Distributed Coalition Formation and Bandwidth Allocation in Ad Hoc Cognitive Radio Networks

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Abstract—In this paper we analyze the problem of distributed coalition formation (CF) and bandwidth allocation in ad hoc cognitive radio networks. We develop a CF algorithm to self organize distributed secondary links into disjoint coalitions and apply the concept of frequency reuse over different coalitions, such that the members of each coalition will transmit over orthogonal sub-bands with the available spectrum being optimally allocated among them. We formulate the problem as a CF game in partition form with non-transferable utility and propose a distributed algorithm through which players may join/leave a coalition based on their individual preferences. We study the convergence properties of the proposed CF rule and present means to guarantee Nash-stability. In addition, we also describe graceful exit procedures if a CF process leads to oscillations. We show that a grand-singleton structure will emerge in the network at very low/high SNR and analytically derive the lower bound on the probability that a general network partition, other than grand-singleton structure, is stable. Performance analysis shows the effectiveness of the proposed coalition formation algorithm with optimal bandwidth allocation over a wide SNR range and indicates a substantial gain in terms of average payoff per link over existing coalition formation techniques.

Index Terms—Cognitive radio, Distributed Coalition Formation, Coalition Formation Cycle, Nash-stable partition, Optimal bandwidth allocation.

I. INTRODUCTION

Cognitive radio networks (CRNs) based on dynamic spectrum access has emerged as a promising technology to meet the insatiable demand for radio spectrum by the emerging wireless applications. The practical implementation of CRNs is facing many challenges and conflicting requirements such as primary user (PU) protection and secondary user (SU) rate maximization [1]. While the last decade of research activities has focused on enhancing spectrum sensing (SS) performance [2], or more recently on jointly optimizing the spectrum sensing and spectrum access parameters [3], [4] as a means of improving the secondary network throughput, their scope has proven to be limited after the FCC ruling [5] which obviated the SS requirement in CRNs. As a result, there has been a dire need to explore stand-alone efficient spectrum access schemes in a competitive environment where SUs do not solely rely on SS performance for their throughput improvement.

Hence, in this paper, we propose a game-theoretic strategy to dynamically share the available spectrum resources among competing SUs under the assumption that available spectrum opportunities are known a priori.

A. Related work and Our contributions

Coalitional game-theoretic tools [6] have been explored for performance improvement of cognitive networks from different viewpoints, [3], [4], [7]-[9]. Most of these works do not consider the role of externalities (effect of SUs outside the coalition) in CF process. For instance, a generic rate allocation problem has been analyzed in [7] as a CF game in characteristic form (without considering the effect of externalities) with transferable utility (TU) which allows the coalition value (coalition sum-rate) to be arbitrarily apportioned among the coalition members. In contrast, in this paper, we aim at self organizing distributed SUs (caring only for their own rate improvement) into disjoint coalitions such that the total available transmission BW is made available to each coalition, while this BW is optimally sub-divided into orthogonal bands within each coalition. This requires us to model the CF game in a partition form since the payoff (transmission rate) of each coalition member is affected by the interference from simultaneous transmissions of members of different coalitions over the same frequency band. Moreover, the proposed game has non-transferable utility (NTU), since the optimal BW allocation restricts the distribution of the coalition value among the coalition members.

The coalitional games in partition form were recently investigated for throughput efficient spectrum access in centralized CRNs [8], and in ad hoc CRNs [3], [9]. Focussing on ad hoc CRNs, the authors in [3], considered the joint optimization of sensing and access where the competing SUs shared the spectrum in chunks of pre-fixed/slotted BW (as defined by PU channels), while in [9], coalition members shared the available BW according to their channel gain ratios. It is evident that these are not optimal BW allocation approaches.

