A 3D Generic Inverse Dynamic Method using Wrench Notation and Quaternion Algebra

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In the literature, conventional 3D inverse dynamic models are limited in three aspects related to inverse dynamic notation, body segment parameters and kinematic formalism. First, conventional notation yields separate computations of the forces and moments with successive coordinate system transformations. Secondly, the way conventional body segment parameters are defined is based on the assumption that the inertia tensor is principal and the centre of mass is located between the proximal and distal ends. Thirdly, the conventional kinematic formalism uses Euler or Cardanic angles that are sequence-dependent and suffer from singularities.

In order to overcome these limitations, this paper presents a new generic method for inverse dynamics. This generic method is based on wrench notation for inverse dynamics, a general definition of body segment parameters and quaternion algebra for the kinematic formalism.

Keywords: 3D inverse dynamics; Wrench notation; Quaternion algebra; Inertial coordinate system

INTRODUCTION

Joint forces and moments obtained from inverse dynamic computation are commonly used in gait analysis for orthopedics, ergonomics or sports analysis [1–6]. The principle of inverse dynamics is to use the Newton–Euler equations of motion in order to compute the net joint forces and moments as a function of the kinematics, body segment parameters and ground reaction forces and moments.

Several 3D inverse dynamic models [3,7–11] have been proposed. However, these models are limited in aspects related to inverse dynamic notation, body segment parameters and kinematic formalism.

First, for inverse dynamic notation, these models [3,7–11] use the conventional inverse dynamic method consisting of three steps:

(i) in the inertial coordinate system (ICS), and according to the first Newton–Euler law, the force vector at the proximal end is computed knowing the force vector at the distal end and the linear acceleration vector of the segment centre of mass.

(ii) in the segment coordinate system (SCS), and according to the second Newton–Euler law, the moment vector at the proximal end is computed knowing the moment vector at the distal end and knowing both force vectors at proximal and distal ends. To do so, the latter force vectors must be transformed from ICS to SCS. Also, the angular velocity and acceleration vectors, as well as the segment inertia tensor, must be known in the SCS.

(iii) for the next adjacent segment and according to the action/reaction principle, the force and moment vectors at the distal end are the opposite of the force and moment vectors at the proximal end of the current segment. Thus, the moment vector must be transformed from the current SCS to the next adjacent SCS.

Therefore, this three-step method yields successive coordinate transformations and manipulations of many attitude matrices [7]. Moreover, it is valid for all segments of a multibody system except for the terminal segment. Actually, for the terminal segment, the ground reaction forces are applied at the center of pressure of the force...
platform instead of the segment distal end. Thus, the
treatment of the terminal segment needs specific notation.
Secondly, in the definition of body segment parameters,
the position of the center of mass is systematically given
as a ratio of the proximal—distal length. Furthermore, the
axes of the SCS are considered as principal axes of inertia.
These two assumptions may become problematic when
personalization is necessary: amputation, hemiplegia,
obesity. Moreover, most studies that analyze the
sensitivity of body segment parameters are still based on
these two assumptions [12].

Thirdly, kinematical formalism generally relies on Euler/
Cardanic angles. The Euler/Cardanic angles are sequence-
dependent and vary for different joints [2,7,10,11,13,14].
Sometimes, the sequence is simply not reported by authors
[12,15]. Furthermore, it is well known that Euler/Cardanic
angles suffer from singularities caused by gimbal locks,
especially when computing angular velocity and angular
acceleration vectors [16].

In order to overcome these limitations, the objective of
this paper is to develop a new generic method for inverse
dynamics. This generic method is based on (i) wrench
notation for the inverse dynamic notation, (ii) a general
definition of body segment parameters and (iii) quaternion
algebra for kinematics formalism.

The conventional method will be summarized first, after
which the generic method will be explained in detail and
discussed.

