Multi-Antenna Techniques in Fixed Wireless Links

Invited Paper

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Abstract—We present a summary of results based on several years of measurements for urban fixed wireless (FW) channels. Our results are aimed at providing reference data for practical applications in FW services. We discuss temporal stability and fade dynamics of outdoor-outdoor single-input single-output (SISO) channels with and without spatial diversity, a relevant aspect when considering the types of modulations that such a channel can support. Our main findings are that nearby vehicular traffic is the main source of temporal variations, but that in most practical situations temporal fades are very shallow. Spatial fades are quite significant and are stable over time. Diversity has the twofold effect of mitigating these fades and increasing temporal stability. We also present practical results on outdoor-indoor multiple-input multiple-output (MIMO) channels. Based on empirical data we contrast spectral efficiencies for diverse types of systems, ranging from those that only use multiple antennas for beamforming to those that take full advantage of channel state information. We find that for low signal to noise ratio (SNR), beamforming achieves in practice spectral efficiencies comparable to that of MIMO and that it retains a noise ratio (SNR), beamforming achieves in practice spectral efficiencies comparable to that of MIMO and that it retains a noise ratio. If this value is low, MIMO transmission may only be able to use a single spatial channel, and beamforming schemes may achieve comparable efficiencies [4]. While several simulation based studies have been reported [5], comparison based on actual data is very scarce. We summarize several years of measurements for urban FW channels. We discuss temporal stability and fade dynamics of outdoor-outdoor SISO channels with and without spatial diversity, a relevant aspect when considering the types of modulations that may be used to achieve a high spectral efficiency. We also present empirical results on outdoor-indoor MIMO channels and contrast spectral efficiencies for diverse types of systems, ranging from those that only use multiple antennas for beamforming to those that take full advantage of channel state information (CSI).

I. INTRODUCTION

Fixed wireless links may be very effective to deliver broadband access to homes and dwellings. However, fixed wireless (FW) links have received relatively little attention, reflecting the only more recent trend in offering wireless broadband services to static users [1]. As has been reported [2], the propagation channel is very different for fixed versus mobile terminals and may permit much higher rates with fixed terminals. Adaptive modulation and coding techniques are required to adjust to the normally broad range of channel conditions. The design of such techniques obviously requires statistical knowledge of the channel. High speed transmission justifies modeling the behavior of narrowband channels, such as those used by a single orthogonal frequency division multiplexing (OFDM) carrier. Frequency response parameters such as the coherence bandwidth, allow the study of broadband channels using multiple carriers. A relatively recent development in the search for greater spectral efficiency, is the broad range of multi-antenna techniques known as multiple-input multiple-output (MIMO). Such systems have been extensively studied over the last decade [3]. They can lead to significant gains over single-input single-output (SISO) transmission schemes, using spatial multiplexing to allow the creation of parallel transmission channels. However, the gains are dependent on the channel conditions and the signal to noise plus interference ratio. If this value is low, MIMO transmission may only be able to use a single spatial channel, and beamforming schemes may achieve comparable efficiencies [4]. While several simulation based studies have been reported [5], comparison based on actual data is very scarce. We summarize several years of measurements for urban FW channels. We discuss temporal stability and fade dynamics of outdoor-outdoor SISO channels with and without spatial diversity, a relevant aspect when considering the types of modulations that may be used to achieve a high spectral efficiency. We also present empirical results on outdoor-indoor MIMO channels and contrast spectral efficiencies for diverse types of systems, ranging from those that only use multiple antennas for beamforming to those that take full advantage of channel state information (CSI).