In comparison to existing work where the available spectrum resources are shared in terms of channels of pre-fixed or equal BW, we propose a continuous BW allocation among the coalition members and provide a closed form expression of rate optimal BW allocation. Note that, our previous work [8], is focused on centralized CRNs and does not take into account the individual preferences of rational players in the CF game. In comparison, in this paper, we develop an efficient, distributed CF algorithm through which rational players are self organized based on their individual rate improvement.
and we study the convergence properties of the proposed selfish CF rule. We introduce the history condition in the CF algorithm to guarantee Nash-stability, and describe graceful exit procedures when a CF cycle is inevitable. Furthermore, we show the effectiveness of the proposed algorithm by evaluating the probability of a general network partition (other than grand/singleton structure) to be stable.

The organization of this paper is as follows. Section II introduces the network model. The optimal BW allocation for a given network partition is presented in Section III, while the proposed CF algorithm to organize distributed links into non-overlapping coalitions is discussed in Section IV. Probabilistic analysis of the proposed CF algorithm is presented in Section V, and some key simulation results are provided in Section VI. Finally, the paper is concluded in Section VII.

II. NETWORK MODEL

We consider an ad hoc cognitive radio network with \( N \) SUs (secondary transmitter-receiver (ST-SR) pairs/links or more popularly known as CRs). We assume that the total available BW for secondary access is \( W \) Hz, and we consider that the channel between any of the secondary transmitters and any of the secondary receivers over this BW follows a quasi-static flat fading model. A representative network model with 6 secondary links arranged in 3 coalitions is illustrated in Fig. 1.

![Fig. 1. A representative network model with 6 secondary links.](image)

Frequency reuse of the available BW is assumed by letting the \( N \) links to arrange themselves into a number of coalitions such that each coalition will be using the total available BW. Moreover, we propose to optimally allocate the total available BW among the coalition members based on the available CSI with the objective of maximizing the coalition sum-rate. It is important to highlight here that the two problems discussed above are coupled, which means that none of them can be solved without solving the other. This renders the problem hard but interesting. We start our analysis by solving the optimal BW allocation problem for a given network partition, and then we discuss how to exploit this optimal bandwidth allocation in the proposed coalition formation algorithm.

III. OPTIMAL BANDWIDTH ALLOCATION

For a given network partition: \( \Pi = \{ S_1, S_2, \ldots , S_{|\Pi|} \} \). In the system model under consideration, each of the \( |\Pi| \) disjoint coalitions will use the total available bandwidth \( W \). Consider Coalition \( S_k \in \Pi \), with \( |S_k| \) representing the number of members (links) in \( S_k \). We will use \( m^k_i \) to refer to Member (Link) \( i \) of Coalition \( S_k \) while \( P^k_i \) will represent the transmission power of this member. Each member \( m^k_i \) of \( S_k \) will be allocated a fraction \( \mu^k_i \) of the total bandwidth, where \( \sum_{i=1}^{|S_k|} \mu^k_i = 1 \). The total interference power affecting the bandwidth \( W \) being used by Coalition \( S_k \) is the sum of all received power from all members in all other coalitions. For simplicity of the analysis, we approximate the interference affecting the band \( \mu^k_i W \) being used by Member \( m^k_i \) to be the average interference affecting this band which is a fraction \( \mu^k_i \) of the total interference power affecting the total bandwidth at the receiver of Link \( m^k_i \); i.e.,

\[
I^S_i = \mu^k_i \sum_{l=1,l \neq k}^{|S|} \sum_{j=1}^{|S_l|} P^j_l |h^j_{i,l}|^2 = \mu^k_i I^S_k .
\]

(1)

where \( h^j_{i,l} \) represents the channel between the transmitter of Link \( j \) in Coalition \( S_l \) and the receiver of Link \( i \) in Coalition \( S_k \) and \( I^S_k \) represents the total interference from all other coalitions in the network affecting the total BW \( W \). The total rate of Coalition \( S_k \) under Partition \( \Pi \) can be written as

\[
R^{S_k,\Pi} = \sum_{i=1}^{|S_k|} R^S_i = \sum_{i=1}^{|S_k|} \mu^k_i W \log \left( 1 + \frac{P^k_i |h^k_{i,k}|^2}{\mu^k_i (N_0 W + I^S_k)} \right).
\]

(2)

In order to maximize the rate achieved by this coalition, the problem can be formulated as

\[
\max_{\mu^k_i} \sum_{i=1}^{|S_k|} \mu^k_i W \log \left( 1 + \frac{P^k_i |h^k_{i,k}|^2}{\mu^k_i (N_0 W + I^S_k)} \right),
\]

subject to \( \sum_{i=1}^{|S_k|} \mu^k_i = 1 \) and \( 0 \leq \mu^k_i \leq 1 \).