Conventional 3D Inverse Dynamics

In general, the net joint force and moment are computed
separately [3,8–11]. From the free body diagram (Fig. 1),
we can write:

\[
\mathbf{F}_{ip} = m_i \mathbf{a}_i - m_i \mathbf{g} - \mathbf{F}_{id}
\]

\[
\mathbf{M}_{ip}^s = \mathbf{H}_i^s - \mathbf{M}_{id}^s - (\mathbf{k}_i \times \mathbf{F}_{ip}) - (\ell_i \times \mathbf{F}_{ip})\]

In Eq. (1), \( \mathbf{F}_{ip} \) is the proximal force vector, \( \mathbf{F}_{id} \) the
distal force vector, \( m_i \) the segment mass, \( \mathbf{a}_i \) the linear acceleration
vector of the center of mass and \( \mathbf{g} \) gravitational
acceleration. Equation (1) is computed in the ICS because
the weight is involved here. The linear acceleration vector
is computed as the second derivative of the center of mass
position vector expressed in the ICS. To do so, the center of
mass position vector is given as a ratio of the proximal to
distal end vector.

In Eq. (2), \( \mathbf{M}_{ip}^s \) is the proximal moment vector, \( \mathbf{M}_{id}^s \) the
distal moment vector, \( \mathbf{k}_i \) and \( \ell_i \) are the lever arm vectors
from the centre of mass to the proximal and distal ends and
\( \mathbf{H}_i^s \) is the time derivative of the angular momentum.
Equation (2) is computed in the SCS (denoted by index \( s \))
and at the center of mass. Then, the derivative of the
angular momentum is given by [3,10,11]:

\[
\mathbf{H}_i^s = \begin{bmatrix}
    \ell_{i}\alpha_{i1}^s - (\ell_{iyy} - \ell_{icz}) \omega_{i1}^s \omega_{i1}^s \\
    \ell_{iyy} \alpha_{i1}^s - (\ell_{icz} - \ell_{ixx}) \omega_{i1}^s \omega_{i1}^s \\
    \ell_{icz} \alpha_{i1}^s - (\ell_{ixx} - \ell_{iyy}) \omega_{i1}^s \omega_{i1}^s
\end{bmatrix}
\]

where \( \mathbf{I}_i \) is the inertia tensor, \( \alpha_i^s \) and \( \omega_i^s \) are the angular
acceleration and velocity vectors, and indexes \( x,y,z \)
correspond to the axes of the SCS. Equation (3) is
valid only if the inertia tensor is principal (i.e. \( \mathbf{I}_i = \text{diag}(I_{ixx}, I_{iyy}, I_{icz}) \)).

To compute Eq. (2), \( \mathbf{F}_{ip} \) and \( \mathbf{F}_{id} \) already computed from
Eq. (1) must be transformed using the attitude matrix \( \mathbf{R}_i \)
of the SCS with respect to the ICS

\[
\mathbf{F}_{id} = \mathbf{R}_i^{-1} \mathbf{F}_{id}
\]

\[
\mathbf{F}_{ip} = \mathbf{R}_i^{-1} \mathbf{F}_{ip}
\]

For the next adjacent segment and at the distal end, the
force vector in the ICS and moment vector in the next
adjacent SCS are computed using the action/reaction
principle

\[
\mathbf{F}_{i+1d} = -\mathbf{F}_{ip}
\]

\[
\mathbf{M}_{i+1d} = - (\mathbf{R}_{i+1}^{-1} \mathbf{R}_i) \mathbf{M}_{ip}^s.
\]

Therefore, the computation of Eqs. (1) and (2) is done in
separate coordinate systems which necessitates successive
coordinate transformations (Eqs. (4), (5) and (7)).

The attitude matrices \( \mathbf{R}_i \) are generally computed with
a pre-determined sequence of Euler/Cardanic angles
(\( \psi_i, \theta_i, \phi_i \)). For example, the \( XYZ \) sequence that converts
the SCS with respect to the ICS, can be computed [2,13]:

\[
\mathbf{R}_i = \begin{bmatrix}
    \cos \theta_i \cos \phi_i & -\cos \theta_i \sin \phi_i & \sin \theta_i \\
    \cos \psi_i \sin \phi_i + \sin \psi_i \cos \phi_i \sin \theta_i & \cos \psi_i \cos \phi_i - \sin \psi_i \sin \phi_i \sin \theta_i & -\sin \phi_i \cos \theta_i \\
    \sin \psi_i \sin \phi_i + \cos \psi_i \cos \phi_i \sin \theta_i & \sin \psi_i \cos \phi_i - \cos \psi_i \sin \phi_i \sin \theta_i & \cos \phi_i \cos \theta_i
\end{bmatrix}
\]

where \( c \) and \( s \) denote cosine and sine, respectively.

