Spectral Leakage Suppression of DFT-based OFDM via Adjacent Subcarriers Correlative Coding

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Abstract—In spectrum pooling applications, the spectral leakage of DFT-based OFDM systems should be divided into two parts, In-Band-Out-of-SubBand (IBOSB) leakage and Out-Of-Band (OOB) radiation. A scheme is proposed to suppress the IBOSB leakage and redistribute OOB radiation through the frequency domain correlative coding with appropriate weighting. It introduces the correlation between adjacent subcarriers, and the DFT-based spectrum is reshaped through mutually cancelation of the sidelobes. The optimum weighting factor is given. Analysis and simulation show that the proposed scheme significantly suppresses IBOSB spectral leakage compared to conventional OFDM. At the same time, Carrier-Interference Ratio (CIR) is increased and Maximum Likelihood Sequence Detection (MLSD) can mostly compensate the BER degradation caused by error propagation at the expense of a little computation complexity.

I. INTRODUCTION

With underlying spectrum shaping capabilities, OFDM is one of the most attractive techniques for spectrum pooling systems [1] or cognitive radio applications [2]. As different spectrum owners (Primary User, abbreviated as PU) bring their frequency bands into a common pool, the Secondary Users (SU) or cognitive radio users may rent and exploit the spectral opportunities. One of the challenges, which SUs must be faced, is to avoid interference with the operating PUs. Unfortunately, the rectangular window, which the conventional DFT-based OFDM systems utilize to operate on the parallel symbol block, has large spectral sidelobes [3]. Consequently, the spectral leakage of OFDM signals is too high to meet the requirements of spectrum pooling applications.

To mitigate this problem, many methods have already been employed, such as the insertion of guard subcarriers [4] or cancelation subcarriers [5], windowing (in time domain) [6][7][8], and using various pulse shaping filter or filtering before transmitting [9]. It is known that the guard subcarriers decrease the spectral efficiency, while windowing reduces the delay spread tolerance and filtering is more complex with latent of distorting the wanted signals.

Several approaches have been proposed to get significant sidelobe suppression recently by linearly precoding each data block [10][11][12]. These spectral precoding schemes, also denoted as correlative coding, introduce correlation among data symbols in frequency domain and thus reshape the power spectrum density (PSD) of the rectangularly pulsed OFDM signal. The computation complexity of these methods cannot be neglected; importantly, they are designed for the pulse-shaping OFDM transmitter, which is modeled on analog representation. According to Lin and Phoong [3], signal spectrum based on the analog representation is not appropriate to analyze the OOB radiation when the system employs the digital implementation which is commonly accepted. Therefore, it is worthwhile to design the spectral correlative coding based on DFT representation, combining with other methods to control the spectral leakage of OFDM systems.

In this paper, we assume that given a frequency band in the pool, the SU OFDM transmitter spans the entire band with its total subcarriers. The spectral leakage is divided into two parts. We denote the spectral leakage of SU data subband (grouped by data subcarriers) as IBOSB radiation, which is partly different to OOB radiation. The IBOSB and OOB radiation have some common features in that they are all spectral leakage from SU to PU, while OOB radiation is spilling over from the SU band to the other PU-occupied band.

A novel method, which uses weighted correlative coding between adjacent subcarriers, is proposed in this letter. The content is organized as follows. Section II starts with the spectrum analysis of the DFT-based OFDM signal after weighted correlative coding. The optimum weighing factor for correlated adjacent subcarriers is given in Section III. Furthermore, Section IV analyzes the CIR of the system using the proposed method. Section V presents the simulation and comparison on spectral leakage, CIR and BER of several OFDM systems. Conclusions are presented in section VI.

