Control of a Class of Discrete Time Nonlinear Systems with a Time-Varying Structure

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Abstract

In this paper we present a control methodology for a class of discrete time nonlinear systems that depend on a possibly exogenous scheduling variable. This class of systems consists of an interpolation of nonlinear dynamic equations in strict feedback form, and it may represent systems with a time-varying nonlinear structure. Moreover, this class of systems allows for the representation of some cases of gain-scheduling control, as well as input-output realizations of nonlinear systems which are approximated via localized linearizations. The results we present here attempt to establish a framework for a wide variety of control applications that fall within the general class of systems we consider.

1 Introduction

The problem of control of nonlinear systems presents fundamental difficulties due to the structural complexity arbitrary nonlinear systems may possess. In order to be able to perform stability analysis, it is of basic importance to first determine a class of systems that is representative of real problems, and yet amenable to analysis. One such class is that of strict feedback systems, which have been studied under the so called “backstepping” methodology [1]. Backstepping currently appears to be the most systematic method for nonlinear control design through step-by-step construction of quadratic Lyapunov functions.

Here, we will develop a theoretical framework and a control methodology which are inspired by backstepping results and which seek to encompass some aspects of gain scheduling control and the results in [2] and [3]. We present a class of discrete time nonlinear systems that have a time-varying structure. This class of systems is a generalization of the class of strict feedback systems traditionally considered in the literature, and it contains a subset of the class of linear time-varying systems considered in [4]. Here, however, no assumptions on rate of change of the system’s dynamics are needed, other than that this rate be bounded. The time-varying structure of the plant is affected by a possibly exogenous scheduling variable (which may include a subset of the states), and may be thought of as an interpolation between nonlinear “subsystems.” The subsystems, or components of the plant, are in strict feedback form, which allows us to methodically develop a control law that guarantees asymptotic stability (which is global under some conditions). To stabilize the plant we consider a control term constructed from laws that are tailored to each of the dynamic components of the nonlinear system (i.e., “local” laws within the scheduling variable space). In addition to a class of nonlinear time-varying systems, other systems that may be represented with the class proposed in this paper include a subclass of the systems considered in [2], as well as those in [3]. In a way similar to [5], a connection with gain scheduling control is also possible. Moreover, the ideas developed in this paper can be applied to the problem of adaptive backstepping [6], but encompassing a larger class of systems than that studied in [1] and [7].

The generalized class of systems and the main control theorem are presented in Section 2, accompanied by a proof of stability. In Section 3 we demonstrate the control method on a nonlinear discrete time system with a time-varying structure. Section 4 concludes the paper.

2 Control of a Class of Discrete Time Systems with a Time-Varying Structure

Here, we will consider the development of a control methodology for a class of nonlinear discrete time systems that may represent a wide variety of control problems, including gain scheduling control, interpolations of linearized input-output dynamics, and time-varying systems. We will present general control results that can be applied to the three cases mentioned, and then present an example of a time-varying nonlinear system. For simplicity, we concentrate on the regulation problem, although the results herein can be readily applied to tracking as well.
Consider the class of systems

\[ x_i(k+1) = \phi_i^c(X_i(k), v(k)) + \psi_i^c(X_i(k), v(k))x_{i+1}(k) \]
\[ x_n(k+1) = \phi_n^c(X_n(k), v(k)) + \psi_n^c(X_n(k), v(k))u(k), \] (1)

where \( X_i(k) = [x_1(k), \ldots, x_i(k)]^\top, v \in \mathbb{R}^q \) is a vector of possibly exogenous scheduling variables, and

\[ \phi_i^c(X_i(k), v(k)) = \sum_{j=1}^R \rho_j(v(k))\phi_i^j(X_i(k)) \]
\[ \psi_i^c(X_i(k), v(k)) = \sum_{j=1}^R \rho_j(v(k))\psi_i^j(X_i(k)). \] (2)

We assume \( \phi_i^c(0, v) = 0 \) and \( \psi_i^c(X, v) \neq 0 \) for all \( X_i \in \mathbb{R}^i \) and \( v \in \mathbb{R}^q \). The \( R \) nonlinear functions \( \rho_j : \mathbb{R}^q \to \mathbb{R} \) are assumed to be piecewise continuous and to satisfy

\[ 0 \leq \rho(\cdot) \leq 1, \]
\[ \sum_{j=1}^R \rho_j(\cdot) = 1. \] (3) (4)

We assume the state vector \( X_n(k) \) and the scheduling vectors \( v(k), v(k+1), \ldots, v(k+n-1) \) to be available for measurement. This assumption may imply that \( v(k) \) is given by an exogenous dynamical system, and this is the point of view we will take here.