Also, the angular velocity and acceleration vectors \( \alpha_i^s \)
and \( \omega_i^s \) are computed as a function of these Euler/Cardanic
angles and their time derivative

\[
\omega_i = \begin{bmatrix} c\theta_i c\varphi_i & s\varphi_i & 0 \\ -c\theta_i s\varphi_i & c\varphi_i & 0 \\ s\theta_i & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_i \\ \dot{\varphi}_i \end{bmatrix}
\]

is the weight wrench and

\[
W_i^{\text{dynamic}}(C_i) = \begin{bmatrix} m_i a_i \\ H_i \end{bmatrix}
\]

is the dynamic wrench. The point \(D_i\) is the distal end of the segment and \(W_i^{\text{distal}}(D_i)\) is the distal wrench.

In a multibody system, the point \(D_i\) corresponds to the point \(P_{i-1}\), which is the proximal end of the previous adjacent segment. The wrench notation of the action/reaction principle is therefore given by

\[
W_i^{\text{distal}}(D_i) = -W_i^{\text{proximal}}(P_{i-1}) = \begin{bmatrix} -F_{i-1} \\ -M_{i-1} \end{bmatrix}.
\]

In the dynamic wrench, the derivative of the angular momentum is given by its general form [17], expressed at the center of mass in the ISC:

\[
H_i = I_i \omega_i + \omega_i \times I_i \omega_i.
\]

Then, the wrench equation of motion can be written at any point of the segment. The sum of the external wrenches (proximal, distal and weight) applied at this defined point equals the dynamic wrench applied at the same point. The segment proximal end \(P_i\) is chosen in order to compute the proximal wrench as a function of the other wrenches:

\[
W_i^{\text{proximal}}(P_i) = W_i^{\text{dynamic}}(P_i) - W_i^{\text{distal}}(P_i) - W_i^{\text{weight}}(P_i).
\]

One property of wrench is that the transformation from one point to another necessitates that the cross product of lever arm and force is added to the moment part of the wrench. For example, the transformation of the weight wrench from the center of mass to the proximal end is given by

\[
W_i^{\text{weight}}(P_i) = \begin{bmatrix} m_i g \\ 0_{3 \times 1} + c_i \times m_i g \end{bmatrix}
\]

**GENERIC METHOD**

The following section presents a one-step inverse dynamic method using wrench as inverse dynamic notation, a general definition of body segment parameters and quaternion algebra for kinematic formalism.

**Inverse Dynamics using Wrenches**

The wrench notation was used in order to compute the force and moment equations of motion at the same time. The wrench is a mechanical notation that represents both force and moment vectors. The wrench is expressed at a defined point location and in a defined coordinate system. In the free body diagram (Fig. 2), all wrenches are expressed in the ISC but at different points of the segment. The point \(P_i\) is the proximal end of the segment and

\[ W_i^{\text{proximal}}(P_i) = \begin{bmatrix} F_i \\ M_i \end{bmatrix} \]

is the proximal wrench. The point \(C_i\) is the center of mass of the segment,

\[ W_i^{\text{weight}}(C_i) = \begin{bmatrix} m_i g \\ 0_{3 \times 1} \end{bmatrix} \]

is the weight wrench. The point \(P_i\) is chosen in order to compute the proximal wrench as a function of the other wrenches:

\[
W_i^{\text{proximal}}(P_i) = W_i^{\text{dynamic}}(P_i) - W_i^{\text{distal}}(P_i) - W_i^{\text{weight}}(P_i).
\]

![FIGURE 2 Free body diagram: wrench notation of inverse dynamics.](image-url)
where \( \mathbf{c}_i = (\mathbf{P}_i, \mathbf{C}_i) \) is the lever arm vector from the proximal end to the center of mass expressed in the ICS.

