstrate that the proposed scheme can suppress the sidelobes significantly in contrast to the uncoded schemes. At the same time, the Bit-Error-Rate (BER) performance of the precoded system over multipath fading channel is improved due to the frequency diversity.

Index Terms—OFDM, DFT, precoding, power spectrum.

I. INTRODUCTION

OFDM is considered to be one of the promising candidates for spectrum pooling systems [1], for its potential of spectrum shaping with flexibility by deactivating some subcarriers. However, the spectral leakage of OFDM signals is too high to guarantee no interference with licensed user (LU).

To mitigate this problem, several effective methods have been proposed recently [2][3][4][5]. They are designed on the basis of the analog representation of OFDM transmitter. As the digital modulation of OFDM subcarriers can be performed using Inverse DFT (IDFT), which can be implemented by the Fast Fourier Transform (FFT) algorithm efficiently, DFT-based OFDM realization is commonly accepted. According to [6], signal spectrum based on the analog representation is not appropriate to analyze the out-of-band radiation when the system employs the digital implementation. Therefore, the aforementioned methods, especially [2] and [3], are not appropriate for DFT-based OFDM.

Through analyzing the spectrum of DFT-based OFDM signal, a new design method of precoding is proposed to suppress the sidelobes by introducing the correlation between the subcarriers. The proposed scheme has the following features: it is robust to the cyclic prefix (CP) length; it has no effect on the peak-to-average-power-ratio (PAPR); and it also improves the system performance over the multipath fading channel at high SNR. Other effective suppression methods, such as [7][8][9], can be combined with the precoding to further suppress the sidelobes.

II. PRECODED OFDM SIGNAL MODEL

A DFT-based OFDM transmitter with precoding is depicted in Fig. 1. The block index is $l$, and $c(l) = [c_{0}^{T} \cdots c_{M-1}^{T}]^{T}$ is the $l$th length-$M$ modulated data column vector, where $(\cdot)^{T}$ denotes the transpose. Map it onto $d(l) = [d_{0}^{T} \cdots d_{N-1}^{T}]^{T}$ via the precoding

$$d(l) = Gc(l),$$

where $G_{N \times M} = [g_{n,m}]$ is precoding matrix with $N \geq M$. Define the precoding efficiency as $p = M/N$. Through IDFT (with DFT-size $N$) and CP insertion (with CP length $\nu$), the resultant discrete time-domain samples constitute a block of length $K = N + \nu$.

$$x_{k}^{(l)} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} d_{n}^{(l)} e^{j2\pi(k-\nu)n/N}, \quad k = 0, \ldots, K - 1. \quad (2)$$

After parallel-to-serial (P/S) conversion, the DAC outputs the complex envelope of the DFT-based baseband OFDM signal $b(t) = \sum_{l=-\infty}^{\infty} \sum_{k=0}^{K-1} x_{k}^{(l)} p(t - (lK + k) T_{s}), \quad (3)$

where $p(t)$ is reconstruction filter and $T_{s}$ is the sampling period. Let $P(j\Omega)$ denotes the continuous time Fourier transform of $p(t)$. The power spectrum of $b(t)$ is given by [6]

$$B(\Omega) = |P(j\Omega)|^{2} E\{ |X^{(l)}(j\Omega)|^{2} \}, \quad (4)$$

where $E[\cdot]$ represents the expectation, and

$$X^{(l)}(j\Omega) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} d_{n}^{(l)} \sum_{k=0}^{K-1} e^{j2\pi(k-\nu)n \Omega} e^{-j2\pi kT_{s}}. \quad (5)$$

which can be described as the sum of the spectrum of individual $N$ subcarriers.

Equation (4) can be explained separately. On one hand, because the cutoff frequency of the reconstruction filter $P(j\Omega)$ is equal to $\pi/T_{s}$, the spectral roll-off of $P(j\Omega)$ directly affects the OOB spectral leakage of OFDM signal. On the other hand, since $X^{(l)}(j\Omega)$ is periodic with a period $2\pi/T_{s}$, it is completely specified by its behavior in the baseband. For the secondary user (SU) using OFDM, the subcarrier indices $n \in \mathcal{Z}$ ($\mathcal{Z} \cup \mathcal{U} = [0, N - 1]$) are corresponding to

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the previously-sensed-as-unused subband $B_u$. Meanwhile, the subcarriers, indexed by $n \in \mathcal{U}$ (corresponding to the LU-using subband $B_u$), are deactivated by SU. The sidelobe power of the SU signal spills to $B_u$ and is denoted as the In-Band-Out-Of-Subband (IBOSB) power.