We will be interested in developing a control strategy for system (1) by treating it as if it were a collection of pieces, or nonlinear dynamical structures, that can be “pulled out” of system (1) for control design purposes. We will consider stabilizing control laws for the \( R \) “pieces,” which we will later use to systematically construct a control law for system (1). In this way, we are adopting a “local” point of view, in that we act as if we were to control each of the \( R \) dynamical structures individually with a corresponding control law. As the structure of system (1) changes with the scheduling variable \( v(k) \), the controller also changes to the corresponding “local” law plus an extra term to eliminate the effects of cross couplings.

Consider the \( j \)th “piece” of (1) \((i = 1, \ldots, n-1),\)

\[ x_i^j(k+1) = \phi_i^j(X_i^j(k)) + \psi_i^j(X_i^j(k))x_{i+1}^j(k) \]
\[ x_n^j(k+1) = \phi_n^j(X_n^j(k)) + \psi_n^j(X_n^j(k))u^j(k), \] (5)

Note that we have labeled the “state” \( X_j^j(k), j = 1, \ldots, R \), of this piece of system (1) for design purposes. Although this labeling may be taken literally (i.e., by considering a system that in fact consists of \( R \) subsystems, each with its own state, which could be the case in a hierarchical system with a supervisor), here we use this separation into subsystems simply as a conceptual tool.

It can be shown that system (5) may be stabilized with \(^1\)

\[ z_i^j(k) = x_i^j(k) - \alpha_{i-1}^j(k) \]
\[ \alpha_{i-1}^j(k) = \frac{1}{\psi_{i-1}^j(X_{i-1}^j(k))} \left( -\phi_{i-1}^j(X_{i-1}^j(k)) + \alpha_{i-2}^j(k+1) \right) \]
\[ u^j(k) = \frac{1}{\psi_i^j(X_i^j(k))} \left( -\phi_n^j(X_n^j(k)) + \alpha_{n-1}^j(k+1) + c^j(k)z_i^j(k) \right), \] (6)

\( i = 1, \ldots, n, \) with \( \alpha_0^j = 0, \) and \( c^j(k) \) to be defined below. Now we will study a combination of the diffeomorphisms and control laws in (6) with the purpose of controlling system (1), and so we will replace \( X_i^j(k) \) by \( X_n(k) \), the real state vector. For convenience, let \( v_i(k) = [v(k), v(k+1), \ldots, v(k+i-1)]^\top. \)

**Theorem 1:** System (1) with the state vector \( X_n(k) \) and the scheduling vectors \( v_n(k) \) measurable, and satisfying the assumptions given in (2), (3) and (4), together with the diffeomorphism given by the definitions in (6) and the recursive equations (letting \( \alpha_0(k) = 0 \) and \( \delta_{12}(k) = 0 \))

\[ \alpha_m(k) = \alpha_m(X_m(k), v_m(k)) = \sum_{j=1}^R \rho_j(k)\alpha_m^j(k) + \alpha_m^a(k), \]
\[ m = 1, \ldots, n-1 \] (7)
\[ \alpha_m^a(k) = \frac{1}{\psi_m^a(k)}(-\delta_{m1}(k) - \delta_{m2}(k)), \]
\[ m = 1, \ldots, n-1 \] (8)
\[ \delta_{m1}(k) = \sum_{i=1}^R \sum_{j=1}^R \rho_i(k)\rho_j(k)\left( \phi_m^i(k) - \psi_m^i(k)\psi_m^j(k) \right), \]
\[ m = 1, \ldots, n \] (9)
\[ \delta_{m2}(k) = \sum_{i=1}^R \rho_i(k)\sum_{j=1}^R \alpha_{m-1}^j(k+1)\rho_j(k)\psi_m^i(k) \]
\[ - \rho_j(k+1) - \alpha_{m-1}^j(k+1), \]
\[ m = 2, \ldots, n \] (10)
\[ z_m(k) = x_m(k) - \alpha_m(k), \]
\[ m = 1, \ldots, n \] (11)

and the control law

\[ u(k) = u(X_n(k), v_n(k)) = \sum_{j=1}^R \rho_j(k)u^j(k) + u^a(k), \] (12)

