Model based foot shape classification using 2D foot outlines

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Abstract

This study introduces a novel technique to identify foot outline characteristics and to classify feet into groups using turning functions and clustering techniques so that shape can complement anthropometry in producing good fitting shoes. The digital 3D foot scans, obtained from 50 Hong Kong Chinese subjects (25 males and 25 females) were processed to generate the foot outlines at heights of 2 mm and 40 mm. The outlines were represented as turning functions and the similarity among shapes was determined using average linkage clustering. The results show that there are two distinct shape groups for the 40 mm foot outlines on both medial and lateral sides of the foot. The presence (46%) or absence (54%) of a medial bulge characterizes the medial side, while the two shape groups on the lateral side are mainly due to the lateral concavity in the mid-foot region. The group with a lateral concavity consists of more females (68%) and thus lateral side of foot outline appears to be gender related. Furthermore, the medial and lateral side clusters are not related to each other. The medial side shape from the 2 mm foot outline is a good indicator of fallen arches. Based on the analyses, four types of feet were identified: feet with (1) lateral concavity and a medial bulge, (2) a medial bulge and no lateral concavity, (3) lateral concavity and no medial bulge and (4) lateral concavity and a medial bulge. These shape differences can be useful in the design of shoe lasts and in the manufacture of compatible footwear so that trial and error fitting can be minimized.

Keywords: Foot outline, Last design, Clustering, Turning function, Arch, Scanning, Digital modeling

1. Introduction

A trend towards mass customized footwear for military personnel, health care workers and even for high-performance sportsmen such as skiers is common today. Such customization requires that good fitting lasts be designed and manufactured to acceptable manufacturing standards. However, the process of generating a customized last has been through trial and error with a series of fitting trials to achieve the required fit. This is primarily because, in most instances, an existing last is transformed to be similar to the person’s foot dimensions and the various intricacies of a foot are neglected. Custom-made footwear requires the dimensions as well as the characteristic features of a foot to be identified.

Characteristics such as arch angle, arch height, rearfoot alignment, presence or absence of a talonavicular bulge and navicular drift, and their similarities and differences among people are well-known [1] even though not all are used in the design of footwear or shoe lasts. The variations in the arch are used to design straight or flared shoes; rearfoot alignments are accommodated with hard or soft or post-type of cushioning and so on. To account for the differences in ball of foot length on the lateral side of foot, Wunderlich and Cavanagh [2] suggested that the fifth metatarsal head expansion in a woman’s last should be at a more proximal location than in a man’s last. Thus, identifying the similarity characteristics and classifying them into groups can be useful in the design of and mass customization of footwear for improved comfort [3–9]. Unfortunately, most of the identified differences among feet have been primarily on the forefoot. Some researchers [10] have claimed that differences in the rearfoot are minimal and suggested that the rear-part of foot lasts can be the same for any given size. This study investigates shape differences among feet and between genders so that shape can supplement anthropometry in the design of lasts so that footwear can be designed and fitted right the first time.

Identifying and classifying foot shapes can have a significant impact on last design [11–13] and the treatment or prevention of foot related disorders [14]. Techniques that can be used to classify shapes include turning functions [15], Fourier descriptors [16], autoregressive coefficient [17], stochastic labeling [18], convolution [19], curve bending function and variations [20], etc. Turning functions are an efficient method for “free” curve matching especially for complex and open shapes [21] that are convex as well as concave [15,22,23].

Thus the objective of this study was to identify the significant characteristics on the lateral and medial sides of the foot.

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2. Methodology

2.1. Participants

Fifty students (25 males and 25 females) from the Hong Kong University of Science and Technology were recruited and consented to participate in this study, which was approved by the institutional research ethics committee. The descriptive statistics of the participants are given in the Table 1. None of them had any visible foot abnormalities or foot illnesses.

2.2. Foot shape determination

The YETITM foot scanner was used to obtain the 3D point cloud of each participant’s foot surface. The scan sections were set to be 1 mm apart and each section had 360 points, so that the total number of scanned points depended on each participant’s foot length.

Prior to scanning, ten anatomical landmarks were identified and marked on the right foot as shown in Fig. 1. Five of those landmarks were on top of the Metatarsal-Phalangeal joints (MPJ), one each on the medial and lateral malleoli, and one at the navicular [24]. Even though we anticipated a need for all of these landmarks, as the main interest of this research was to examine the foot outlines, only two landmarks were used in the final analysis.

The participants were asked to stand on the laser scanner and then the participant’s right foot was aligned and scanned when each foot bore half of the body weight. The scanned data included the 3D coordinates of the points on the foot surface and the 3D coordinates of the landmarks that were placed on the foot.

