NEW INSIGHTS INTO OPTIMAL ACOUSTIC FEEDBACK CANCELLATION
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Overview

- Feedback cancellation approach where canceler bases estimation on a probe signal.
- Optimum solution is biased. The solution is the product of the feedback path and the sensitivity function.
- Conditions to obtain an unbiased estimate: lower gains, or adequate forward path delay.
- Solution may still be biased if probe signal is spectrally shaped/masked.
- If canceler converges, bias term decreases over time.

System Description

Fig. 1: canceler’s estimation based on probe signal.
- where $G(q)$ is the feedback path,
- $K(q) = q^{-1} \cdot K$ is the forward path,
- $S(q) = \frac{1}{1-K(q)G(q)}$ is the sensitivity function,
- $m(n)$ is the microphone signal,
- $g(n)$ is the loudspeaker signal,
- $w_{in}(n) = M(q)w(n)$,
- $M(q)$ is some masking filter, and
- $w(n)$ is the injected probe.

2014 ICASSP - Florence, Italy

Biased Optimal Solution

Minimizing the mean square error cost function results in the Wiener filter

$$\hat{g}_w = \frac{E \{ w_{in}(n)w_m^T(n) \}^{-1} E \{ w_{in}(n)m(n) \} }{E \{ w_{in}(n)m(n) \} } \quad (1)$$

where $w_{in}(n) = [w_{in}(n), \ldots, w_{in}(n-L_0+1)]^T$, $L_0$ is the canceler’s length, and $\hat{g}_w$ is the set of optimal coefficients.

Expanding $m(n)$ in (1)

$$\hat{g}_w = \frac{E \{ w_{in}(n)w_m^T(n) \}^{-1} E \{ w_{in}(n)w_{in}(n) \} }{E \{ w_{in}(n)w_{in}(n) \} } \quad (2)$$

where $w(n)$ and $w_{in}(n)$ are uncorrelated, $w_{in}(n) = A(q)w(n)$, and $A(q) = G(q)S(q)$.

Expanding $w_{in}$ as

$$w_{in}(n) = A_{FIR}(q)w_{in}(n) + q^{-d_k}A_k(q)w_{in}(n) \quad (3)$$

where $A_{FIR}(q)$ is first $L_0$ coefficients of $A(q)$, and $q^{-d_k}A_k(q)$ is the residual impulse response.

Then, the optimal solution can be written as

$$\hat{g}_w = A_{FIR}(q)w_{in}(n) + q^{-d_k}A_k(q)w_{in}(n)$$

where $w_{in}(n) = A(q)w(n)$, and $A_{FIR}(q)$ is the coefficients for $A_{FIR}(q)$.

If $w_{in}(n)$ is white noise, then

$$\hat{g}_w = A_{FIR}(q)w_{in}(n) \quad (5)$$

Therefore, assuming that $w_{in}(n)$ is white noise, the optimal solution of the feedback canceler is not the feedback path but the product of the feedback path and the sensitivity function.

The work in [1] studies the impact of $M(q)$ on the canceler’s performance.

Conditions for Identifiability

Conditions for identifiability where the desired solution $\hat{G}(q) = G(q)$ can be obtained. If we write $A(q)$ as

$$A(q) = G(q) + q^{-d_k}K \cdot A(q)E(q) \quad (6)$$

where $E(q) = G(q) - \hat{G}(q)$, and
- noting that a delay, $d_k$, is contained in $K(q)E(q)$, then
- the first $d_k$ coefficients of $A(q)$ coincide with the impulse response of the feedback path.

If $d_k \geq L_0$ then $G(q)$ can be completely obtained from the first $L_0$ coefficients of $A(q)$, such as,

$$A(q) = g_0 + q^{-1}g_1 + \ldots + q^{-L_0+1}g_{L_0-1} + \ldots + q^{-d_k}a_{d_k-1} + \ldots \quad (7)$$

Reduce or remove bias term by

- using adequate delay $d_k$, and/or
- having lower gains (contradicts high gain requirement).

Let

$$\hat{A}(q) = \hat{G}(q)S(q) \quad (8)$$

be defined as the canceler, then $\hat{A}(q)$ is an unbiased estimate of $A(q)$.

Consider solution in (5) as $\hat{A}(q) \rightarrow A(q)$. From multiplying both sides of $E(q)$ by $S(q)$, $E(q)$ becomes

$$E(q) = \frac{A(q) - \hat{A}(q)}{1 + K(q)[A(q) - \hat{A}(q)]} \quad (9)$$

- if $\hat{A}(q) \rightarrow A(q)$ then $E(q) \rightarrow 0$, thus
- the bias term is reduced over time if the system converges, i.e. $A(q) \rightarrow G(q)$.

Simulation Results

- NLMS algorithm, $\mu = 0.01$, white Gaussian noise (WGN) as probe signal. Presented: average of 50 simulation runs w/ a new realization of WGN sequence for each run.

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Figure 1: Probe signal approach for feedback cancellation.

Figure 2: Varied gain $K$: misalignment between $G(q)$ & $A(q)$.

Figure 3: Varied $d_k$: Misalignment between $G(q)$ & $A(q)$.

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