II. FADE STATISTICS AND DYNAMICS FOR SISO FW LINKS

The behavior of FW channels differs significantly from that of mobile links. While for mobile channels, small scale spatial fades result in temporal fades, this is not the case when the terminal remains at a fixed position. The conventional concept of fades being modeled by Rayleigh/Ricean distributions, depending on the existence or lack of a dominant propagation path is not applicable for FW channels, where temporal fluctuations are due to the movement of scatterers. These fluctuations will often be much smaller than those of mobile channels, allowing the use of higher order modulations, to increase spectral efficiency. We have performed extensive outdoor measurement in urban environments, in the frequency bands of 3.5 and 5.8GHz, to determine the nature and origin of temporal fluctuations. We used a CW channel sounder with both directive and omnidirectional antennas as described in detail in [6]. Our study included line-of-sight (LOS) and obstructed (NLOS) links in residential urban environments in Valparaíso and Viña del Mar, Chile. The area contains a mix of one and two story houses and apartment buildings. Trees not higher than 5m line most streets. Pedestrian traffic is sparse and wind is moderate. We estimated the temporal $K$-factor from the data, by the procedure described in [7]. For most practical locations of a subscriber terminal, power fluctuations...
were quite shallow when compared to the small scale spatial fades. The conclusion that temporal fades originate in vehicular traffic was arrived at by classifying links according to the exposure of the user terminal antenna to a direct view of traffic. To this effect we considered whether or not the subscriber antenna beam had an unobstructed path to a nearby street with vehicular traffic. We further took into account the existence of a direct path (LOS) between transmitter and receiver, since this affects the strength of the signal not scattered by moving objects. A clear pattern of behavior emerged from this classification. Only NLOS links, where the remote antenna beam was exposed to traffic, exhibited fades that would in practice be of concern from a link budget perspective. This is seen in Fig. 1 where CDFs of temporal $K$-factors are plotted. We however note that even shallow temporal fluctuations must be dealt with through adequate amplitude tracking when using high order modulations [8]. Our measurements also revealed that spatial fades were typically quite large even in LOS cases, which is not surprising considering the presence of large nearby reflecting surfaces. Thus it may be speculated that small movements of the remote terminal antenna can have a significant impact on the static part of the received power, without much changing the time varying part. Assuming Ricean temporal fading, the signal envelopes at 2 separate antennas can be described by $c_i(t) = A_i + n_i(t)$, $i = 1, 2$, where $c_1(t)$, $c_2(t)$ are the complex envelopes of the signals consisting of the time invariant components $A_1$, $A_2$ and the zero-mean complex Gaussian random processes $n_1(t)$, $n_2(t)$ with variances $2\sigma_1^2$ and $2\sigma_2^2$ respectively. The Rice-distributed real envelopes $r_i(t) = |c_i(t)|$ have $K$-factors $K_i = \frac{A_i^2}{2\sigma_i^2}$, $i=1,2$. Spatial fades cause differences in the values of $A_1$ and $A_2$. If we assume the effect of moving scatterers to be the same for both antennas, we may write $\sigma_1^2 = \sigma_2^2 = \sigma^2$. The average envelope powers $\{P_i\}_{i=1,2}$ can be written as $P_i = 2(K_i + 1)\sigma^2$ [6]. We define $\Delta K = 10 \log \left( \frac{K_i+1}{K_i+2} \right)$ and $\Delta P = 10 \log \left( \frac{P_i}{\sigma^2} \right)$ and observe that under our assumption we will have

$$\Delta K = \Delta P. \quad (1)$$

One would therefore expect that $K_i$ will increase in proportion to the received power. We found this indeed to be the case as illustrated in Fig. 2. We thus conclude that finding a favorable position for the remote antenna will not only increase the received power but also the temporal stability (i.e. $K$-factor).

The above conclusion breaks down when movement of the antenna covers a range that affects its exposure to traffic. In such cases we found, as reported in [6], that there is virtually no correlation between the temporal $K$-factor and the average received power. Since the “best” position may vary over time due to long term changes in the environment, a spatial diversity system will add long term robustness. The spatial fade reduction achieved by separate antennas has been extensively discussed before [9–11]. However studying fade dynamics for FW links introduces a difference with mobile links where all antennas are sequentially exposed to the same environment and thus receive equal average power. For a FW link, spatial fades will cause average power imbalance of the receive channels. The question then arises how this impacts on the statistics of the output of a diversity combiner. We consider the case of a maximal ratio combiner (MRC). Our aim is to characterize the output dynamics of the combiner, an important aspect when considering high order modulations. Following [11] we may write the output of an $M$-branch combiner as:

$$r(t) = \sum_{i=1}^{M} r_i^2(t). \quad (2)$$

For equal noise power in all branches and uncorrelated fades, $r(t)$ is proportional to the instantaneous signal to noise ratio (SNR) of the combiner output and its statistics will determine those of the outages due to inadequate SNR. We assume as before, that all branches are exposed to the same fluctuation power $2\sigma^2$. It has been shown [12] that under this condition, for a combiner with branches characterized by individual $K$-factors $K_i$, the probability density function of the output signal $f_R(r)$ is:

$$f_R(r) = \frac{1}{2\sigma^2} \left( \frac{r}{\sigma^2} \right)^{M-1} 2^{\sum_{i=1}^{M} K_i} \exp \left\{ -\frac{r}{2\sigma^2} - \frac{M}{2} \sum_{i=1}^{M} K_i \right\} \left( \phi_{M-1} \left( \frac{2r}{\sigma^2} \sum_{i=1}^{M} K_i \right) \right. \quad (3)$$

where $\phi_{M-1}$ is the Marcum Q-function.
It follows from (3) that fade margins for a MRC will depend on the sum of the branch temporal $K$-factors. Our empirical results show that in most practical cases the individual $K$-factors are greater than 10 and in these cases a simpler treatment is possible. We note that for large values of the $K$-factors, the complex envelopes $\{c_i(t)\}_{i=1}^M$ may be approximated by their real part, i.e., $c_i(t) \approx A_i + n_i(t)$ and are thus approximately Gaussian. For the analysis that follows it is convenient to use, instead of (2), the square root of the MRC output which is proportional to the combiner output signal voltage to r.m.s. noise voltage. In the literature both formulations have been used, see for example [9]. We thus write the square root output as:

$$v = \sqrt{\sum_{i=1}^{M} r_i^2}.$$  

(4)

Using the fact that for large $K$-factors, most of the time $A_i \gg n_i(t)$, it is easy to show [12] that $v$ can be approximated by a Gaussian random process with expectation and variances given by $E[v] \approx \frac{1}{2} \sum_{i=1}^{M} (A_i^2)$ and $\text{Var}[v] \approx \sigma^2$. The closeness of the Gaussian and Rice distributions implies that $v$ is also approximately Rice distributed with a $K$-factor given by:

$$K_{MRC} = \frac{(E[v])^2}{2\text{Var}[v]} = \sum_{i=1}^{M} K_i.$$  

(5)

This of course agrees with the more general expression (3). These results can be used for the calculation of combiner output dynamics in the form of the level crossing rate (LCR). To this effect the joint statistics of the output $v$ and its time-derivative $\dot{v}$ have to be obtained to evaluate the expression for the crossing rate of a level $R$ [11]:

$$\text{LCR}(R) = \int_{0}^{\infty} \dot{v}(t) f(v(t) = R, \dot{v}(t)) \, dv.$$  

(6)

It can be shown [12] that under the assumption of independence between $r_i$ and $\dot{r}_i$ for the combiner branches (which is justifiable as discussed later):

$$\text{LCR}(\rho_v) = \sqrt{2} B_{\text{WRMS}} \exp \left\{ - \left( \frac{\rho_v}{R_{\text{WRMS}}} \left[ K_{MRC} + \frac{1}{2} \right] \right)^\frac{1}{2} \right\},$$  

(7)

where we have defined the normalized voltage threshold as $\rho_v = \frac{R}{R_{\text{WRMS}}}$ with $R_{\text{WRMS}}$ the r.m.s value of the combiner output. The r.m.s. bandwidth $B_{\text{WRMS}}$ is calculated from the power spectrum $S(f)$ of the branch autocovariance fade processes (which we assume to be equal) as:

$$B_{\text{WRMS}} = \sqrt{\int_{0}^{\infty} f^2 S(f) \, df \over \sigma^2}.$$  

(8)