II. POWER SPECTRUM OF DFT-BASED OFDM WITH CORRELATIVE CODING

Since the analog representation of OFDM signal is not suitable to analyze the spectral roll-off of DFT-based OFDM transmitter, Lin’s model in [3] is modified here to study the correlative-coding-added effect on the signal PSD. Fig.
1 depicts the typical OFDM system in conjunction with the correlative coding. The data block index is \( l \), and \( \{c_{n}^{(l)}\}_{n=0}^{N-1} \) is the \( n \)th original data sequence, which is mapped into complex data sequence \( \{d_{n}^{(l)}\}_{n=0}^{N-1} \) via the weighted correlative coding

\[
d_{n}^{(l)} = \frac{\sqrt{3}}{2} (e^{j\theta} c_{n-1}^{(l)} + c_{n}^{(l)}), \quad n = 0, \ldots, N-1, \tag{1}
\]

where \( N \) is sequence length, \( e^{j\theta} \) is the weighting coefficient and \( c_{n}^{(l)} \) is defined to be 0. In order to guarantee the transmitted signals of systems with and without coding have the similar energy per bit, the signal is multiplied by \( \sqrt{2}/2 \). Through IDFT (with DFT-size \( N \)), Cyclic Prefix (CP) attaching (with CP length \( \nu \)), and windowing, the resultant discrete time-domain samples (at point \( \oplus \) in Fig. 1) constitute a data block of length \( L = N + \nu \).

\[
x^{(l)}[k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} d_{n}^{(l)} e^{2\pi(j-k\nu)n/N} w_L[k], \quad k = 0, \ldots, L-1, \tag{2}
\]

Note that \( w_L[k] \) is a discrete time-domain window function, and it is commonly defined as a discrete rectangular window, i.e., \( w_L[k] = 1 \) for \( 0 \leq k \leq L-1 \) and 0 else.

The parallel symbols in block \( x^{(l)} = [x^{(l)}[0], x^{(l)}[1], \ldots, x^{(l)}[N-1]] \) are converted to a symbol sequence and fed into DAC (Digital-Analog Converter), where the sequence is extended to an impulse train by sample-and-hold circuit, with each symbol duration being \( T_s \) and the corresponding block duration being \( T = NT_s \). Afterwards, the impulse train is filtered by a lowpass filter \( h(t) \). Finally, the DAC output at point \( \oplus \) is complex envelope of the baseband OFDM signal and can be expressed as

\[
b(t) = \int_{-\infty}^{\infty} h(t - \tau) \left\{ \sum_{k=-\infty}^{+\infty} \delta(\tau - kT_s) x[k] \right\} d\tau, \tag{3}
\]

where \( \delta(\tau) \) is the Dirac impulse function, and \( \{x[k]\} \) is a concatenation of the blocks \( x^{(l)} \).

Quoting [3][7], the PSD of \( b(t) \) is described as

\[
\Phi(\Omega, \theta) = |H(j\Omega)|^2 \mathbb{E} \left[ \sum_{n=0}^{N-1} d_n W_L \left( e^{j(\Omega T_s - 2\pi n/N)} \right)^2 \right], \tag{4}
\]

where \( \mathbb{E}[\cdot] \) and \( H(j\Omega) \) represent expectation and frequency response of \( h(t) \), respectively. \( W_L \left( e^{j\Omega T_s} \right) \) is Discrete Time Fourier Transform (DTFT) of \( w_L[k] \) and can be written as

\[
W_L \left( e^{j\Omega T_s} \right) = e^{-j\Omega T_s / 2} \frac{\sin(L \Omega T_s / 2)}{\sin(\Omega T_s / 2)},
\]

which is periodic in \( \Omega \) with a period \( \Omega_s = 2\pi/T_s \). The fundamental period in the range \( \Omega < |\Omega_s|/2 \) is sometimes referred to as the baseband. In the case that \( c_{n}^{(l)} \) are uncorrelated and have a variance \( \mathbb{E}[|c_{n}^{(l)}|^2] = \sigma^2 \), (4) can be rearranged as

\[
\Phi(\Omega, \theta) = |H(j\Omega)|^2 \sigma^2 \sum_{n=0}^{N-2} \left| W_L + e^{j\theta} W_L^{n+1} \right|^2, \tag{5}
\]

where \( W_L^{n} = W_L \left( e^{j(\Omega T_s - 2\pi i/n)} \right) \).