The 3D point cloud data obtained from the scanner were used to generate the foot outline using the algorithm proposed by Witana et al. [9]:

1. All scanned points below the pre-determined cut-height were selected and projected on to the $X-Y$ scanner surface plane.

2. The two points on the boundary of the $XY$ projection (points with minimum $Y$ coordinate and maximum $Y$ coordinate) were selected as representing the foot outline.

Two foot outline profiles at cut heights of 2 and 40 mm from the bottom surface of the foot were determined (Fig. 2). The 2 mm profile was to simulate a barefoot footprint because a footprint is traditionally used to characterize the different types of foot arches [25] and in the design of foot lasts. The 40 mm height was chosen to represent the “top-view” of the foot excluding the malleoli, as the foot top view of the foot is commonly used to match the shoe last outline when customizing footwear [8].

Registration is important in shape comparisons [26]. With reference to feet, alignment based on the principal component (PC) method [27,28] has been shown to be more robust than the commonly used reference axes such heel-center line [29]. Hence, the 40 mm foot outline data were processed with the PC method as this outline gives an overall representation of the foot. Once aligned, the 2 mm outline was set to the same alignment.

To compare the shape on the lateral and medial sides, the outline data were re-sampled so that the number of points ($N = 250$) from the pternion to M-MPJ and pternion to L-MPJ were the same for all subjects. The MatLab “resample” function, which is based on Newton interpolation was used with an anti-aliasing (low-pass) filter and compensator to eliminate any noise generated during re-sampling.

2.3. Representation of foot shape using turning function

The “Turning Function” [15] is the counter-clockwise angle between the tangent at any point on the shape of concern and the $x$-axis. The function is generally represented with respect to the length along the boundary (Fig. 3). The function tracks the degree of turn, increasing with left-hand turns and decreasing with right-hand turns [30]. The algorithm to generate the turning function is as follows:

Let

\[ N = \text{Number of data points on the periphery of the foot}. \]

\[ X_i, Y_i \text{ are } i\text{th data point of the foot shape } i = 1, 2, \ldots, N. \]

\[ S_i = \text{accumulated arc length at ith point}. \]

\[ \theta_i = \text{the counterclockwise angle between the line from } (X_i, Y_i) \text{ to } (X_{i+1}, Y_{i+1}) \text{ and x-axis, where } i = 1, 2, \ldots, (N - 1). \]

1. The cumulative arc length $l_i$ is calculated as:

\[ l_i = \begin{cases} 0 & \text{if } i = 1 \\ l_{i-1} + \sqrt{(X_i - X_{i-1})^2 + (Y_i - Y_{i-1})^2} & \text{if } i \neq 1. \end{cases} \]  \hspace{1cm} (1)

2. The normalized arc length,

\[ S_i = \frac{l_i}{l_N}. \]  \hspace{1cm} (2)
3. The turning angle $\theta_i$ is calculated using the equation

$$\theta_i = \begin{cases} 
\text{sgn}(Y_{i+1} - Y_i) \cdot \frac{1}{2} & \text{if } (X_{i+1} - X_i) > 0 \\
\text{sgn}(Y_{i+1} - Y_i) \cdot \left( \pi - \text{sgn}(Y_{i+1} - Y_i) \cdot \frac{\pi}{2} \right) & \text{if } (X_{i+1} - X_i) < 0 \\
\text{undefined} & \text{if } (X_{i+1} - X_i) = 0
\end{cases}$$

If $(Y_{i+1} - Y_i) = 0$;

$$\theta_i = \begin{cases} 
0 & \text{if } (X_{i+1} - X_i) > 0 \\
\pi & \text{if } (X_{i+1} - X_i) < 0 \\
\text{undefined} & \text{if } (X_{i+1} - X_i) = 0
\end{cases}$$

where $\text{sgn}(z)$ is the sign function defined as

$$\text{sgn}(z) = \begin{cases} 
-1 & \text{if } z > 0 \\
0 & \text{if } z < 0 \\
1 & \text{if } z = 0.
\end{cases}$$

4. Then the turning function $\Phi(s)$ is given by;

$$\Phi(s) = \sum_{i=1}^{N-1} \theta_i \chi_{[i,i+1]}(S)$$

where $\chi_{[i,i+1]}(S) = \begin{cases} 
1 & \text{if } S \in [i, i+1] \\
0 & \text{otherwise.}
\end{cases}$

To understand the shape of feet, the lateral part of the foot from the MPJ to the pternion and the medial part from the MPJ to pternion were investigated separately. The toe region was excluded from the analysis as differing characterizations exist for toes. The respective turning functions ($\Phi(S)$) for the medial and lateral sides of a foot were calculated with M-MPJ and L-MPJ as the starting (or reference) point respectively (Fig. 4). When comparing shapes, it is important that the reference point be the same as otherwise, there will be a shift in the turning function which makes comparisons difficult.

