Abstract—The Shannon Capacity equation defines the total data transmission capacity of a transmission channel as a function of receiver SNR and bandwidth. When frequency-overlapped (FO) duplex mode of operation with an echo canceller is implemented on a digital subscriber loop (DSL) channel, then the near-end receiver SNR that determines the inbound capacity is not only a function of the far-end transmitted signal power, but also of the residual near-end transmit-signal power that leaks through the echo canceller. This ‘coupling’ occurs on both ends of the copper loop for upstream and downstream reception. Consequently, maximization of the data transmission capacity is a “bi-directional” problem. This problem has not been analyzed in the literature. In this paper, we define the notion of the total bi-directional data capacity of a copper loop and formulate the maximum bi-directional capacity that can be achieved when a power budget is allocated to each of the two transmitters. We then compute the channel capacity by an integer linear programming (ILP) technique for various channel and noise conditions and also compare the results with the data rates that are obtained by a modified “water-filling” algorithm designed for FO duplex operation and with the channel capacities of the conventional frequency-division (FD) simplex transmission.

I. INTRODUCTION

The Shannon bound equation expresses the capacity of a channel in terms of the receiver signal-to-noise ratio $SNR(f)$, and the channel bandwidth $W$.

$$\text{Capacity} = \int_{f = f_{\text{min}}}^{f_{\text{min}} + W} \log_2 \left( 1 + SNR(f) \right) df$$

(1)

Application of this equation is quite straightforward for the calculation of the capacity of a copper loop on which the simplex mode of transmission is used, i.e., the transmitted signals in the two directions are separated either by time division (TD) or by frequency-division (FD) multiplexing. In the duplex mode of transmission in which the upstream (US) and downstream (DS) signals share the bandwidth at the same time, echo cancellation must be implemented to minimize the leakage of the transmitted signal into the received signal through the hybrid circuit. If perfect echo cancellation could be implemented, then frequency-overlapped operation would result in double the bidirectional capacity than that can be obtained in simplex operation. However, due to practical limitations (such as finite word size in sampling, fixed-point operations and inexact estimation of the echo channel transfer function), there always is a residual echo signal that leaks into the received signal. The residual echo is independent of the inbound received signal and hence is additional interference for the receiver. Moreover, the residual echo signal is proportional to the outbound transmitted signal power level. Therefore, the near-end leakage of the DS signal transmitted by the CO reduces the SNR of the received US signal at the central office (CO). Similarly, the near-end leakage of the US signal transmitted by the CP reduces the SNR of the received DS signal at the customer premise (CP). Due to the interaction between the transmitted signal and the received signal at each of the two ends (CO and the CP), the problem of determining the bi-directional Shannon capacity of a duplex channel is coupled between the CO and the CP [1] (see Fig. 1). While the bi-directional capacity of the channel for given SNR profiles at the two ends can be calculated simply by (1), the problem of calculating the maximum bi-directional capacity for any given power budget is more complex.

In the simplex mode of operation, given the noise power spectral density (PSD), the channel transfer function and the transmission bandwidth, only the transmit power PSD determines the capacity. It can be seen from (1) that the simplex capacity increases monotonically with respect to the transmit power. In frequency-overlapped duplex mode, the notion of channel capacity should be extended to the bi-directional capacity that is the sum of the capacities in the two directions. In this paper we formulate the problem of maximizing the bi-directional data capacity in frequency-overlapped duplex operation. We demonstrate that the optimization problem fits the framework of the classic integer linear-programming (ILP) problem that can be solved by conventional optimization algorithms designed for such problems. We present the optimal bi-directional data capacities for some specific realistic sets of parameters and then contrast the optimal capacities with the FD data rates and with data rates that are generated for identical sets of parameters by a sub-optimal bit-loading algorithm that is designed for FO duplex operation.