We observe from (7), that since typically $K_{MRC} \gg 1$, the crossing rate of the zero level, which we denote as ZCR allows us to easily obtain the r.m.s bandwidth from empirical data: $ZCR = \text{LCR}(0\text{dB}) = \sqrt{2} B_{\text{WRMS}}$. Although the Doppler spectra for mobile and FW are totally different [13], the expression for the LCR as a function of the threshold is the same. The only difference is the numerical value of the r.m.s. bandwidth. Theoretical models and empirical data for the Doppler Spectra of FW links can be found in [14]. The above derivation relies on several approximations and thus it is important to test if empirical results can confirm their validity in practical cases. A crucial assumption is Rice’s condition for independence of $r_i$ and $\dot{r}_i$. Theoretical conditions for this have been discussed in [11]. We instead evaluated the empirical correlation between $r_i$ and $\dot{r}_i$ to verify if this often made assumption holds. The correlation calculated for over 100 links was found to have an average value of -0.25. Though this implies some dependence, the effect on the accuracy of our results as described below was of no significance. The validity of the assumption of equal fluctuation power i.e. $\sigma_r^2 = \sigma^2$ is justified by the empirical results, that confirmed that (1) holds within a reasonable degree of accuracy. We present below typical LCRs for a MRC. From the empirical data we estimated the r.m.s. bandwidth using $ZCR = \sqrt{2} B_{\text{WRMS}}$ and the $K$-factors as in [7]. We refer fade depth to average power. We found that the r.m.s. bandwidths of the individual branches were in almost all cases practically equal. The actual values ranged from 0.5 to 5Hz, the higher values usually associated with closeness to vehicular traffic. Links with low $K$-factors represent the most critical cases for the accuracy of our approximations and are also those which are relevant considering fade depth. We thus show an example where the estimates of the branch $K$-factors were found to be in the low range of measured values. Fig. 3 shows the results. The continuous line corresponds to expression (7). For the curves, the parameters estimated from the measurements are: r.m.s. bandwidth = 1.3Hz, $K$-factor of MRC output = 74. As seen there is a very good match to the empirical data.

III. MIMO CHANNELS VS. BEAMFORMING FOR FW LINKS

Higher spectral efficiencies than obtainable with SISO systems as discussed above are possible when taking advantage of the rich multipath characteristics of urban environments. We investigated the actual gains achievable when both transmitter
and receiver use multiple antennas. The channel is then described by an $N \times M$ matrix or “H matrix” where the element $h_{ij}$ is the gain for the channel between the $j$-th receiver and $i$-th transmitter. We evaluated short range outdoor-indoor links. To estimate the propagation channel matrix a 2.4GHz evaluated short range outdoor-indoor links. To estimate the propagation channel matrix a 2.4GHz narrowband channel sounder was used. Its features, and details about the measurement scenarios have been described before [15]. We measured at night, to exclude temporal variations due to pedestrians. Links included offices and aisles in the campus of the Universidad Técnica Federico Santa María, Valparaíso, Chile, with link lengths in the range of 30-100m. The building is a steel reinforced concrete structure with particleboard interior divisions. At the outdoor (base station, BS) location, a horizontal linear array of four vertically polarized, 2λ spaced sector antennas was installed on a 4th floor rooftop. To simulate a subscriber unit (SU), a horizontal linear array of four vertically polarized λ spaced omnidirectional antennas was placed in positions representative of user locations. At each SU location the array was rotated in steps of 10°. From the empirical data, the spectral efficiencies of various transmit/receive schemes were calculated and compared in terms of the complementary cumulative distribution functions (CCDF). The system SNR was assumed to be constant for each SU array position. Two values of SNR were considered, 5dB and 20dB, representing conditions of high and low interference by other users. The theoretical expressions for spectral efficiency are reviewed below.

A. Spectral efficiency expressions for the considered systems

For the comparison each $H$ was normalized using the Frobenius norm $\|H\|_F$ so that $\|H\|_F = \sqrt{M \cdot N}$. At the receiver we consider a system SNR $= E_s/\sigma_n^2$, with $E_s$ being the total transmitted power adjusted to take into account the average propagation losses, and $\sigma_n^2$ the noise power in each receive channel. We will consider the case $M = N = 4$.

1) MIMO with perfect CSI at the transmitter and receiver: The spectral efficiency is given by [3, 10]

\[
C = \sum_{i=1}^{P} \log_2 \left( 1 + \frac{E_s \gamma_i}{\sigma_n^2} \lambda_i \right) \text{[bps/Hz]},
\]

where $\lambda_i, i = 1 \ldots P$ are the eigenvalues of $HH^\dagger$, with $(\cdot)^\dagger$ denoting the conjugate transpose operator and $\gamma_i$ is the optimal power allocation factor for the $i$-th subchannel (waterfiling), subject to constant transmit power, i.e. $\sum_{i=1}^{P} \gamma_i = 1$.