Transforming the distal and dynamic wrenches in the same way as the weight wrench in Eq. (13) yields the following relation:

\[
\begin{bmatrix}
\mathbf{F}_i \\
\mathbf{M}_i
\end{bmatrix}
=
\begin{bmatrix}
m_i \mathbf{a}_i \\
\mathbf{I}_i \mathbf{a}_i + \omega_i \times \mathbf{I}_i \omega_i + \mathbf{c}_i \times m_i \mathbf{a}_i
\end{bmatrix}
-
\begin{bmatrix}
m_i \mathbf{g} \\
\mathbf{c}_i \times m_i \mathbf{g} - \mathbf{F}_{i-1}
\end{bmatrix}
- \mathbf{F}_{i-1}
- \mathbf{M}_{i-1} - \mathbf{d}_i \times \mathbf{F}_{i-1} \tag{15}
\]

where \( \mathbf{d}_i = (\mathbf{P}_i, \mathbf{D}_i) = (\mathbf{P}_i, \mathbf{P}_{i-1}) \) is the lever arm vector from the proximal end to the distal end expressed in the ICS.

We can notice that when expressing the equation of motion at the proximal end, it is no longer necessary to transform the inertia tensor at this point with the complex parallel axis theorem. The dynamic wrench can be easily transformed with the lever arm \( \mathbf{c}_i \).

Representing the proximal and distal wrenches as 6D-vectors, Eq. (15) can be written in the following compact form:

\[
\begin{bmatrix}
\mathbf{F}_i \\
\mathbf{M}_i
\end{bmatrix}
=
\begin{bmatrix}
m_i \mathbf{E}_{3 \times 3} & 0_{3 \times 3} \\
\mathbf{m}_i \mathbf{\hat{c}}_i & \mathbf{I}_i
\end{bmatrix}
\begin{bmatrix}
\mathbf{a}_i - \mathbf{g} \\
\omega_i \times \mathbf{I}_i \omega_i
\end{bmatrix}
+
\begin{bmatrix}
0_{3 \times 1} \\
\mathbf{E}_{3 \times 3}
\end{bmatrix}
\begin{bmatrix}
\mathbf{F}_{i-1} \\
\mathbf{M}_{i-1}
\end{bmatrix}
+ \begin{bmatrix}
\mathbf{E}_{3 \times 3} & 0_{3 \times 3} \\
-\mathbf{d}_i & \mathbf{E}_{3 \times 3}
\end{bmatrix}
\begin{bmatrix}
\mathbf{F}_{i-1} \\
\mathbf{M}_{i-1}
\end{bmatrix}
\tag{16}
\]

where \( \mathbf{E}_{3 \times 3} \) is the identity matrix, \( 0_{3 \times 3} \) and \( 0_{3 \times 1} \) are matrix and vector of zeros, \( \mathbf{\hat{c}}_i \) and \( \mathbf{d}_i \) are the skew-symmetric matrix of lever arms \( \mathbf{c}_i \) and \( \mathbf{d}_i \) (see Appendix).

The right hand \( 6 \times 6 \) matrix represents the mass and inertia terms that multiply the 6D-vector of linear and angular acceleration (including the acceleration of gravity). The left hand \( 6 \times 6 \) matrix (with ones on the diagonal) represents the contribution of the distal actions.

Equation (16) is an explicit form of the wrench equation of motion (Eq. 13) that includes both first and second Newton–Euler laws. Moreover, the distal wrench notation (Eq. 11) directly takes into account the action/reaction principle. Therefore, Eq. (16) is a one-step inverse dynamic method.

In order to generalize this method, a generic segment notation is proposed (Fig. 3). In this generic segment notation, only the proximal wrench of every segment is designated. The distal wrench is not explicitly needed because the action/reaction principle is already included in Eq. (16). Moreover, when considering the terminal segment, there is simply no distal wrench. Even though the distal end of the foot (or the hand) can be defined from an anatomical point of view, it is not appropriate in dynamics because the ground reactions are exerted at another point, the center of pressure. In the generic segment notation, the ground (denoted by \( i = 0 \)) follows the same notation as the other segments. The point \( P_0 \) corresponds to the center of pressure and

\[
\mathbf{W}_0^{\text{ground}}(P_0) = \begin{bmatrix}
\mathbf{F}_0 \\
\mathbf{M}_0
\end{bmatrix}
\]

is the ground reaction wrench expressed in the ISC. The point \( P_0 \) is equivalent to the “proximal end” of a “ground segment” and \( \mathbf{W}_0^{\text{ground}}(P_0) \) is equivalent to the proximal wrench exerted on this “ground segment”. Therefore, the one-step inverse dynamic method (Eq. 16) is still valid for the terminal segment.
From a practical point of view, the position of the center of pressure and the ground reaction wrench are determined according to the platform data.