(3)

It can be shown that this problem is concave, and hence the globally optimal solution is guaranteed to be obtained numerically. However, here we provide a closed form expression for the optimal solution by arguing that since the log function is concave, and since \( \sum_{i=1}^{|S_k|} \mu^k_i = 1 \) and \( 0 \leq \mu^k_i \leq 1 \), the objective function can be upper bounded by

\[
W \sum_{i=1}^{|S_k|} \mu^k_i \log (1 + \frac{x^k_i}{\mu^k_i}) \leq W \log (1 + \sum_{i=1}^{|S_k|} x^k_i),
\]

(4)

where \( x^k_i = \frac{P^k_i |h^k_{i,k}|^2}{N_0 W + I^S_k} \). It can be shown that this upper bound can be achieved by choosing

\[
\mu^k_{i,\text{opt}} = \frac{x^k_i}{\sum_{m=1}^{|S_k|} x^k_m}.
\]

(5)
and hence, the closed form expression of optimal rate $R_{i,\text{opt}}^{S_i,\Pi}$ for any $i \in S_k$ is given by:
\[
R_{i,\text{opt}}^{S_i,\Pi} = \mu_{i,\text{opt}} \log \left(1 + \sum_{m=1}^{|S_k|} x_m^k \right).
\]

IV. DISTRIBUTED COALITION FORMATION

In the previous section, we assumed that network partition is known a priori. In this section, we address the problem of finding a Nash-stable (NS) partition of individually rational (caring only for their own rate) distributed secondary links in an ad hoc network. We use a game-theoretic (GT) framework to maintain a balance between the spectrum reuse and interference avoidance by evaluating the individual preferences of each link whether it should operate in a non-cooperative manner and utilize the total available $W$ Hz at the cost of high interference, or, it should make coalition with other player(s) and share BW to avoid interference from them.

A. Game Formulation:

We model the problem of finding a NS network partition, as a CF game in partition form with non-transferable utility (NTU) [6], defined as:

**Definition 1:** A CF game in partition form with NTU is specified by a pair $(N, \mathcal{V})$, where $N$ is the set of players, and $\mathcal{V}$ is a mapping such that for every partition $\Pi \in \mathcal{P}$, where $\mathcal{P}$ is the set of all possible partitions of $N$, and every coalition $S_k \subseteq N$, $S_k \in \Pi$, $V(S_k, \Pi)$ assigns a set of $|S_k|$-dimensional vectors, representing the possible payoffs that the players in $S_k$ can achieve when the partition $\Pi$ is in place. Formally, $V(S_k, \Pi)$ is a closed, convex subset of $\mathbb{R}^{|S_k|}$.

In the proposed CF game, the $N$ secondary links act as the players of the game constituting the set $N = \{1, 2, ..., N\}$, and each player, $i \in N$ is identified by its unique global index $i$, $i \in \{1, 2, ..., N\}$. For any coalition $S_k \subseteq N$, $S_k \in \Pi$, $V(S_k, \Pi)$ contains a single vector $v(S_k, \Pi) \in \mathbb{R}^{|S_k|}$, where each element $v_i \in v(S_k, \Pi)$ represents the payoff of player $i \in S_k$ and is given by its respective rate $R_{i,\text{opt}}^{S_i,\Pi}$ in (6).