The discrete multiple-tone (DMT) technology as it applies to DSL application is well known [2] and its description is not included in this paper. Section II introduces the DMT-based notation used in the paper. Section III includes a formulation of the single-bin duplex capacity maximization problem and then we extend the concept to the DMT capacity maximization in section IV. Section V includes a description of the simulation results and section VI summarizes the conclusions and the contributions of this paper.

II. NOTATION

$\Delta f = \text{Frequency bin width and } N = \text{no. of frequency bins.}$

$E_{rk} = \text{Power allocated at the CP in frequency bin number } k.$

$E_{ck} = \text{Power allocated at the CO in frequency bin number } k.$

$E_{cs}$ and $E_{rc} = \text{Power budgets at the CO and the CP.}$

$P_{mask_{rk}}$ and $P_{mask_{ck}} = \text{Power spectral density (PSD) masks at the CP and the CO that limit the maximum power that may be transmitted in frequency bin } k.$

$N_{rk}$ and $N_{ck} = \text{Noise power in bin } k \text{ at the CP and the CO.}$

$b_{g_k} \cdot b_{c_k} = \text{No. of bits allocated to bin } k \text{ at the CP and the CO.}$

$C_{g_k} \cdot C_{c_k} = \text{US and DS date rates in bits per second (bps) bin } k.$

$\varepsilon_{c_k} \cdot \varepsilon_{g_k} = \text{Total Echo Return Loss Enhancement (ERLE) in bin } k \text{ at the CO and the CP.}$

These fractions represent the total attenuation of the transmit power that leaks into the receiver.
\( g_c, g_r = \) Power attenuation values due to the channel transfer function in bin \( k \) as observed at the CO and at the CP, respectively. The attenuation values can also incorporate the coding gain of the modem receiver. That is, if the channel transfer function at frequency \( f_k \) is \( H(f_k) \), and the coding gain is \( G \), then \( g_{ck} = H(f_k) \cdot G \).

We define the following channel to noise (CNR) and ERLE to noise ratios (ENR) to simplify subsequent expressions.

\[
M_{ck} = \frac{g_{ck}}{N_{ck}}, M_{rk} = \frac{g_{rk}}{N_{rk}}, \alpha_{ck} = \frac{\varepsilon_{ck}}{N_{ck}}, \alpha_{rk} = \frac{\varepsilon_{rk}}{N_{rk}}. \tag{2}
\]

III. CAPACITY OF ONE FREQUENCY BIN

Based upon the notation defined above, equation (3) expresses the data rate capacities for one frequency bin in the DS and the US directions.

\[
b_c = \frac{C_r}{\Delta f} \cdot \log_2 \left( 1 + \frac{E_r \cdot g_r}{N_r + E_r \cdot \varepsilon_r} \right) = \log_2 \left( 1 + \frac{E_r c}{1 + E_r \alpha_r} \right)
\]

\[
b_r = \frac{C_r}{\Delta f} \cdot \log_2 \left( 1 + \frac{E_r \cdot g_r}{N_r + E_r \cdot \varepsilon_r} \right) = \log_2 \left( 1 + \frac{E_r c}{1 + E_r \alpha_r} \right) \tag{3}
\]

The level of the signal received at the CP \( (E_r \cdot g_r) \), and the level of the leakage of the transmitted power level \( (E_c \cdot g_c) \) in the upstream direction, influence the transmission capacity \( (C_r) \) in the downstream direction. Similarly, the leakage of the transmitted power level \( (E_r \cdot g_r) \) in the downstream direction and the level of the signal received at the CO \( (E_c \cdot g_c) \) influence the capacity \( (C_r) \) in the upstream direction. Consequently, if the power \( E_r \) is increased with the objective of increasing the data rate \( C_c \) in the downstream direction, the date rate \( C_r \) in the upstream direction decreases due to the additional noise caused by the residual echo at the CO. This “coupling” (Fig. 1) between the power allocation at the CO and the power allocation at the CP reduces as the echo canceller performance improves, i.e., as \( \varepsilon_{ck} \) and \( \varepsilon_{rk} \) become smaller. In the limit, as \( \varepsilon_{ck} \) and \( \varepsilon_{rk} \) approach null values, the problem simplifies to the uncoupled simplex problem.

