Empirical Comparison of MIMO and Beamforming Schemes

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Abstract—We present an empirically based comparative study of spectral efficiency for a variety of transmission systems applicable to a fixed or repositionable wireless environment, in the context of Wi-Fi, WiMAX or MuniNet systems. A narrowband 4x4 Multiple Input Multiple Output (MIMO) channel sounder has been constructed and a series of outdoor to indoor measurements have been carried out, in multiple locations and with different array configurations. The channel measurements have been used to compute the spectral efficiency of different systems that could be deployed in such scenarios, ranging from a full MIMO system with perfect Channel State Information (CSI) at both ends to simple diversity schemes such as classical beamforming. We show comparisons between the different transmit/receive configurations operating in a representative variety of indoor receive locations. Our results indicate that for low values of Signal to Noise Ratio (SNR, in the range of 5dB), simple schemes can achieve capacities as high as 80% of that of MIMO with complete CSI.

I. INTRODUCTION

MIMO systems have been extensively studied over the last decade [1]. Using antenna arrays, they offer the possibility of dramatically increasing spectral efficiency by adding a new dimension - space - to Single-Input Single-Output (SISO) transmission schemes. MIMO spatial multiplexing techniques allow the creation of multiple parallel transmission channels if appropriate signal processing techniques are employed [2–4]. The highest spectral efficiency of MIMO systems is reached when CSI is available at the transmitter [2, 5]. Without CSI at the transmitter, a significant portion of the highest capacity can still be achieved at the cost of increased computational load at the receiver [3, 4, 6, 7]. Beamforming techniques reduce the required processing efforts at both ends [8–10], but the achievable transmission rate can be lower due to the lack of spatial multiplexing.

The main objective of this work is to empirically evaluate capacity gains of MIMO systems in comparison to several simpler beamforming schemes in typical settings representing the operation of outdoor-indoor wireless networks. This is useful when considering the tradeoff between achievable capacity, cost, computational complexity and feedback rate for CSI.

The reasoning behind our approach is that achieving high MIMO capacity depends on the availability of multiple modes in the channel propagation matrix [2, 5]. At low SNR, MIMO transmission may only be able to take advantage of a single mode. The multiantenna MIMO channel then only benefits from antenna diversity and simple beamforming schemes may achieve comparable capacities. On the other hand, for the same ensemble of channels, the advantage of MIMO will become significant when SNR is large. Our empirical approach allows us to quantify the potential gains of MIMO processing under realistic propagation conditions.

II. MEASUREMENT SETUP AND PROCESSING OF EMPirical DATA

To estimate the achievable capacity in typical fixed outdoor-indoor scenarios, a 4 × 4 narrowband channel sounder has been constructed. Its features have already been described in [11]. It operates in the 2.4 GHz band, simultaneously transmitting 4 continuous wave (CW) carriers, one per antenna, separated by 2 kHz. The downconverted outputs of the corresponding four receivers are processed via FFT to separate the signals originating at each transmit antenna, yielding the complex 4 × 4 propagation channel matrix or “H matrix”. We tested outdoor-indoor scenarios at night, to minimize temporal variations due to pedestrian traffic. Links included offices and aisles in the campus of the Universidad Técnica Federico Santa María, Valparaíso, Chile, with link lengths in the range of 30-100m. The building is a steel reinforced concrete structure with windows of approximately 2.5 × 2m and particleboard interior divisions. The distances measured are representative of a Wireless Local Area Network scenario or a Wireless Metropolitan Area Network system such as WiMAX, considering a micro-cell arrangement with the base station (BS) deployed on lamp posts. In order to emulate these scenarios, we used a horizontal linear array of four vertically polarized, 2λ spaced sector antennas as the outdoor BS. It was placed at a position located 2 m above a 4th floor rooftop, and aimed in the general direction of the indoor measurement positions. The half power beamwidths of each antenna is 60° azimuth and 8° vertical. Wind was typically moderate (< 5m/sec). The indoor measurements were made in a third floor. To simulate a remote terminal or subscriber unit (SU), a horizontal linear array of four vertically polarized λ spaced omnidirectional antennas was placed at a height of around 1.5 m in positions that

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would represent possible user locations. At each SU location the remote antenna array was rotated with respect to its own center in steps of $10^\circ$. A variety of scenarios were considered, as shown schematically in Fig. 1. In most of them the BS was blocked from view of the SU, either by concrete walls or by indoor panels, with a few instances where a direct path existed through a window. More than 30 combinations of transmit and receive locations were tested.