The proposed CF game has a non-transferable utility since the payoff of player $v_i(S_k, \Pi), \forall i \in S_k$ cannot be arbitrarily assigned to another player $j \in S_k, j \neq i$, while the CF game is in partition form since the payoff $v_i(S_k, \Pi) = R_{i,\text{opt}}^{S_i,\Pi}$ of every player $i \in S_k$ depends on the players in $S_k$ as well as on the players outside $S_k$; i.e. the players in $N \setminus S_k$.

B. Preference Relation and CF Rule

We use the notation $(S_1, \Pi) \succ_i (S_2, \hat{\Pi})$ to indicate that player $i$ strictly prefers (has a higher rate in) Coalition $S_1$ under partition $\Pi$ over Coalition $S_2$ when $\hat{\Pi}$ is in place. Based on this preference relation, we propose the following CF rule:

**Definition 2:** Given the network partition $\Pi = \{S_1, S_2, \ldots, S_{|\Pi|}\}$ of $N$, a player $i \in N$ decides to leave its current coalition $S_k \in \Pi$ and join a new coalition $S_l \in \Pi \cup \{\emptyset\}, S_l \neq S_k$ and hence making a transition from $\Pi$ to $\hat{\Pi} = \{\Pi \setminus \{S_k, S_l\}\} \cup \{S_l \setminus \{i\}, S_l \cup \{i\}\}$, if and only if $(S_l \cup \{i\}, \hat{\Pi}) \succ_i (S_k, \Pi)$; i.e. $R_{i,\text{opt}}^{S_l,\Pi} > R_{i,\text{opt}}^{S_k,\Pi}$.

### Algorithm 1: Distributed CF algorithm based on proposed selfish CF rule.

The proposed distributed CF algorithm is summarized in Algorithm 1. Under the assumption that each player in the network is made aware of the average external interference it experiences through measurements fed back from its receiver over a control channel, we propose to start the CF game in a grand coalition, $\Pi_1 = N$. We define $h(i)$ as the coalition history set which is a set of all coalitions that player $i$ was a member of in the past but did not remain its member, because it, or some other coalition member, left the coalition. We initialize $h(i)$ with $N, \forall i \in N$.

The proposed CF rule can be viewed as a selfish decision made by individual player to move from its current coalition to a new coalition, regardless of the effect of its move on other players.

C. Proposed CF Algorithm:

The proposed distributed CF algorithm is summarized in Algorithm 1. Under the assumption that each player in the network is made aware of the average external interference it experiences through measurements fed back from its receiver over a control channel, we propose to start the CF game in a grand coalition, $\Pi_1 = N$. We define $h(i)$ as the coalition history set which is a set of all coalitions that player $i$ was a member of in the past but did not remain its member, because it, or some other coalition member, left the coalition. We initialize $h(i)$ with $N, \forall i \in N$.

The CF algorithm is invoked by distributed players in the ascending order of their global index $i$, where at each iteration, only one player can move from its current coalition to a new coalition to improve its rate. Each player first evaluates the splitting (moving to an empty coalition $\emptyset$) possibility. Player $i$ splits, if and only if, it improves its rate by becoming singleton, provided, it was never singleton before; i.e. $\{i\} \not\in h(i)$. However, if a player $i \in N$ finds out that it cannot split, it evaluates the possibility of switching from its current coalition $S_k$ to another coalition $S_l \in \Pi$, under the history condition; i.e. $S_l \not\in h(i)$. In this regard, we consider the player to be opportunistic; i.e. a player $i$ checks all the coalitions $S_l \in \Pi \setminus \{\emptyset\}, l \neq k, S_l \not\in h(i)$, one after the other in a round-robin fashion and switches to the first coalition $S_l \in \Pi \cup \{\emptyset\}$ which offers rate improvement to player $i$. Hence, the CF algorithm completes its iteration for player $i \in N$ when it finds a suitable coalition $S_l \in \Pi \cup \{\emptyset\}$ to switch to, or, when after checking all switching possibilities,
it decides to stay in its current coalition \( S_k \in \Pi \). Player \( i \) updates its coalition history set by adding its current coalition to \( h(i) \) at the end of each iteration. After iterating over all the players in the network, one round of the proposed CF algorithm concludes. The algorithm keeps on iterating over all the players in the network until all players decide to stay in their current coalition, which indicates that the algorithm has converged to a final Nash-stable network partition \( \Pi_f \).