Fig. 2 and Fig. 3 include plots of the total bi-directional capacity in one frequency bin, \( C_c, C_r \), on the x-axis for two values of ERLE, \( 10^{-2} \) and \( 10^{-4} \), respectively. In each figure the power \( E_r \) is plotted on the x-axis and the power \( E_c \) is plotted on the y-axis as the allocated power value is varied from 0 to 100 mWatts. Clearly, when the ERLE is good \( (\varepsilon_{ck} = \varepsilon_{rk} = 10^{-7} \) in Fig. 2), the capacity continues to increase as the power is increased, albeit slowly. However, for worse values of ERLE, \( (\varepsilon_{ck} = \varepsilon_{rk} = 10^{-4} \) in Fig. 3), maximum capacity is obtained with the FD solution! That is, when either \( E_c \) or \( E_r \) is set to zero, the bi-directional capacity increases to values greater than the capacity achieved when \( E_c \) and \( E_r \) is each equal to 100 mWatts. The actual value of ERLE at which the choice between FD or FO operation would result in higher capacity depends upon the channel attenuation, which in turn, depends upon the frequency. Therefore, the implication of these observations is that even though frequency-overlapped operation may be permitted, the maximum capacity solution could be a mix of FD and FO operation over the full bandwidth that is available to the DSL modem. This observation indicates that the maximum capacity solution at low frequencies would allow the US and DS bandwidths to overlap, but at higher frequencies the US and DS signals would result in the frequency-division ‘zipper’ solution described in [3].

IV. TOTAL CAPACITY MAXIMIZATION PROBLEM

The maximization of total bi-directional capacity over the available frequency bandwidth is a constrained optimization problem that maximizes the following figure of merit in the DMT context, subject to constraints (5) and (6).

\[
J = J = \Delta f \cdot \sum_{k=1}^{N} \left[ \log_2 \left( 1 + \frac{E_r c}{1 + E_r \alpha_r} \right) + \log_2 \left( 1 + \frac{E_r c}{1 + E_r \alpha_r} \right) \right] \tag{4}
\]

\[
\sum_{k=1}^{N} E_{c_k} \leq E_{c_r}, \tag{5}
\]

\[
\sum_{k=1}^{N} E_{r_k} \leq E_{r_r}, \tag{6}
\]

\[
0 \leq E_{c_k} \leq P_{mask_{c}}, \tag{7}
\]

\[
0 \leq E_{r_k} \leq P_{mask_{r}}. \tag{8}
\]

It is possible to solve this problem by calculus of variations to determine the two functions \( E_{c_k} \) and \( E_{r_k} \) of frequency, \( f \), that maximize the functional, \( J \), subject to the power constraints. Another approach would be to treat this problem as an ordinary maximization problem with 2N variables subject to the power constraints.

A. Case 1:

For this case we ignore the power mask constraint to determine the theoretical total capacity constrained only by the power budgets such that, the inequality power budget constraint actually becomes the equality constraint, since the maximization of the capacity will necessarily use all the available power, since scaling up all of the power variables increases the capacity. Then we define the constrained maximization problem with a Lagrangian.

\[
\mathcal{F}(E_r, E_c) = J + \lambda_c \cdot \left( E_{c_r} - E_c^T \cdot U \right) + \lambda_r \cdot \left( E_{r_r} - E_r^T \cdot U \right), \tag{9}
\]

where \( U = \begin{bmatrix} 1 & 1 & \ldots & 1 \end{bmatrix} \) is a column vector of N-ones, and \( E_r \) and \( E_c \) each is a N-dimensional column vector. The value of \( N \) is equal to 256 for ADSL-like DMT parameters. Differentiating the quantity \( \mathcal{F} \) with respect to the vectors \( E_c \) and \( E_r \), and the scalars \( \lambda_c \) and \( \lambda_r \) yields \( 2N+2 \) equations in \( 2N+2 \) unknowns.