2) MIMO with CSI at the receiver only: Defining $I$ as the $N \times N$ identity matrix, the achievable spectral efficiency assuming isotropic transmission is [16]

\[
C = \log_2 \left( \det \left[ I + \frac{E_s}{N\sigma_n^2} HH^\dagger \right] \right) \text{[bps/Hz].}
\]

3) Beamforming schemes: We considered classical eigen-beamforming (BF-E) which results in a spectral efficiency given by [17]:

\[
C = \log_2 \left( 1 + \frac{E_s}{\sigma_n^2} \lambda_{\text{max}} \right) \text{[bps/Hz],}
\]

where $\lambda_{\text{max}}$ is the dominant eigenvalue. Several alternatives requiring less CSI exist. We here consider 2 of them. The first consists in choosing the transmit weighting factors from the channel data of the receive antenna that maximizes the spectral efficiency. If the selected target antenna corresponds to the $j$-th row of the matrix $H$ only the row $\hat{H}_j$ needs to be fed back to the transmitter. As a further simplification, we use only the phase data (BF-P), under the assumption that this is the most relevant aspect for properly aiming the antenna beam. Although the BS-array is “pointed” at the selected antenna, the rest will also receive some signal power. We here chose to apply MRC at the receiver [9]. If we denote by $v$ the transmit weighting coefficients, then

\[
C = \log_2 \left( 1 + \frac{E_s}{\sigma_n^2} \|Hv\|^2 \right) \text{[bps/Hz].}
\]

The second, even simpler option is sub-sectorization (SBS) which requires no CSI. Narrow beams aimed in fixed directions are generated at the base. We considered 4 sub-sectors, using beams that have the same radiation pattern as that of the original antennas but scaled in width by a factor of 4. Each beam has a 6dB gain advantage on boresight but is allocated one fourth of the total power $E_s$. The placement of the SU is assumed to be random, with uniform distribution within the coverage sector. At the SU, a MRC receiver is assumed. The corresponding spectral efficiency for base antenna $i$ is calculated from the same ensemble of channels using

\[
C_i = \log_2 \left( 1 + \frac{G \cdot E_s}{\sigma_n^2} \|\hat{h}_i\|^2 \right) \text{[bps/Hz]},
\]

where $\|\hat{h}_i\|$ is the $i$-th column of the $H$ matrix and $G (G \leq 1)$ is the antenna gain factor in the direction of the SU. The normalization of the $H$-matrix is not changed.

B. Empirical results

We found that spatial fading statistics were well described by Ricean distributions with $K$-factors, not larger than 3. Power correlation was only significant for adjacent transmit antennas sharing a single receiver, with values in the range 0.4–0.6. Other combinations rarely exceeded 0.1. Thus the channels cannot be assumed to be i.i.d. Rayleigh, and degradation of MIMO spectral efficiency with respect to this condition is to be expected. The empirically obtained CCDF of the spectral efficiency for the systems is shown in Fig. 4a for a SNR of 5dB. As a reference we indicate for Rayleigh i.i.d. channels the CCDFs for MIMO with full CSI and for eigen-beamforming. As expected, measured MIMO efficiencies are lower than the reference i.i.d. case. In contrast beamforming performance is higher for measured channels. This suggests that the channels exhibit limited angular spread and non-zero $K$-factors as also reported in [5, 18], and several references quoted therein. The overall best case is MIMO with full CSI,
but the MIMO advantage over beamforming is significantly less for the measured channels than for the Rayleigh i.i.d. case. Beamforming spectral efficiency will actually exceed that of MIMO with no CSI in some locations, a consequence of MIMO allocating power to eigenmodes which are too weak to make a significant contribution. While eigen-beamforming is the best of the beamforming options, its advantage is not very significant. Fig. 4b shows the CCDFs of spectral efficiency for 20dB SNR. In this case MIMO techniques offer significant gains as they can take advantage of the multiple channels available. However at 10% outage probability, the spectral efficiency of the beamforming schemes is in practice still within 70% of that of MIMO with full CSI. In the high outage range, which corresponds to the case where the user is assumed to have searched for a favorable location, the curves show little improvement for beamforming, but considerable gains in the MIMO case, which can take advantage of the existence of regions where the H matrix has several significant eigenmodes.

**IV. CONCLUSIONS**

Fixed wireless channels exhibit in general very high temporal stability, but their behavior can vary significantly with the remote antenna position. Antenna diversity not only reduces spatial fade depth but very effectively contributes to temporal stability, allowing the use of the high order modulations required to approach channel capacity. Multiantenna systems making full use of CSI offer the possibility of further increasing spectral efficiency. However the achievable gains are in practice considerably less than those associated with the often assumed Rayleigh i.i.d. model. For low SNR, very simple beamforming schemes can often provide comparable benefits.

**REFERENCES**