From the empirical data, the theoretical capacities of various transmit/receive schemes were calculated. We compare these options in terms of the Complementary Cumulative Distribution Functions (CCDF) of the spectral efficiency, obtained from the set of measured channels over the range of positions tested. The SNR was assumed to be constant for each receiver location and antenna array orientation. This was done normalizing the channel matrix using the Frobenius norm. Two values of SNR were considered, 5dB and 20dB, representing respectively conditions of high and low interference by other users. The theoretical expressions for spectral efficiency are briefly reviewed in the next section.

III. SPECTRAL EFFICIENCY EXPRESSIONS FOR THE CONSIDERED SYSTEMS

We consider a narrowband MIMO system with $M$ transmitter and $N$ receiver antennas. To compute the spectral efficiency, we use a general linear baseband model given by [1]:

$$r(n) = Hs(n) + \eta(n),$$  

where $r(n) = [r_1(n) \ldots r_N(n)]^T$ is the received vector at time $n$, $s(n) = [s_1(n) \ldots s_M(n)]^T$ is the transmitted symbol vector, $(\cdot)^T$ denoting the transpose operator, and $H$ is the $M \times N$ channel matrix, whose complex entries represent the gain between each transmitter and receiver pair. $\eta(n)$ is a zero mean circularly symmetrical complex gaussian noise (ZMCSG) vector with covariance matrix $I_N \sigma^2_n$ ($I_N$ being the identity matrix of size $N$ and $\sigma^2_n$ the noise power for each channel). $s(n)$ is obtained in general by processing the data symbol vector $s(n)$ as $\hat{s}(n) = Vs(n)$, where $V$ is a linear transformation applied at the transmitter. At the receiver, a linear transformation $U$ is applied to the received signal to obtain a symbol estimation $y(n)$ as $y(n) = Ur(n)$. We assume a system SNR $\frac{E_s}{\sigma^2_n}$, with $E_s$ being the total transmitted power. In this article we will only consider the case $N = M$.

We present in what follows several possible systems whose differences correspond to the signal processing techniques performed at the transmitter and receiver, given the CSI available at each end.

A. MIMO with Perfect CSI at the transmitter and receiver

In this case the transmitter and receiver processing matrices are obtained from the Singular Value Decomposition (SVD) of $H = U \Sigma V$ [12]. To distribute the available power among the antennas, the waterfilling algorithm is used [12]. This system is equivalent to having $P$ parallel channels with $P \leq \text{min}(M, N)$. The spectral efficiency is then given by [2, 12]

$$C = \max_{\gamma_i} \sum_{i=1}^{P} \log_2 \left(1 + \frac{E_s \gamma_i \lambda_i}{M \sigma^2_n}\right) \text{ [bps/Hz]},$$  

where $\lambda_i, i = 1 \ldots P$ are the eigenvalues of $HH^\dagger$, with $(\cdot)^\dagger$ denoting the conjugate transpose operator, and $\gamma_i$ is the optimal power allocation factor for the $i$-th subchannel, subject to constant transmit power, i.e. $\sum_{i=1}^{P} \gamma_i = 1$. The drawback of this scheme is the high rate feedback channel needed to inform the transmitter of the channel parameters, as often as required according to the channel coherence time.