\( \Phi(\Omega, \theta) \) can be explained separately. According to the periodicity of \( W_L \left( e^{j\Omega T_s} \right) \), the function \( \Phi^{i}(\Omega, \theta) \) in \( \Omega \geq |\Omega_s|/2 \) (i.e. OOB range of baseband signals) is periodic replications of its In-Band part. Reshaping \( \Phi^{i}(\Omega, \theta) \) is mainly used to control IBOSS power and redistribute the OOB power, while in the circumstances of discontinuous subband OFDM applications, reconstruction filter \( H(j\Omega) \) directly controls the spectral leakage out of the band of OFDM signal.

III. OPTIMUM CORRELATIVELY WEIGHTING COEFFICIENTS

This section will give the optimum weighting coefficient for two adjacent subcarriers to correlative encoding in order to suppress the IBOSS spectrum leakage. It is necessary to consider \( \Phi^{i}(\Omega, \theta) \) only in the frequency interval \( \Omega < |\Omega_s|/2 \). As the spectrum is combined for adjacent related subcarriers, the variable \( \theta \) plays an important role in the extent to which the sidelobes can be canceled. Provided that the total energy of \( \Phi^{i}(\Omega, \theta) \) within a period is fixed, minimizing the sidelobe energy, by selecting optimum \( \theta \), is equal to maximizing the mainlobe energy. In brief, the design problem of the optimum \( \theta \) can be stated as follows:

\[
\max_{\theta} \left\{ \int_{\Omega, t=\Omega, n=2}^{\Omega, t=\Omega, n=N-2} \Phi^{i}(\Omega, \theta) \frac{d\Omega}{2\pi} \right\}, \quad \text{for } i = 0, \ldots, N-2,
\]

Define \( \varphi = \frac{\Omega T_s - \pi i}{2} \) and \( \xi(\varphi) = \frac{\sin(L\varphi)}{\sin(\varphi)} \), therefore

\[
\Phi^{i}(\varphi, \theta) = \left| W_L^{n} + e^{j\theta} W_L^{n+1} \right|^2 = \xi^2(\varphi) + \xi^2(\varphi - \pi) + 2\xi(\varphi)\xi(\varphi - \pi) \cos(\theta + (L - 1) \frac{\pi}{N}). \tag{6}
\]

Differentiate (6) with respect to \( \theta \) and make it to zero, and resultanty the solution arrives at \( \theta = - (L - 1) \pi/N \) or \( \theta = \pi - (L - 1) \pi/N \). According to the polarity of the derivative, the minimum sidelobe leakage of adjacent subcarriers pair is achieved if and only if

\[
\theta = - (L - 1) \pi/N, \tag{7}
\]

which is the optimum weighting factor. In addition, for non-CP situation, the optimum \( \theta \) is attained by substituting \( L = N \).

IV. INFLUENCE ON SYSTEM PERFORMANCE

As the correlation between subcarriers is introduced, the statistical characteristic of the OFDM signal is changed. It is necessary to analyze the influence of weighted correlative coding on the system performance.
A. CIR Analysis

Since the proposed method resembles Zhao’s CIR enhancement scheme with (1-D) correlative coding [13] in that the same symbol modulates a pair of adjacent subcarriers simultaneously, it is possible for transceiver to be less sensitive to Carrier Frequency Offset (CFO). Assume that $b(t)$ is transmitted on an Additive White Gaussian Noise (AWGN) channel. In the $l$th block period, the received signal sample on the $k$th subcarrier after FFT demodulation can be written as [13]

$$r^{(l)}[k] = d^{(l)}_k S_0 + \sum_{p=0, p \neq k}^{N-1} d^{(l)}_p S_{p-k} + n^{(l)}[k],$$

where the function $S_{p-k}$ reflects the interfering strength from the $p$th subcarrier to the $k$th subcarrier and its detail is identical to [13, (2b)] as

$$S_{p-k} = \frac{1}{N \sin(\pi (p-k + 3)/N)} \exp(j \frac{\pi}{N} ((N-1) - (p-k))),$$

where $\varepsilon$ represents the normalized CFO with respect to the OFDM subcarrier spacing $\Delta f = 1/NT_s$.

In (8), the first term is the desired part of the received signal, the second term (denoted as $I^{(l)}_k$) represents the sum of interferences from other subcarriers, and $n^{(l)}[k]$ is the sample of a zero-mean Gaussian distributed random variable.