**General Definition of Body Segments Parameters**

In addition, in the generic segment notation, the body segment parameters are expressed in the most general definition. As the distal end is not explicitly designated in the proposed inverse dynamic method, the center of mass is not given as a proximo–distal ratio. This proximo–distal ratio is a restrictive assumption in 3D inverse dynamics. Therefore, the generic segment notation includes the center of mass vector \( \mathbf{c}_i \) in the SCS. Also, the generic segment notation includes all of the elements of the inertia tensor. Actually, compared to Eq. (3), Eq. (12) is valid for any inertia tensor, principal or not.

From a practical point of view, the center of mass 3D vector \( \mathbf{c}_i \) and the inertia tensor (principal moments of inertia and orientation of the principal axes of inertia with respect to the SCS axes) can be obtained from literature [18,19].

**Kinematic Formalism using Quaternion Algebra**

Besides, in the generic segment notation (Fig. 3), the segment position is given by the generalized coordinates [20]

\[
\begin{bmatrix}
\mathbf{p}_i \\
q_i
\end{bmatrix}
\]

The 3 \( \times \) 1 vector \( \mathbf{p}_i \) is the position of the segment proximal end \( P_i \) expressed in the ICS, and the 4 \( \times \) 1 quaternion \( q_i \) represents the attitude of the SCS with respect to the ICS. This 7 \( \times \) 1 generalized coordinate is determined according to the SCS construction: (i) following the International Society of Biomechanics standardization [21], the SCS origin is constructed at proximal end and vector \( \mathbf{p}_i \) is the position of the origin (ii) the quaternion \( q_i \) is obtained from the orientation of the SCS axes with respect to the ICS axes by a singularity-free extraction [22]. By extension to the ground, vector \( \mathbf{p}_i \) expressed in the ICS is the position of the center of pressure \( P_0 \). Therefore, the lever arm vector \( \mathbf{d}_i \) is given for every segment by

\[
\mathbf{d}_i = [\mathbf{p}_{i-1} - \mathbf{p}_i].
\]

Equation (19) allows the computation of the linear velocity vector \( \mathbf{v}_i \) of the segment center of mass expressed in the ICS by direct differentiation:

\[
\begin{bmatrix}
0 \\
\mathbf{v}_i
\end{bmatrix} = \begin{bmatrix}
0 \\
\dot{\mathbf{p}}_i \\
q_i \otimes [\mathbf{c}_i]^\ast 
\end{bmatrix} \otimes q_i \ast + q_i \otimes [\mathbf{c}_i] \otimes \dot{q}_i \ast \tag{20}
\]

Likewise, Eq. (20) gives the linear acceleration vector \( \mathbf{a}_i \) of the segment center of mass expressed in the ICS by direct differentiation:

\[
\begin{bmatrix}
0 \\
\mathbf{a}_i \\
\dot{\mathbf{a}}_i \\
\ddot{\mathbf{a}}_i
\end{bmatrix} = \begin{bmatrix}
0 \\
\dot{\mathbf{p}}_i \\
0 \\
\dot{\mathbf{v}}_i
\end{bmatrix} + q_i \otimes [\mathbf{c}_i] \otimes q_i \ast + 2 \begin{bmatrix}
0 \\
\dot{\mathbf{c}}_i \\
\dot{\mathbf{q}}_i \ast \\
\dot{\mathbf{c}}_i \otimes \dot{q}_i \ast
\end{bmatrix} + q_i \otimes [\mathbf{c}_i] \otimes \ddot{q}_i \ast \tag{21}
\]

The segment angular velocity vector expressed in the ICS is given by [16,23]:

\[
\begin{bmatrix}
0 \\
\omega_i
\end{bmatrix} = 2\dot{q}_i \otimes q_i \ast. \tag{22}
\]
Equation (22) allows the computation of the segment angular acceleration vector expressed in the ICS by direct differentiation

\[
\begin{bmatrix}
0 \\
\alpha_i
\end{bmatrix} = 2(\ddot{q}_i \otimes \dot{q}_i^* + \dot{q}_i \otimes \ddot{q}_i^*). \tag{23}
\]

Finally, the attitude matrix \( R_i \) can be computed from the attitude quaternion

\[
R_i = [(q_i^2 + \dot{q}_i^2)E_{3x3} + 2q_i \dot{q}_i^* + 2\dot{q}_i \dot{q}_i^*], \tag{24}
\]

where \( E_{3x3} \) is the identity matrix and \( \dot{q}_i \) is the skew-symmetric matrix of vector \( \dot{q}_i \).