2.4. Similarity between shapes

If two shapes, $P$ and $Q$, have turning functions of $\theta_P(S)$ and $\theta_Q(S)$ (Fig. 5), then the degree to which the shapes $P$ and $Q$ are similar can be measured using various measures. Euclidean distance between two functions or distance function, $D(P, Q)$, is a common measure defined as follows [15,31]:

$$D(P, Q) = \|\theta_P(S) - \theta_Q(S)\|_2$$

$$= \left( \int_0^1 |\theta_P(s) - \theta_Q(s)|^2 ds \right)^{1/2}. \tag{5}$$

This distance function has the following properties [32]:

1. $D(P, Q) \geq 0$ for all $P$ and $Q$.
2. $D(P, Q) = 0$ if and only if $P = Q$.
3. $D(P, Q) = D(Q, P)$ for all $P$ and $Q$ (Symmetry, in other words the order of comparison should not matter); and
4. For a third shape, $R$, $D(P, Q) + D(Q, R) \geq D(P, R)$ for all $P$, $Q$, and $R$ (Triangle Inequality). This means that, if we have $n$ number of shapes to compare, we will get a $(n \times n)$ symmetrical matrix with zeros along the diagonal as the distance function matrix.

The normalized form of the distance function, similarity, $S(P, Q)$ can be defined as follows:

$$S(P, Q) = \left( 1 - \frac{D(P, Q)}{D(\max)} \right) \times 100 \tag{6}$$

where $D(\max) = \text{maximum value of the distance function matrix.}$

The above equation gives the relative similarity among two shapes with respect to the largest distance function. A higher value of similarity indicates a closer match between the shapes.

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2.5. Shape clustering

The medial and lateral distance function matrix of the fifty foot outlines were further analysed using the average linkage hierarchical clustering method. This method of clustering is based on the average of the single linkage, which depends on minimum distance and complete linkage which evaluates maximum distance [33,34]. It is superior to both the single and complete linkage methods as it can minimize the within cluster variations and maximize between cluster variations [35]. In addition, the average linkage possesses some features of both Ward’s and Centroid linkage techniques such as minimizing of variance within clusters and finding the smallest distance between centroids. The average linkage hierarchical clustering method produces a very robust nested series of partitions and it can handle very large data sets quite efficiently [36] as well.

In average linkage clustering, the distance, $d(r,s)$ between any two clusters, $r$ and $s$, is defined as the mean of distances between all pairs of objects, where each pair is made up of one object from each group. The $d(r,s)$ is computed as $d(r,s) = \frac{1}{N_r \times N_s} \sum_{i=1}^{N_r} \sum_{j=1}^{N_s} d(i,j)$, where $N_r$ is the sum of all pairwise distances between clusters $r$ and $s$, $N_r$ and $N_s$ are the sizes of the clusters $r$ and $s$ respectively. At each stage of clustering, the clusters $r$ and $s$, for which $d(r,s)$ is a minimum, are merged.

3. Analysis and results

A cluster analysis was performed on the similarity indices for the medial and lateral side of 2 and 40 mm foot outlines separately. The number of combinations for the distance comparisons were $50C_2 = 1225$, as there were fifty subjects and the dissimilarity calculations were between any two subjects. The descriptive statistics of the distance functions among subjects are in Table 2 and it was found that the distance functions follow a normal distribution (Fig. 6). The maximum distance is highest for the 2 mm medial side distance function (Table 2), mainly due to a larger range of shape differences in the arch of the foot.

Table 2

<table>
<thead>
<tr>
<th></th>
<th>2 mm foot outline</th>
<th>40 mm foot outline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum distance</td>
<td>0.45300</td>
<td>0.19200</td>
</tr>
<tr>
<td>Minimum distance</td>
<td>0.08020</td>
<td>0.01900</td>
</tr>
<tr>
<td>Mean distance</td>
<td>0.22992</td>
<td>0.09096</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.07590</td>
<td>0.02434</td>
</tr>
</tbody>
</table>

Fig. 7a shows a dendrogram of the 2 mm foot outline clusters with a 60% similarity level. The corresponding mean foot outline shapes of each of the clusters is shown in Fig. 7b.