\(^1\)We will keep the time index \( k \) in the discrete time derivations, but we will generally omit the state dependence unless clarification is necessary.
with the definitions

\[ u^*(k) = \frac{-1}{\psi^c_i(k)} (\delta_{n1}(k) + \delta_{n2}(k) + \delta_{n3}(k)) \]  

(13)

\[ \delta_{n3}(k) = \sum_{i=1}^{R} \sum_{j=1,j \neq i}^{R} \rho_i(k) \rho_j(k) \psi_j^n(k) c_j^i(k) z_1(k) \]  

(14)

\[ c_j^i(k) = \frac{1}{\Pi_{k=n-i+1}^{n-1} |\psi_i^c(k+n-i)|} \text{ where } 0 \leq c_j^i < 1, \]  

(15)

has an asymptotically stable equilibrium at the origin under the scheduling variable \( \nu(k) \). If, in addition, the functions \( \psi_i^c \) satisfy for all \( k \geq 0 \)

\[ |\psi_i^c(X_i(k))| \geq \beta_i > 0 \]  

(16)

for some constants \( \beta_i, i = 1, \ldots, n \), the stability results become global. If (16) is not satisfied, the closed loop system is locally asymptotically stable.

**Proof:** The proof is by induction, and it requires \( n \) steps. We start by letting \( z_1(k) = \sum_{j=1}^{R} \rho_j(k) z_1^j(k) = x_1(k) \). Notice that this equality holds because of assumption (4) and due to the fact that we are replacing the conceptual “state” \( x_1 \) in (6) by the actual state, \( x_1 \).

Furthermore, let

\[ z_2(k) = \sum_{j=1}^{R} \rho_j(k) z_2^j(k) - \alpha_1^c(k) = x_2(k) - \alpha_1(k), \]  

(17)

with \( \alpha_1(k) = \sum_{j=1}^{R} \rho_j(k) \alpha_3^j(k) + \alpha_1^c(k) \), where \( \alpha_1^c(k) \) is a stabilizing term canceling the effects of cross-terms that will be given later. Note that

\[ z_1(k+1) = \phi_1^c(k) + \psi_1^c(k)(z_2(k) + \alpha_1(k)) \]

\[ = \psi_1^c(k) z_2(k) + \delta_{11}(k) + \psi_1^c(k) \alpha_1^c(k), \]  

(18)

with

\[ \delta_{11}(k) = \sum_{i=1}^{R} \sum_{j=1,j \neq i}^{R} \rho_i(k) \rho_j(k) \phi_1^i(k) - \psi_1^c(k) \phi_2^c(k). \]  

(19)

Choose \( \alpha_1^c(k) = -\frac{\delta_1(k)}{\psi_1^c(k)} (-\delta_1(k)) \), which yields the dynamic equation \( z_1(k+1) = \psi_1^c(k) z_2(k) \). Note that in the special case where \( \psi_1^c(k) = \cdots = \psi_1^N(k) \), only \( \delta_1(k) \) (k) vanishes, whereas \( \delta_2(k) \) depends on the difference between \( \rho(k) \) and \( \rho(k+1) \), which is not necessarily zero. Also note that \( \alpha_2(k) = \alpha_2(X_2(k), \nu_2(k)) \). To complete this step, let \( V_2(k) = V_1(k) + \psi_1^c(k) z_2^2(k) \), whose difference is

\[ \Delta V_2(k) = \Delta V_1(k) + \psi_1^c(k) \psi_2^c(k) z_2^2(k) - \psi_1^c(k) z_2^2(k) \]

\[ = (\Pi_{i=1}^{n-1} \psi_1^c(k+2-i)) z_2^2(k) - z_1^2(k). \]  

(25)

We may continue this process until the \( n \)th state, where the system in \( z \)-coordinates is given by

\[ z_i(k+1) = \psi_i^c(k) z_{i+1}(k), \quad i = 1, \ldots, n-1 \]

\[ z_n(k+1) = \phi_n^c(k) + \psi_n^c(k) u(k) - \alpha_{n-1}(k+1) \]  