D. Convergence and stability of final network partition:

A CF game leads to a cycle when one or more players in the network switch from their coalition in such a way that the network partition at the end of \( r \)-th CF round, \( \Pi_{r+1} \), is identical to an already encountered partition at the end of any of the previous rounds including the initial network partition \( \Pi_1 \); i.e. \( \Pi_{r+1} \in \mathcal{P}_r = \{ \Pi_1, \ldots, \Pi_r \} \). Therefore, a necessary but not sufficient condition for a CF cycle to occur is that at least one player \( i \) revisits a coalition in its coalition history set \( h(i) \), and hence, the proposed CF game is guaranteed to converge (without oscillating in a cycle) to a final network partition \( \Pi_f \) after finite CF rounds.

The stability of the final network partition \( \Pi_f \) is evident from the fact that the proposed CF process continues until no player in the network prefers to switch from its current coalition, and hence, the resulting network partition \( \Pi_f \) is a Nash-stable partition.

Remark 1: Although using the history condition guarantees the stability of the algorithm, in Section VI, we propose some exit procedures to exit cycles in cases where history condition is not used, and we compare the average payoff per player with and without the history condition.

V. Probabilistic Analysis of Coalition Formation

In this section, we evaluate the probabilities that a grand-singleton structure is stable and use these probabilities to find a lower bound on the probability that a network partition, that is different from grand and singleton structures, is stable. Due to space limitation, we present the final expressions for the required probabilities without going through the detailed derivations.

Grand structure (GS) would be stable if no player \( i \in \mathcal{N} \) is capable of improving its rate by splitting from grand structure \( \Pi = \mathcal{N} \) to act as a singleton Coalition \( \{i\} \in \Pi_1, \Pi_1 = \{\{i\}, \mathcal{N}\setminus\{i\}\} \); i.e.

\[
P(\text{GS is stable}) = P(R_{i}^{\{i\},\Pi_1} < R_{i}^{\mathcal{N},\Pi_1}) \quad \forall \, i \in \mathcal{N} \quad (7)
\]

Considering equal BW allocation among coalition members, the rate equations can be expressed in a simplified form as:

\[
R_i^{\{i\},\Pi_1} = W \log(1 + \frac{X_i}{1 + Y_{\{i\}}}) , \quad R_i^{\mathcal{N},\Pi_1} = \frac{1}{N}W \log(1 + NX_i)
\]

where, \( X_i \sim \alpha_i \exp(-\alpha_i x_i) \) with \( 1/\alpha_i \) representing the mean direct link SNR for player \( i \in \mathcal{N} \) and \( Y_{\{i\}} \) is the total interference observed at the receiver of link \( i \in \{i\} \) given by \( \sum_{j=1,j\neq i}^{N} Y_{ji} \) with \( Y_{ji} \sim \beta_{ji} \exp(-\beta_{ji} y_{ji}) \) where \( 1/\beta_{ji} \) represents the mean SNR observed at the receiver of link \( i \) due to the interference caused from link \( j \in \mathcal{N}, j \neq i \).

It can be shown that for the most general case of different \( \alpha_i \, \forall \, i \in \mathcal{N} \) and different \( \beta_{ji} \, \forall \, i, j \), the required probability is given by:

\[
P(\text{GS is stable}) = \prod_{i=1}^{N} \int_{0}^{\infty} \left( 1 - \left( \prod_{j=1,j \neq i}^{N} \beta_{ji} \right) \right) \left( \sum_{j=1,j \neq i}^{N} 1 - \exp \left( - \beta_{ji} (\frac{x_i}{1 + \alpha_i x_i}) (\frac{1}{N}) - 1 \right) \right) \alpha_i \exp(-\alpha_i x_i) dx_i \quad (8)
\]