Based on the cluster analysis it is clear that that there are three distinct groups (Figs. 7a and 7b) and these clusters comprise 74%, 20% and 6% of the subjects in cluster 1, 2 and 3 respectively. The identical subjects were classified in a different study [37] and
Cluster 1
Cluster 2
Cluster 3
Fig. 7a. Dendrogram of 2 mm foot outlines on medial side with 60% of similarity level (M/F denotes male/female).

Fig. 7b. The mean shapes corresponding to the three clusters of the 2 mm medial side foot outline.

Table 3
Arch classification within the three clusters based on [10]. The values correspond to the number of subjects categorized as low, normal and high arch with dorsal angle. The number corresponding to the footprint arch index (AIF) is within parentheses. An angle less than 22.68° was considered a low arch while an angle greater than 26.188° was categorized as a high arch. The corresponding values of AIF were ≥0.26 and ≤0.21 respectively.

<table>
<thead>
<tr>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>Normal</td>
<td>High</td>
</tr>
<tr>
<td>Cluster 1</td>
<td>2 (2)</td>
<td>8 (9)</td>
</tr>
<tr>
<td>Cluster 2</td>
<td>4 (6)</td>
<td>1 (0)</td>
</tr>
<tr>
<td>Cluster 3</td>
<td>0 (0)</td>
<td>1 (1)</td>
</tr>
</tbody>
</table>

Note: Out of 50 subjects, the data of two subjects within cluster 1 are excluded as they had unusable footprint data.

The foot arch related measures of these subjects are reported in Table 3.

Based on the widely accepted foot arch index [38] and the recently reported dorsal angle measure [37], it is clear that cluster 2 is predominantly low-arched feet (80%) and cluster 1 is representative of both high and normal arch (31.4% of high arch and 60% of normal arch). The foot outlines (Fig. 7b) clearly show that cluster 2 has a wider mid-foot area and hence it is not surprising that cluster 2 represents low-arched subjects as the footprint arch index depends on the ratio of the mid-foot area to the area of the entire foot, excluding the toes.

The dendrogram for the cluster analysis of 60% of similarity level on the lateral side of 2 mm foot outlines and corresponding mean of foot outline shapes of each clusters are shown in Figs. 8a and 8b respectively.

The lateral side foot outline of 2 mm shows two distinct clusters at a 60% of similarity level. Cluster 1 has 48% of the subjects and the remaining 52% belong to cluster 2 showing a somewhat equal distribution between these two clusters.

Table 4
Gender distribution of 2 mm lateral side foot outlines within clusters.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster 1</td>
<td>09 (36%)</td>
<td>15 (60%)</td>
<td>24</td>
</tr>
<tr>
<td>Cluster 2</td>
<td>16 (64%)</td>
<td>10 (40%)</td>
<td>26</td>
</tr>
</tbody>
</table>

The gender distribution of the 2 mm lateral side foot outlines (Table 4) shows more females (60%) in cluster 1. This cluster represents outline shapes that have a lateral concavity (inverted ramp) in the mid-foot region when compared to the shapes in cluster 2. In contrast, cluster 2 comprises more male subjects (64%) with a straighter lateral side shape.

The cluster compositions for the 40 mm foot outlines are in Table 5. The dendrograms of the 40 mm medial and lateral side foot outlines are shown in Figs. 9a and 10a. The mean shapes are shown in Figs. 9b and 10b.

Subjects are somewhat equally distributed between the two clusters in the lateral side and nearly equally (46% in cluster 1 and 54% in cluster 2) distributed in the medial side. Cluster 1 has a significant medial bulge compared to cluster 2 (Fig. 9b); with a similar proportion of males and females in cluster 1 (48% of males and 52% of females), the probability of having a medial bulge is similar between males and females.

Cluster 1 of the 40 mm lateral outline has 68% males with 68% of females in cluster 2. Thus, it is no surprise that the mean outline of cluster 1 is outside the mean outline of cluster 2 (Fig. 10b) even though cluster 2 has a higher lateral concavity, because males tend to have wider feet than females [13]. This result is consistent with the 2 mm lateral outline as well.

The inter-correlations between clusters are in Table 6. The only significant correlation is between the 2 mm lateral side cluster and the 40 mm lateral side cluster (correlation coefficient = 0.642; p < 0.001).
### Table 5
Cluster composition for 40 mm foot outlines.