(26)

via the global diffeomorphism given by the recursive equations (11), together with the auxiliary control laws (7) and the stabilizing terms (8), (9) and (10). If we let

\[ V_{n-1}(k) = V_{n-2}(k) + (\Pi_{i=1}^{n-2} \psi_1^c(k+n-2-i)) z_{n-1}^2(k) \]  

(27)

(with \( V_1(k) \) and \( V_2(k) \) as defined above, and \( V_{n-2}(k) \) defined recursively), its difference at time \( k \) is given by

\[ \Delta V_{n-1}(k) = \left(\Pi_{i=1}^{n-1} \psi_1^c(k+n-1-i)\right) z_{n-1}^2(k) - z_n^2(k). \]  

(28)
Further, lim}_{\infty} are to one, the slower the convergence is. makes the controller act as a deadbeat controller, bring-

To complete the proof, note that

\[ z_n(k + 1) = \phi_n^c(k) + \psi_n^c(k)u(k) - \alpha_{n-1}(k + 1) \]

\[ = \sum_{j=1}^{R} \rho_j(k)\phi_j^c(k) + \sum_{j=1}^{R} \rho_j(k)\phi_j^c(k) - \phi_n^c(k) \]

\[ + \alpha_{n-1}^c(k + 1) + c^j(k)z_1(k) + \psi_n^c(k)u(k) \]

\[ = \delta_{n1}(k) + \delta_{n2}(k) + \delta_{n3}(k) \]

\[ + \sum_{j=1}^{R} \rho_j^2(k)c^j(k)z_1(k) + \psi_n^c(k)u(k) \]

\[ = \delta_{n1}(k) + \delta_{n2}(k) + \delta_{n3}(k) \]

\[ + \sum_{j=1}^{R} \rho_j^2(k)c^j(k)z_1(k) + \psi_n^c(k)u(k) \]

where we have used the fact that \( z_1^c(k) = z_1(k) \) by contraction, together with definition (14). Using the defini-
tion of the design terms (15), the control choice (12) yields

\[ z_n(k + 1) = \frac{\sum_{j=1}^{R} \rho_j^2(k)c^j(k)}{\sum_{j=1}^{R} \rho_j^2(k)c^j(k)}z_1(k). \] (30)

Pick the Lyapunov candidate

\[ V(k) = V_{n-1}(k) + \left( \sum_{i=1}^{R} \rho_i^2(k)c^i(k) \right) \]

\[ z^2_1(k), \] (31)

and note that, by design, \( 0 \leq \sum_{i=1}^{R} \rho_i^2(k)c^i(k) < 1 \), so that

\[ \Delta V(k) = \left( \sum_{i=1}^{R} \rho_i^2(k)c^i(k) \right)^2 - 1 \]

\[ z^2_1(k) \leq 0. \] (32)

Therefore, the system in \( z \)-coordinates is stable, and

since the transformation (11) is a global diffeome-

rphism, system (1) is globally stable. The argu-

tment to show asymptotic stability of the closed-

loop system uses LaSalle’s theorem. The negative

semidefiniteness of \( V(k) \) implies \( \lim_{k \to \infty} z_1(k) = 0 \), so that \( \lim_{k \to \infty} x_1(k) = 0 \) and \( \lim_{k \to \infty} \alpha_1(k) = 0 \). \n
Furthermore, \( \lim_{k \to \infty} z_1(k + 1) = 0 \), which implies \( \lim_{k \to \infty} z_2(k) = 0 \) and \( \lim_{k \to \infty} z_2(k) = \lim_{k \to \infty} \alpha_1(k) = 0 \), so that \( \lim_{k \to \infty} \alpha_2(k) = 0 \). We

may continue this argument until \( \lim_{k \to \infty} z_n(k) = 0 \), which implies \( \lim_{k \to \infty} x_n(k) = \lim_{k \to \infty} \alpha_{n-1}(k) = 0 \). Further, \( \lim_{k \to \infty} u(k) = 0 \). In this manner we have

then shown that control law (12) guarantees asymptotic stability of system (1), i.e., \( \lim_{k \to \infty} \| X_n(k) \| = 0 \). Observe that satisfaction of (16) makes stability global, since then the terms (15) are bounded away from in-

finity for all \( k \). Finally, letting the parameters \( c^1 = 0 \) makes the controller act as a deadbeat controller, bringing

the system states to zero in \( n \) steps. The closer \( |c^1| \)

are to one, the slower the convergence is.