This allows the transformation of the inertia tensor from the SCS into the ICS:

\[
I_i = R_i \Gamma R_i^{-1}. \tag{25}
\]

**DISCUSSION**

The generic method proposed in this paper, is based on wrench for the inverse dynamic notation, a general definition of body segment parameters and quaternion algebra for kinematical formalism.

Concerning the dynamic notation, the wrench notation was used in order to compute the force and moment equations of motion in the same time (Eq. (15) vs. Eqs. (1) and (2)). To do so, all of the wrenches exerted on the segment must be expressed in the same coordinate system. The ICS is chosen because the weight of the segment is involved and because the action/reaction principle can be directly taken into account in the distal wrench notation (Eq. (11) vs. Eqs. (6) and (7)). Therefore, the proposed method is a one-step inverse dynamic method (Eq. (16)). Compared to the conventional three-step method, no successive coordinate system transformations are required (Eqs. (3), (4) and (7)). Moreover, the ground reactions are also noted as a wrench, so that the one-step inverse dynamic method is valid for every segment, including the terminal segment.

Setting the inverse dynamics in a compact one-step form and using the same notation for the ground and all segments may be convenient for numerical implementation.

The transformative property of wrenches from one point to another is advantageous in inverse dynamics where different points are considered: proximal end, distal end, center of mass, center of pressure etc. Notably, when computing the equation of motion at the proximal end, the non trivial transformation of inertia tensor (parallel axis theorem) is replaced by a simple wrench transformation bringing a “so-called” second inertial term \((\mathbf{c} \times m_i \mathbf{a})\) [24].

In the conventional method, the moment equation of motion (Eq. 2) is expressed in the SCS and the center of mass because the derivative of the angular momentum can be simplified (Eq. 3). However, Eq. (3) is only valid if the inertia tensor is principal.

Concerning the body segment parameters, the generic method proposed here includes a general definition. All of the components of the inertia tensor are featured in Eq. (12) and the center of mass is positioned by a 3D vector \( c_i^* \) in the SCS. The definition of the center of mass as a proximal–distal ratio [2,10] formally comes from the classical 2D inverse dynamics method [6] which is restrictive in a 3D model.

Concerning the kinematics formalism, the present method takes advantage of quaternion algebra for the computation of both linear and angular kinematics. The linear acceleration is conventionally determined as the second time derivative of \( r_i \), the position vector of the center of mass in the ISC [3,10,25]. However, the only way to obtain the center of mass in the ISC without referring to the moving SCS is to use a proximal–distal ratio. Using a 3D vector \( c_i^* \) in the SCS, the generic method proposes to compute the angular acceleration vector with the generalized coordinate and its time derivative (Eq. 21). Furthermore, the angular velocity and acceleration vectors are computed with the quaternion and its time derivative (Eqs. 22 and 23). Therefore, the computation of the linear acceleration vector and of the angular velocity and acceleration vectors is done without sequence pre-determination that would specify the model, and without singularities that may be encountered with Euler/Cardanic angles [16].

The use of quaternions vs. Euler/Cardanic angles is still debated in biomechanics [26,27]. In order to facilitate a clinical interpretation, Euler/Cardanic angles were formally introduced to quantify, for example, the knee joint movement [28]. However, Euler/Cardanic angles are not only applied to joint movements but also to the global attitude of segments [10,13,29]. The global attitude of segments is not required for clinical interpretation but for the computation of the velocity and acceleration vectors. Therefore, quaternions can be preferred for computational convenience: no sequence determination and no singularities. For clinical interpretation, the force \( \mathbf{F}_i \) and moment \( \mathbf{M}_i \), computed in the ISC (Eq. 16) can be further transformed in the SCS using the attitude quaternion \( q_i \) (or matrix \( R_i \)) without referring to the Euler/Cardanic angles.

Therefore, this paper presents a useful one-step inverse dynamic method compared to the conventional three-step method. This one-step inverse dynamic method is based on wrench notation and avoids successive coordinate system transformations.