Now proceeding as in the proof of [13], the original symbol sequence $\{r^{(l)}[k]\}_{k=0}^{N-1}$ is assumed to fulfill the independence condition, as indicated in section II. Thereby, the correlatively coded sequence gets the correlation conditions as

$$E\left[d^{(l)}_k d^{(l)*}_p S_0\right] = \begin{cases} \sigma^2 & k = p \\ \frac{\sigma^2}{2} e^{j\pi \text{sgn}(k-p) \varepsilon} & k = p \pm 1 \\ 0 & \text{else} \end{cases}. \quad (10)$$

where $x^*$ denotes conjugate of $x$ and $\text{sgn}(x)$ indicates the sign of $x$. Note that actually $d^{(l)}_0$ has the variance $\sigma^2/2$, this difference can be omitted without affecting the analysis result obviously. Accordingly, the average carrier power is

$$E\left[|d^{(l)}_k S_0|^2\right] = \sigma^2 |S_0|^2. \quad (11)$$

Meanwhile, the average interfering power is

$$E\left[I^{(l)}_k^2\right] = \frac{\sigma^2}{2} \left\{ \sum_{p=1}^{N-1} |S_p|^2 + \sum_{p=2}^{N-1} \left( e^{-j\varepsilon} S_p S^*_p S_{p-1} + e^{j\varepsilon} S^*_p S_p S_{p-1} \right) \right\}. \quad (12)$$

Without loss of generality, it is assumed that $k = 0$ because of $S_p$’s periodicity. Thereby, theoretical CIR is given by

$$\text{CIR} = \frac{2 |S_0|^2}{\sum_{p=1}^{N-1} |S_p|^2 + \sum_{p=2}^{N-1} \left( e^{-j\varepsilon} S_p S^*_p S_{p-1} + e^{j\varepsilon} S^*_p S_p S_{p-1} \right)},$$

which is verified by simulations presented in the next section.

B. Optimum original data recovery

Provided that $c_{n}^{(l)}$ is chosen from a Gray-labeled symbol set, the squared Euclidean distance between adjacent symbol points is $\sigma^2$. Then after weighted correlative coding, the minimum squared Euclidean distance becomes

$$\delta_{\text{min}} = \frac{1}{2} \left| (c^{(l)}_i + c^{(l)}_j e^{j\theta}) - (c^{(l)}_j + c^{(l)}_i e^{j\theta}) \right|^2 \leq \sigma^2 (1 + \cos \theta).$$

Recall (7), $\cos \theta$ is always negative, so the minimum squared Euclidean distance is decreased.

On the other hand, because of the correlation between adjacent subcarriers, the error propagation across symbol-by-symbol-detected data tends to occur. These two characteristics will inevitably result in BER degradation. To conquer this problem, the received correlatively coded OFDM signal $\{r^{(l)}[k]\}_{k=0}^{N-1}$ can be coherently decoded by applying the Maximum-Likelihood Sequence Detection (MLSD) method [14] on each received useful signal block. The MLSD rule is to find $\{\hat{c}^{(l)}_k\}_{k=0}^{N-1}$ which yields the minimum squared Euclidean distance, that is,

$$\{\hat{c}^{(l)}_k\}_{k=0}^{N-1} = \min_{\{c^{(l)}_k\}_{k=0}^{N-1}} \sum_{k=0}^{N-1} \left| r^{(l)}[k] - \sqrt{2} (e^{j\theta} c^{(l)}_{k-1} - c^{(l)}_k) \right|^2. \quad (14)$$

The decoding procedure can be efficiently realized by the Viterbi algorithm. Generally speaking, MLSD is too computing-complex to be practically implemented. But in this case, the number of correlatively coded symbol levels is merely $M^2$, where $M$ is the size of the original symbols set. Meanwhile the memory length is just 2. Therefore, the additional computation complexity is proportional to $NM^2$, and the extra memory is proportional to $M$. Both are insignificant for system realization.

V. Simulation Results

In this section, some numerical and simulation results are presented to show the effect of the weighted correlative coding on the performance of OFDM systems.