B. MIMO with Perfect CSI at the receiver only

For this case we assume that only the receiver has full knowledge of the channel matrix and the noise statistics. Since the transmitter has no knowledge of the CSI, the power is equally distributed among the antennas. In this case we have that [1]

$$C = \log_2 \left(\det \left[I_N + \frac{E_s}{M \sigma^2_n} HH^\dagger\right]\right) \text{ [bps/Hz].}$$  

C. Transmission beamforming schemes

As an alternative to MIMO we consider several simpler schemes, all of which are generalizations of beamforming for a scattering environment.

1) Beamforming, dominant eigenmode: This system which we will call “Beamforming-Dominant” (BF-D) was discussed in [9]. Here, a single stream is transmitted over the $M$ transmitter antennas using the dominant eigenmode of the $H$ matrix. Considering that all the power is concentrated in the dominant mode (with associated eigenvalue $\lambda_{\text{max}}$), the spectral efficiency is readily obtained from (2) as

$$C = \log_2 \left(1 + \frac{E_s}{\sigma^2_n \lambda_{\text{max}}}\right) \text{ [bps/Hz]}.$$  

### Fig. 1. Measurement location map
2) Beamforming, single antenna: We propose various simple beamforming schemes similar to the one presented above. The main difference is that instead of transmitting over the dominant eigenmode, the transmitter will select the weighting coefficients based on channel data for a single receive antenna of the array. In the case of a non-scattering environment, this collapses to aiming the transmit beam in the geometrical direction of the corresponding antenna.

Let \( h_j \) be the \( j \)-th row of the matrix \( H \), whose elements correspond to the complex gains between the transmitter antennas and a selected receiver antenna. If the \( j \)-th receiver antenna is selected as the beamforming target, the transmitter weighting coefficients are given by \( v = \frac{h_j}{\|h_j\|} \). Using this transmission scheme the SNR at the selected antenna branch will be \( \frac{\|h_j\|^2 E_s}{\sigma_n^2} \). Although the BS-array is pointed at the selected antenna, the rest of the receiving antennas at the SU will also receive some signal power. We note that since the transmitter weighting coefficients were calculated without regard to the rest of the channel gains, the joint optimization of weighting coefficients at both transmitter and receiver used in eigen-beamforming is not applicable. As an obvious choice, we here chose to apply Maximum Ratio Combining (MRC) at the receiver, which in this case corresponds to multiplying the received signal vector by \( (Hv)^\dagger \). The combiner output SNR is easily found to be \( \frac{E_s \|Hv\|^2}{\sigma_n^2} \). The spectral efficiency can then be expressed as

\[
C = \log_2 \left( 1 + \frac{E_s \|Hv\|^2}{\sigma_n^2} \right) \text{[bps/Hz]} \tag{5}
\]

The advantage of this system is its simplicity and the reduced feedback information needed. An important issue is how to select the receiver antenna that the beam is pointed at. The optimal solution is to select the antenna that maximizes the spectral efficiency, which, according to (5), is equivalent to selecting the \( j \)-th receiver antenna, where \( j \) is the antenna index that maximizes the expression \( \|Hv\| / \|h_j\| \). We denote this method as “Beamforming-Best” (BF-B). Another possibility is to randomly choose any receive antenna, under the assumption that in most practical cases the difference with respect to the optimum will not be very significant. The advantage is the simplification of the computations. We denote this option “Beamforming-Random” (BF-R). Another possible variation to this method is to use only the phases of the \( v \) vector, under the assumption that the phase information is the most relevant aspect for properly pointing the antenna beam. Notice that this procedure cuts the feedback information needed at the transmitter in half. We denote this method “Beamforming-Phase” (BF-P).

IV. EMPIRICAL RESULTS

We found that spatial fading statistics were well described by Rayleigh or Ricean distributions with typically quite low \( K \) factors (\( K < 3 \)) in all cases, as in [13]. An attempt to classify links as line of sight when one antenna was visible from the location of the other failed to predict higher \( K \) factors. Consequently this classification was not further pursued. Channel correlation, calculated using the method discussed in [14], was only significant for adjacent transmit antennas sharing a single receiver, which resulted in values in the range 0.4-0.6. All other combinations rarely exceeded 0.1. This is consistent with the fact that the presence of scatterers is concentrated in the vicinity of the receiver.