<table>
<thead>
<tr>
<th></th>
<th>Medial side</th>
<th></th>
<th></th>
<th></th>
<th>Lateral side</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male Female</td>
<td>High arch</td>
<td>Normal arch</td>
<td>Low arch</td>
<td>Male Female</td>
<td>High arch</td>
<td>Normal arch</td>
<td>Low arch</td>
</tr>
<tr>
<td>Cluster 1</td>
<td>12</td>
<td>11</td>
<td>5</td>
<td>9</td>
<td>9</td>
<td>17</td>
<td>08</td>
<td>5</td>
</tr>
<tr>
<td>Cluster 2</td>
<td>13</td>
<td>14</td>
<td>7</td>
<td>15</td>
<td>3</td>
<td>08</td>
<td>17</td>
<td>9</td>
</tr>
</tbody>
</table>

*Note: Out of 50 subject data, 2 data belonging to cluster 2 on medial side and cluster 1 on lateral side were discarded as the arch index could not be calculated.

### Table 6
Cross-correlations between clusters; the numbers shown are the Pearson correlation. The *p*-value is in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>2 mm medial side</th>
<th>2 mm lateral side</th>
<th>40 mm medial side</th>
<th>40 mm lateral side</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 mm lateral side</td>
<td>-0.071</td>
<td>(0.633)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40 mm medial side</td>
<td>-0.165</td>
<td>(0.262)</td>
<td>0.125</td>
<td>(0.397)</td>
</tr>
<tr>
<td>40 mm lateral side</td>
<td>-0.207</td>
<td>(0.157)</td>
<td>-0.642</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

### 4. Discussion

Feet can be classified in many different ways, but most methods require extensive data gathering and analysis [37]. The pattern recognition method adopted here is easy to use, especially, with the availability of many types of scanners and imaging methods these days.

The distance functions among subjects showed a normal distribution for each type of outline. The medial side of the 2 mm outline showed higher distances in the distance function matrix (mean = 0.2299, SD = 0.0759) than other outlines, mainly because of the larger range of shape differences in the mid-foot region due to the different arch types [38]. The 2 mm outlines show good agreement of the low-arch classifications with dorsal angle and the footprint arch index (Table 3). Interestingly, the low arch classification of dorsal angle for females is in perfect agreement with the proposed method, except for two cases in cluster 3. The mean shapes of cluster 3 are somewhat in-between clusters 1 and 2 and are possibly a transition from cluster 1 to cluster 2. In other words, this may correspond to the initiation of a fallen arch. None of the previous methods has been able to identify this transition from a normal arch to a fallen arch. The lateral sides of the outlines at 2 and 40 mm clearly show marked differences in concavity (Figs. 9b and 10b) and it can be attributed to the gender differences. The clusters with more females (60%) have a concavity in the mid-foot region while the other cluster exhibits a wider and straighter lateral shape. This result is consistent with previous reports [2,13] where researchers have shown that males generally have longer and broader feet when compared to females and that the dimensional differences between genders are significant.
particularly on the lateral sides of feet. The lateral concavity may be partly due to the abduction of talus and calcaneus bones relative to tarsometatarsal bones [11]. The differences in the lateral sides should be accounted for when designing shoes rather than scale the male lasts to fit females.

The 40 mm outlines clearly indicate the presence or absence of a medial bulge. Those with a noticeable medial bulge (Fig. 10b) are primarily due to an overhang of the navicular bone [11]. The cluster with the medial bulge comprises 52% of males and 48% of females indicating that the presence of fallen arches is not specific to any one gender. The medial bulge and the lateral concavity are not correlated (Table 6) and consistent with Kouchi [11]. This lack of correlation is indicative that the foot functioning in the different regions can be independent of each other due to the numerous bones and joints. The match of the proposed method with the dorsal angle is best for the females with low arches while the proposed method has a good match with the arch index for males.

Even though there are many variations in the outline shapes, the primary differences allow the identification of four critical groups that should be considered when designing shoes or lasts. These groups appear to be related to the differences in shapes in the mid-foot region identified between clusters. These groups are medial concavity with or without medial bulge and a straight (no concavity) with or without a medial bulge. These differences if not identified, can result in a greater mismatch between foot and last (see Fig. 11) and related to fit, because these locations have been identified as critical locations in previous fit studies [9]. Thus, considering the shape characteristics can improve footwear fit in the mid-foot and rearfoot regions.

A limitation of the proposed method is that it is unable to classify the normal and high arched feet. These two categories are clustered together and whether they need to be separated is debatable. If needed, then more research and analysis is needed to identify a related characteristic.

Another limitation is the load on the foot when scanning. To standardize the procedure and to minimize postural effects, the feet of young adults were scanned when bearing half body-weight. However, foot deformation is known to vary with different weight bearing conditions [24] and age [25]. Hence the shape differences should be investigated under differing load-bearing conditions as well and for different age groups.

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References