A. Comparison of spectral leakage

A simple spectrum pooling system is considered to hold a frequency band, which is divided into four identical subbands. The PU transceivers are using the 1st subband and the 3rd subband. The SU spans total 4 subbands with 128 OFDM subcarriers, and 32 subcarriers each subband. The subcarriers of SU, falling into the 2nd subband and the 4th subband, are data subcarriers, while the others are deactivated to avoid interference to the working PUs. Moreover, SU transmitter usually takes $h(t)$ as a rectangular pulse reconstruction filter, and its frequency response $H(j\Omega)$ is given by

$$H(j\Omega) = T_s e^{-j\pi \Omega T_s/2} \sin \left( \frac{\Omega T_s}{2} \right), \quad \Omega \in (-\infty, +\infty).$$

(15)
Fig. 2. PSD of three OFDM systems is compared, such as conventional OFDM without coding, OFDM with correlative coding, and OFDM with subcarrier weighting. Additionally, $|H(j\Omega)|^2$ in (5) is for reference.

Fig. 3. CIR comparison among conventional OFDM (without coding), the proposed scheme (with correlative coding) and Zhao’s CIR enhancement scheme (with (1-D) correlative coding).

Fig. 4. Bit Error Rate characteristics of conventional OFDM and the proposed scheme, with $N = 128$, QPSK mapping, and CP length is $N/4$.

B. CIR simulation

Fig. 3 presents the theoretical and simulated CIR, versus the normalized CFO $\varepsilon$, of three schemes such as conventional OFDM without coding, the proposed scheme and Zhao’s CIR enhancement scheme with (1-D) correlative coding. All three systems use only a band of 32 subcarriers and QPSK symbol mapping. The theoretical CIR of the proposed (i.e. (13)), conventional and Zhao’s schemes (both from the reference [13]) agree with the computer simulations perfectly. Although the proposed scheme is a little inferior to Zhao’s method, which does aim at intercarrier interference (ICI) cancelation and CIR enhancement, it is obvious that it receives 3 dB CIR improvements relative to conventional OFDM for $0 < \varepsilon < 0.5$.

C. BER performance on AWGN and multipath fading channel

The proposed scheme and conventional OFDM without coding are compared here. Then $2 \times 10^7$ OFDM blocks are
It is assumed that the fixed normalized frequency offset $\epsilon$ to-noise-density ratio of both systems on AWGN channel. The conventional OFDM, the proposed scheme only suffers propagation-induced SNR loss. As indicated, compared with is augmented. But the MLSD can compensate this error-resides between the transmitter and receiver. For the weighted correlative coding and MLSD reception. The residual performance degradation of the proposed scheme degrades less than conventional OFDM. Note that the correlative coding is a little more robust to the frequency offset $f_d$, but the penalty $\Delta f$ is expressed independent of reconstruction filter of DAC. By introducing the correlation between adjacent subcarriers and weighting, the spectrum sidelobes of different subcarriers can be mutually canceled, and the proposed scheme achieves more than $10$dB sidelobes suppression in contrast to the conventional OFDM in the case of CP attachment. Furthermore, the proposed scheme achieves the same performance on spectral leakage suppression as the SW, but needs less computation complexity. At the same time, the system CIR is improved by $3$dB due to coding in frequency domain, thus it is a little more robust to the ICI. Additionally, the system BER characteristic, over AWGN channel and multipath fading channel, achieves no obvious degradation due to the correlative coding and MLSD reception.

VI. CONCLUSION

Through analyzing the power spectrum of DFT-based OFDM baseband signal and separating it to two parts, it is possible to reshape the spectrum and suppress the IBOSB spectral leakage by correlative coding independent of reconstruction filter of DAC. By introducing the correlation between adjacent subcarriers and weighting, the spectrum sidelobes of different subcarriers can be mutually canceled, and the proposed scheme achieves more than $10$dB sidelobes suppression in contrast to the conventional OFDM in the case of CP attachment. Furthermore, the proposed scheme achieves the same performance on spectral leakage suppression as the SW, but needs less computation complexity. At the same time, the system CIR is improved by $3$dB due to coding in frequency domain, thus it is a little more robust to the ICI. Additionally, the system BER characteristic, over AWGN channel and multipath fading channel, achieves no obvious degradation due to the correlative coding and MLSD reception.

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REFERENCES