Remark 1: It is also possible to design a control law for system (1) by a direct application of the backstep-

ping methodology. We refer to such a control law as a

“global” design, in the sense that it treats system (1)
as a unit (within the scheduling space of \( v \)), and disre-
gards its underlying structure. For this reason, such a

law has only one design constant. The control strategy

in Theorem 1 may be referred to as using “local” de-
signs, since it is indeed composed of a combination of

control laws suited for each of the individual “pieces”
of system (1). For this reason, it allows for a greater

design flexibility, and is perhaps conceptually more ap-

pealing than the “global” law.

Remark 2: One may think of the functions \( \rho_j(v(k)) \),
j = 1, \ldots, R, as performing an interpolation between

the “subsystems” of the form (5). No assumptions are

made on differentiability of these functions, so they

can be anything that satisfies (3) and (4). In particu-
lar, these functions may specify the left-hand side of a

Takagi-Sugeno (TS) fuzzy system [8], as has been con-

sidered in [2, 3]. However, in this paper the right-hand

side of the TS fuzzy system would be given by non-

linear systems in strict feedback form, whereas in the

mentioned works it is given by linear systems. More-

over, the functions \( \rho_j \) may be discontinuous at isolated

points as well, so that the class (1) includes systems

with a discontinuously time-varying structure. These

two possibilities will be exemplified in the next section.

Remark 3: The scheduling functions \( \rho_j(k) \) are assumed

generally to depend on a scheduling vector \( v(k) \)
of exogenous variables. Since the controller needs ac-

cess to the value of this vector at time index \( k + n - 1 \), it

is possible to include \( x_1(k) \) as part of the dependen-
cies of \( \rho_j(k) \), \( j = 1, \ldots, R \). For systems in input-output

form, tough, it is possible to schedule on the complete

state vector, so that \( v(k) \) may be equal to \( X_n(k) \) or it

may include it.

Remark 4: One may question the applicability of the

results presented in this paper, since the plant (1) is as-

sumed to have a very particular structure, with its un-
derlying pieces in strict feedback form. In addition to a

class of time-varying nonlinear plants, systems that can

be represented by (1) include some of those studied un-
der gain scheduling control. Moreover, systems given

by input-output equations which are locally approxi-
mated by, e.g., linearizations of the dynamic equation

(as studied in [3]), can also be put in the form (1). Fi-
nally, a subclass of the systems considered in [2] where

the linear systems in the right hand side of the TS fuzzy

system’s rules are, e.g., in controllable canonical form,
is also represented by the class (1). We pay the price

of restricting the linear systems in the right-hand side

of the rules to be in strict feedback form in order to

be able to guarantee the existence of a control law that
does not depend on LMI conditions.

3 Illustrative Example

Consider the dynamic equations

\[
\begin{align*}
    x_1^1(k+1) &= x_1^1(k) + e^{-|x_1^1(k)|}x_2(k) \\
    x_2^1(k+1) &= \sin(x_1(k))(x_1^1(k) + x_2^1(k)) \\
    &\quad - (1 + |x_1^1(k)x_2(k)|)x_3(k) \\
    x_3^1(k+1) &= \sqrt{|x_1^1(k)x_2(k)x_3(k)|} \\
    &\quad + (1 + x_1^2(k)x_2^2(k)x_3^2(k))u(k) \quad (33)
\end{align*}
\]

and

\[
\begin{align*}
    x_1^2(k+1) &= -3.2\frac{x_1(k) - x_3^1(k)}{1 + x_1^2(k)} \\
    &\quad + (2 + \cos(x_1(k))x_1^2(k))x_2(k) \\
    x_2^2(k+1) &= x_1(k)x_2(k) - \frac{x_2(k)}{|x_1(k)| + e^{-|x_2(k)|}} \\
    &\quad - (1 + x_1^2(k) + x_2^2(k))x_3(k) \\
    x_3^2(k+1) &= -x_1^2(k) - 2.7x_3(k) + x_2(k)x_3(k) \\
    &\quad + e^{x_1(k) - x_3(k)}u(k). \quad (34)
\end{align*}
\]