Substituting (1) into (5), the calculation is proceeded as

$$X^{(l)}(j\Omega) = \frac{1}{\sqrt{N}} \sum_{m=0}^{M-1} c_m^{(l)} \sum_{n=0}^{N-1} g_{n,m} \sum_{k=0}^{K-1} e^{j\pi(k-n)\Omega} e^{-jkm\Omega}.$$  

Thus the precoding introduces the correlation to the subcarriers, and helps the spectrum sidelobes to cancel each other if well designed. Resultantly, the IBOSB power is suppressed and the OOB power is redistributed.

III. DESIGN OF PRECODING MATRIX

Alternatively, (6) can be written in vector-matrix form as $X^{(l)}(j\Omega) = e(j\Omega) F^H D G C^{(l)}$, where $e(j\Omega)$ is a $1 \times K$ vector with the $k$th entry $e^{-jk\Omega T_s}$, $F_{N \times K}$ is a Fourier matrix with the $(n,k)$th entry $[F]_{n,k} = (1/\sqrt{N}) e^{-j\pi nk/N}$, $(\cdot)^H$ denotes conjugate transpose, and $D = \text{diag} \{ 1, e^{-j2\pi/n}, \ldots e^{-j2\pi (N-1)/N} \}$.

Meanwhile, assume the modulated data $c^{(l)}$ contains the complex variables from a given MPSK (Multiple Phase-Shift-Keying) constellation. It has the mean $E\{e^{(l)}\} = 0$ and covariance $\text{Var}(c^{(l)}) = \sigma^2 I_M$ with $I_M$ being $M \times M$ identity matrix. Thus the second part of (4) is equal to

$$E\{ |X^{(l)}(j\Omega)|^2 \} = \sigma^2 \text{tr} (G^H D^H F H(j\Omega) e(j\Omega) F^H D G),$$

where $\text{tr}(\cdot)$ denotes trace. To measure the characteristic of (7), define the contrast energy ratio (CER), which is the ratio between the energy of IBOSB (i.e. in $B_u$) and that of In-Band-In-Subband (IBIS) (i.e. in $B_s$)

$$\text{CER} = \frac{\int_{B_u} E\{ |X^{(l)}(j\Omega)|^2 \} \, d\Omega / 2\pi}{\int_{B_s} E\{ |X^{(l)}(j\Omega)|^2 \} \, d\Omega / 2\pi}.$$  

Define two matrices $\tilde{P} = \int_{B_u} e^{H(j\Omega)} e(j\Omega) d\Omega$ and $P = \int_{B_u} e^{H(j\Omega)} e(j\Omega) d\Omega$. Thus, in order to reduce the IBOSB power, we construct a CER minimization problem

$$\min_G \left( \frac{\text{tr} (G^H A G)}{\text{tr} (G^H B G)} \right) \quad \text{subject to} \quad G^H G = I,$$

where $A = D^H F \tilde{P} F^H D$ and $B = D^H F P F^H D$ are two Hermitian matrices. It can be treated as a generalized Hermitian eigenvalue problem, and $G$ provides a basis for the invariant subspace of eigenvectors corresponding to the $M$ smallest eigenvalues of $B^{-1}A$. We appeal to the singular value decomposition (SVD) [10] (named in honor of Stiefel and Grassmann) and follow its trace-minimization example to derive $G$. The output $G$ satisfies the orthogonality constraint, which guarantees that the power is invariant during the precoding. Thus, the PAPR of the system is expected to be invariant (simulation not presented here confirms this).

Furthermore, since some subcarriers are nulled and the corresponding rows of $G$ are zeros, $A$ and $B$ can be deflated.

Fig. 2. Power spectrum comparison in the case of $\rho = 52/64$ and $\nu = N/4$.

Meanwhile, both the transmitter and the receiver can calculate the precoding matrix only once by themselves respectively, as long as the available subbands are identified. Therefore, neither block-wise optimization methods nor handshakes between transmitter and receiver, as [3][4][5], are needed. As a result, the precoding scheme requires less computation complexity.