The empirically obtained CCDF of the spectral efficiency for the systems considered is shown in Fig. 2. The system SNR is 5dB. As a reference we indicate the CCDF for a MIMO system with full CSI at both ends considering independent identically distributed (i.i.d.) Rayleigh channels, which represents the best case scenario. The corresponding measured capacities are lower, yet the case of MIMO with full CSI represents, as also expected, the best case for the measured capacities. The spectral efficiency of MIMO with CSI at the receiver only (MIMO CSI Rx) is lower and does not provide very significant gains over the beamforming schemes. In fact the measured beamforming capacities outperform MIMO with CSI at the receiver only, when considering 10% outage. This is due to the fact that a MIMO system without CSI at the transmitter will allocate power equally to all eigenmodes, some of which will in fact often be so weak as to only contribute negligible capacity. As seen in Table I, beamforming schemes actually perform close to MIMO with full CSI at 10% outage, with the advantage that they may be simpler to implement. As expected among the beamforming schemes, BF-D has the highest spectral efficiency, and BF-R the lowest, but the differences are significant only for very low outage probabilities. The maximum spectral efficiency of all beamforming options converges to the same value. This happens when the channel matrix has a single dominant eigenmode, i.e. the theoretical capacities of all beamforming schemes become equal.

Fig. 3 shows the CCDF of capacities for the considered systems when a 20 dB SNR is assumed, which models the case of a low interference scenario, as for instance users in isolated hotspots. In the high SNR case, MIMO techniques offer significant spectral efficiency gains compared to beamforming.
schemes. This is due to the fact that in this SNR range MIMO systems can take advantage of the multiple channels available. The difference between MIMO with full CSI and MIMO with CSI at the receiver only, becomes relatively small; hence the latter would be a good choice for this kind of scenarios, considering that the small spectral efficiency loss is compensated by the feedback channel becoming unnecessary. In the beamforming schemes the spectral efficiency improvement is solely due to the SNR increase. At 10% outage probability, the spectral efficiency of beamforming schemes however is still within 70% of the MIMO spectral efficiency with full CSI, as seen in Table I. This suggests that, considering the implementation complexity and feedback rate required, beamforming schemes may still be a good choice at high SNR values, for the kind of environment studied.

V. CONCLUSIONS

We have presented an empirical comparison between several multiantenna schemes, based on outdoor to indoor channel measurements. We found that under certain conditions, quite simple transmit receive schemes achieve capacities which are close to those of considerably more complex systems, at the advantage of less transmission overhead for CSI. We observed that for a SNR of 5 dB (or high interference level) the beamforming schemes, can perform close to the MIMO system with singular value decomposition and waterfilling, and can even outperform a MIMO system with CSI only at the receiver. The proposed beamforming configurations achieve around 80% of the median spectral efficiency of MIMO with full CSI. Moreover, the results at this SNR level indicate no appreciable difference in median capacities between the various beamforming configurations and a MIMO system with CSI at the receiver only. This makes beamforming an attractive option for Wi-Fi/Max where the subscriber unit should be kept simple but with adequate performance in high interference scenarios. If the feedback rate is to be kept small, then the BF-P scheme can be used to cut the feedback information in half, without significant loss of spectral efficiency. At higher SNR, the MIMO configurations outperform the beamforming schemes. However in the kind of environment studied, the achievable MIMO capacities are significantly lower than those of the often assumed the Rayleigh i.i.d. model. In the high SNR scenario MIMO schemes can take advantage of their spatial multiplexing capabilities by exploiting the multiple channel modes available. For beamforming schemes, the spectral efficiency improvement compared with the low SNR case is solely due to the improvement in SNR (5 bps/Hz for a 15 dB increase). However, if the designer wants to guarantee performance with high probability (10% outage), and low CSI feedback and complexity are a relevant consideration, then beamforming can still be a good choice since these schemes can perform within 70% of MIMO with full CSI.

REFERENCES