Moreover, this paper presents a practical generic notation of segments. This generic notation allows the one-step inverse dynamic method to be valid at every segment including the terminal segment. Also, this generic notation allows us to take into account a general definition
of body segment parameters. Additionally, this generic notation of segments includes quaternions, so that the kinematic formalism, compared to conventional Euler/Cardan angles, is free from sequence determination and singularities.

NOMENCLATURE

\( i \) \hspace{1cm} \text{segmental index}

SCS \hspace{1cm} \text{segment coordinate system (denoted by index \( i \))}

ICS \hspace{1cm} \text{inertial coordinate system}

\( m_i \) \hspace{1cm} \text{segment mass}

\( \mathbf{c}_i^s / \mathbf{c}_i \) \hspace{1cm} \text{position vector of segment center of mass expressed in the SCS/ICS}

\( \mathbf{I}_i^s / \mathbf{I}_i \) \hspace{1cm} \text{inertia tensor at segment center of mass expressed in the SCS/ICS}

\( \mathbf{F}_{ip}^s / \mathbf{F}_{ip} \) \hspace{1cm} \text{force exerted on segment proximal end expressed in the SCS/ICS}

\( \mathbf{F}_{id}^s / \mathbf{F}_{id} \) \hspace{1cm} \text{force exerted on segment distal end expressed in the SCS/ICS}

\( \mathbf{M}_{ip}^s / \mathbf{M}_{id}^s \) \hspace{1cm} \text{moment exerted on segment proximal/distal end expressed in the SCS}

\( k_i^s / k_i \) \hspace{1cm} \text{lever arm from segment center of mass to proximal/distal end expressed in the SCS/ICS}

\( \mathbf{F}_i \) \hspace{1cm} \text{wrench exerted on segment proximal end expressed in the ICS}

\( \mathbf{p}_i \) \hspace{1cm} \text{position vector of segment proximal end expressed in the ICS}

\( \mathbf{r}_i \) \hspace{1cm} \text{position vector of segment center of mass expressed in the ICS}

\( \mathbf{v}_i \) \hspace{1cm} \text{linear velocity vector of segment center of mass expressed in the ICS}

\( \mathbf{a}_i \) \hspace{1cm} \text{linear acceleration vector of segment center of mass expressed in the ICS}

\( q_i \) \hspace{1cm} \text{attitude quaternion of the SCS with respect to the ICS}

\( \mathbf{R}_i \) \hspace{1cm} \text{attitude matrix of the SCS with respect to the ICS}

\( \psi_{ij}, \theta_i, \phi_i \) \hspace{1cm} \text{Euler/Cardan angle sequence of rotation of the SCS with respect to the ICS}

\( \omega_i^s / \omega_i \) \hspace{1cm} \text{segment angular velocity vector expressed in the SCS/ICS}

\( \alpha_i^s / \alpha_i \) \hspace{1cm} \text{segment angular acceleration vector expressed in the SCS/ICS}

\( \mathbf{d}_i \) \hspace{1cm} \text{lever arm from segment proximal end to distal end expressed in the ICS}

\( \mathbf{H}_i^s / \mathbf{H}_i \) \hspace{1cm} \text{time derivative of the segment angular momentum expressed in the SCS/ICS}

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References


APPENDIX

The quaternion product of two quaternions \( q_1 = [q_{s1}, q_{v1}]^T \) and \( q_2 = [q_{s2}, q_{v2}]^T \) is given by:

\[
q_1 \otimes q_2 = \begin{bmatrix}
q_{s1}q_{s2} - q_{v1}^T q_{v2} \\
q_{s1}q_{v2} + q_{s2}q_{v1} + q_{v1} \times q_{v2}
\end{bmatrix}
\] (A1)

where symbol “\( T \)” denotes the transpose, and symbol “\( \times \)” denotes the cross product.

The conjugate of a quaternion \( q = [q_s, q_v]^T \) is given by

\[
q^* = \begin{bmatrix}
q_s \\
-q_v
\end{bmatrix}.
\] (A2)

The skew-symmetric matrix of a 3D vector \( \mathbf{e} = [e_1, e_2, e_3]^T \) is given by

\[
\mathbf{\tilde{e}} = \begin{bmatrix}
0 & -e_3 & e_2 \\
e_3 & 0 & -e_1 \\
-e_2 & e_1 & 0
\end{bmatrix}.
\] (A3)