IV. SYSTEM PERFORMANCE

In this section, some simulation results are presented to show the effect of the precoding on the performance of OFDM system. A simple spectrum pooling scenario is considered to hold a frequency band, which is divided into four equal-wide subbands. The LU’s are using the 1st and the 3rd subband, while the SU’s span all 4 subbands with $N = 128$ OFDM subcarriers, i.e., 32 subcarriers each subband. The subcarriers of SU falling into the 2nd and the 4th subband are data subcarriers, while the others are deactivated.

A. Sidelobe suppression

The proposed scheme is compared here with three sidelobe suppression schemes. The first is the conventional OFDM with guard subcarriers. The second scheme leaves some subcarriers on each edge of the available subbands as the cancelling subcarriers (CC) [4]. In simulations, the power of CC is limited to $1/5$ of the total transmission power. To guarantee the same spectrum efficiency, smaller $\rho$ for precoding means more guard or cancelling subcarriers at each end of the subbands for conventional and CC respectively. The third scheme is Subcarrier Weighting (SW) [5] with the ratio of the maximum to minimum weighting factor less than 3/2. For the latter two schemes, the optimization is based on (4), but not the models in [4][5], which use the analog representation of OFDM signal.

Fig. 2 presents the power spectrum in the range of $\Omega \in [-2\pi/T_s, 2\pi/T_s]$. All results are averaged over $10^5$ blocks and for each subcarrier space, there are 2 samples. We take $p(t)$ as a rectangular pulse, and its frequency response is also shown in Fig. 2 for reference. We can observe that the mainlobe of the frequency response of $p(t)$ covers the two periods of
TABLE I
CER COMPARISON (IN dB) WITH RESPECT TO ρ AND ν

<table>
<thead>
<tr>
<th></th>
<th>variable ρ, fixed ν = N/4</th>
<th>variable ν, fixed ρ = 56/64</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>60/64</td>
<td>56/64</td>
</tr>
<tr>
<td>Conventional</td>
<td>-22.3</td>
<td>-22.8</td>
</tr>
<tr>
<td>SW</td>
<td>-26.0</td>
<td>-26.4</td>
</tr>
<tr>
<td>CC</td>
<td>-24.0</td>
<td>-28.6</td>
</tr>
<tr>
<td>Precoding</td>
<td>-28.9</td>
<td>-33.9</td>
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</tbody>
</table>

B. Decoding and BER performance over fading channel

Of particular interest is the decoding of the precoded data at the receiver. Assume the lth received block after CP removal and FFT processing can be written as [11]

\[ y^{(l)} = FH^{(l)}F^HGe^{(l)} + n^{(l)}, \]  

(10)

where \( y^{(l)} \) is an \( N \times 1 \) vector; \( H^{(l)} \) is the channel matrix with the first column \( h(0) \cdots h(L) \cdots 0 \) and \( h(\cdot) \)'s denote the \( L+1 \) i.i.d. Rayleigh distributed taps, and \( n^{(l)} \) is the \( N \times 1 \) complex AWGN vector with zero mean and variance \( \sigma_n^2 \). A simple MMSE detector can be used and the detected vector is given as

\[ c_{\text{MMSE}}^{(l)} = QH(\sigma_n^2I_N + QQ^H)^{-1}y^{(l)}, \]  

(11)

where \( Q = FH^{(l)}F^H \).

For simulation, each OFDM block includes 56 QPSK-mapped data and \( \nu = N/4 \). Two multipath fading channels generated by Jakes model are 6 taps and 4 taps for channel A and B respectively. Besides, assume the maximum delay spread is less than the CP length and the receiver knows the perfect channel state information. The transmitting power is identical under the condition of same spectrum efficiency because \( G \) is unitary. Therefore, the comparison is fair.

As Fig. 3 presents, the precoding improves the performance in BER considerably when bit SNR is above a threshold in contrast to the conventional OFDM with guard subcarriers. By spreading the power of individual \( c_m \) onto all the active subcarriers, the proposed scheme takes advantage of the frequency diversity [11]. In the case of low SNR, the BER performance can be guaranteed by the channel coding.

V. CONCLUSION

To suppress spectral leakage of the DFT-based OFDM in the spectrum pooling scenario, precoding ahead of the IDFT is considered. Under both conditions of the identical spectral efficiency and CP length, the precoding scheme can achieve better capability of suppressing IBOSS radiation than conventional OFDM, CC and SW. Also, precoding does not increase the PAPR of the OFDM signal, and the precoded system achieves better BER performance on multipath fading channel than the uncoded systems.

REFERENCES