Similarly, by representing the value of \( R \cdot V Y_{\{i\},k} \) as \( Y_{\{i\}} - Y_{ki} \) by \( \hat{y}_i \), we can find the probability of singleton structure (SS) being stable as:

\[
P(\text{SS is stable}) = \prod_{i=1}^{N} \prod_{k=1,k \neq i}^{N} \int_{\hat{y}_i=0}^{\infty} \int_{x_i=0}^{\infty} \left( 1 - \exp \left( \beta_{ki} (\frac{x_i}{1 + \alpha_i x_i}) 10.5 - 1 - \frac{1}{\beta_{ki}} \hat{y}_i \right) \right) \alpha_i \exp(-\alpha_i x_i) \prod_{j=1,j \neq i,k}^{N} \exp(-\beta_{ji} \hat{y}_i) \frac{\prod_{l=1,l \neq i,k}^{N} (\beta_{li} - \beta_{ji})}{\sum_{j=1,j \neq i,k}^{N} \prod_{l=1,l \neq i,k}^{N} (\beta_{li} - \beta_{ji})} \right) dx_i d\hat{y}_i \quad (9)
\]

Using (8) and (9), a lower bound on the probability that a network partition (different from GS/SS) is stable can be found as \( 1 - P(\text{GS is stable}) - P(\text{SS is stable}) \).

VI. Performance Evaluation

In this section, we evaluate the performance of the proposed CF algorithm by observing the average payoff (rate in Mbps) per link. We assume \( W \) to be 5 MHz. All the channels are assumed to follow a quasi-static Rayleigh flat fading model, and the results are averaged over 100,000 runs. The transmit power \( P_t \) and noise power spectral density \( (N_0) \) are normalized to 1, and their effects are included in the channel coefficients. We analyze the performance for \( N = 10 \) randomly distributed links over a wide range of average direct link SNR with the interference power (dB) having mean \( \sim U[-10,0] \).

Fig. 2 shows the average payoff (rate in Mbps) per link offered by the proposed (selfish) CF algorithm with optimal BW allocation and compares it with three benchmark cases: (1) always singleton/grand, (2) CF based on selfish-approval rule, where, a player switches from its current coalition to a new coalition if it can strictly improve its own rate, without decreasing the rate of any member of the new coalition, as proposed in [3], and (3) CF with equal BW allocation. Our results indicate that at very low/high average direct link SNR values, distributed players prefer to organize into GS/SS, respectively, while for moderate SNR values (between \( -5 \) dB to \( 5 \) dB), the proposed CF algorithm yields other structures. It is observed that under GS initialization, players prefer to leave GS for SNR > \( -5 \) dB. Fig. 2 also quantifies the gain.
in the average payoff per link when the proposed (selfish) CF rule is compared with the selfish-appraisal rule combined with optimal BW allocation. Our results show that (selfish) rule with equal BW allocation offers 62% more average payoff per link, as compared to (selfish-appraisal) CF rule at $-5$ dB SNR, which is further increased by 24%, reaching up to 86% using optimal BW allocation.

Fig. 3. Average payoff per link for single/multiple operating points.

Fig. 4 shows a lower bound on the probability that a general network partition, other than GS and SS is stable against a wide range of average direct link SNR. Our results indicate that a general network partition will emerge with the probability $> 0.5$ over the wide SNR range from $-2.5$ dB to $23$ dB which shows the effectiveness of proposed CF algorithm for moderate operating SNR.

VII. CONCLUSIONS

In this paper, the problem of joint coalition formation and bandwidth allocation in distributed cognitive radio networks, was considered. An efficient CF algorithm with history condition, to guarantee stability, was proposed. Moreover, three exit procedures were suggested in case of CF cycles which may result if the history condition is not used. Furthermore, probabilistic analysis to study the stability of GS and SS was performed. The proposed algorithm, with optimal bandwidth allocation, was shown to provide substantial payoff gains over other fixed CF structures and over CF algorithms with suboptimal bandwidth allocation.

REFERENCES