The plant will have its structure changing dynamically between subsystems (33) and (34). Note that we have only labeled the states in the left-hand side of the equations, in order to avoid notational confusions. Let \( X_3^1(k+1) = [x_1^1(k+1), x_1^1(k+1), x_1^1(k+1)]^T \) and \( X_3^2(k+1) = [x_1^2(k+1), x_2^2(k+1), x_3^2(k+1)]^T \), where the superscripts are indexes, denote the right-hand side of (33) and (34), respectively. Letting \( X_3(k) = [x_1^1(k), x_2^1(k), x_3^1(k)]^T \) be the state vector of the plant, its dynamics are given by

\[
X_3(k+1) = \rho_1(k)X_3^1(k+1) + \rho_2(k)X_3^2(k+1). \quad (35)
\]

Let the scheduling variable \( v(k) \in \mathbb{R} \) be given by a discrete-time system, where we denote \( v_1(k) = v(k), v_2(k) = v_1(k+1) \) for convenience, and

\[
\begin{bmatrix}
    v_1(k+1) \\
    v_2(k+1)
\end{bmatrix} =
\begin{bmatrix}
    0 & 1 \\
    0.3 & -0.4
\end{bmatrix}
\begin{bmatrix}
    v_1(k) \\
    v_2(k)
\end{bmatrix} +
\begin{bmatrix}
    0 \\
    0.4\sin(v_1(k)+v_2(k))
\end{bmatrix} \quad (36)
\]

We first assume the smooth scheduling functions \( \rho_1 \) and \( \rho_2 \) are given by

\[
\rho_i(v(k), x_1(k)) = \frac{w_i(v(k), x_1(k))}{\sum_{j=1}^2 w_j(v(k), x_1(k))} \quad (37)
\]

\[
w_i(v(k), x_1(k)) = \prod_{j=1}^2 \exp\left(-0.5\left(\frac{v(k) - c_j^i}{\sigma_j^i}\right)^2\right), \quad (38)
\]

with \( c_1^1 = -4, c_2^2 = 4, c_2^1 = 4, c_2^2 = -4, \sigma_1^1 = 3.6, \sigma_1^2 = 2.7, \sigma_2^1 = 2.4, \) and \( \sigma_2^2 = 3.8 \). As mentioned in Remark 2, these definitions correspond to the left-hand side of a TS fuzzy system, where the scheduling inputs are \( v(k) \) and \( x_1(k) \); i.e., the scheduling depends nonlinearly both on the exogenous input and on the first plant state (see Remark 3).

Figures 1 and 2 contain the results for this configuration of the plant, where the design parameters of the controller have been picked as \( c_1^1 = 0.8 \) and \( c_2^2 = 0.3 \). Note in Figure 1 that the plant and transformed states are regulated as desired. Furthermore, observe the behavior of the interpolation functions \( \rho_1 \) and \( \rho_2 \) in Figure 2, where we see that the structure of the plant (35) changes dynamically and is a “mixture” between (33) and (34).

Next we show how it is also possible to deal with a system with discontinuous interpolation functions. As-
sume that

\[ \rho_1(k) = \begin{cases} 
1 & \text{if } v(k) \geq 0 \\
0 & \text{if } v(k) < 0 
\end{cases} \]  

\[ \rho_2(k) = \begin{cases} 
0 & \text{if } v(k) \geq 0 \\
1 & \text{if } v(k) < 0 
\end{cases} \]  

Now there is a discontinuity at the origin of \( v(k) \) in the interpolation functions. We observe in Figure 3 how the proposed method handles this situation. Up to time index \( k = 4 \) the plant is composed exclusively of system (33). After this point the structure starts to alternate discontinuously between (33) and (34), as seen in Figure 4. Note that regulation is achieved, nevertheless, as shown in Figure 3.

4 Conclusions

In this paper we have presented a control methodology for a class of discrete time nonlinear systems that depend on a possibly exogenous scheduling variable. This class of systems consists of an interpolation of nonlinear dynamic equations in strict feedback form, and it may represent systems with a time-varying nonlinear structure, as was shown here via an example. In addition, this class of systems allows for the representation of some cases of gain-scheduling control, as well as input-output realizations of nonlinear systems which are approximated via localized linearizations. Thus, the framework presented here establishes the foundations for methodologies dedicated to a wide variety of applications.

References