\[
\frac{\partial \mathcal{F}}{\partial E_c} = \frac{\partial J}{\partial E_c} - \lambda_c \cdot U = 0, \tag{10}
\]

\[
\frac{\partial \mathcal{F}}{\partial E_r} = \frac{\partial J}{\partial E_r} - \lambda_r \cdot U = 0, \tag{11}
\]

These two equations (8) can be written as the following after taking the partial derivatives of (7).
solved with the original inequality constraint given by (5), in

where

constraint (13) implies that each bin has only one choice of

the power budget constraint would not necessarily become equality

US in frequency bin k, and

1 represents the choice of transmitting

subject to the constraints,

is generally referred to as the Power Mask. In presence of a mask,

power that can be allocated to each frequency bin. Such constraint

bit/power allocation algorithm could be compared. A tighter upper

bound would result for the non-negative PSD constraint,

some power values becoming negative. A more useful optimum

result since nothing prevents a solution that could result in

substituting the solutions in (4).

Equations (9) and (10) together constitute 2N non-linear

equations and two linear equations in the 2N+2 unknowns that can

be solved for the 2N elements of \( E_c \) and \( E_r \) and the Lagrangian

constants, \( \lambda_c \) and \( \lambda_r \). After obtaining the solutions for \( E_c \) and

\( E_r \), the theoretical optimum capacity for the specified power

budgets and the measured system parameters can be calculated by

substituting the solutions in (4).

B. Case 2: Positive power constraint

The simplified approach described above may not yield a very

useful result since nothing prevents a solution that could result in

some power values becoming negative. A more useful optimum

bound would result for the non-negative PSD constraint,

\[
0 \leq E_{ck} \quad 0 \leq E_{rk}
\]

C. Case 3: The PSD constraint

The solution to Case-2 would provide a useful upper bound on

the capacity of the duplex channel against which the results of any

bit/power allocation algorithm could be compared. A tighter upper

bound would result if the solution were obtained with the power

spectral density constraint that also includes the upper bound on the

power that can be allocated to each frequency bin. Such constraint

is generally referred to as the Power Mask. In presence of a mask,

the power budget constraint would not necessarily become equality

for all loop lengths and the optimization problem would have to be

solved with the original inequality constraint given by (5), in

addition to the original PSD constraint given by (6).

D. Case 4: The ILP Formulation

It is possible to cast the optimization problem in the form of an

ILP problem and to solve it using the branch and bound method[4].

Maximize

\[
\sum_{k=1}^{N} \sum_{b_c=0}^{b_{\text{max}}} \sum_{b_r=0}^{b_{\text{max}}} v(k,b_c,b_r)(b_c + b_r)
\]

subject to the constraints,

\[
v(k,b_c,b_r) = 0 \quad or \quad 1, \text{ where the value 1 represents the choice of transmitting } b_c \text{ bits DS and } b_r \text{ bits US in frequency bin } k, \text{ and}
\]

\[
\sum_{b_c=0}^{b_{\text{max}}} \sum_{b_r=0}^{b_{\text{max}}} v(k,b_c,b_r) = 1 \quad \text{for every } k = 1 \ldots N
\]

where, constraint (13) implies that each bin has only one choice of

the selected pair of bits, \( b_c \) and \( b_r \). Equations (14) express the

constraints that the total transmitted power at the CO and at the CP

is each limited by the respective power budget,

\[
\sum_{k=1}^{N} \sum_{b_c=0}^{b_{\text{max}}} \sum_{b_r=0}^{b_{\text{max}}} v(k,b_c,b_r)(b_c + b_r) E_c(\cdot,\cdot)
\]

subject to the constraint,

\[
v(k,b_c,b_r) = 0 \quad \text{if } E_c(\cdot,\cdot) > P_{\text{mask}}(\cdot,\cdot)
\]

for all, \( k = 1 \ldots N \), \( 0 \leq b_c, b_r \leq b_{\text{max}} \).

The ILP problem can be solved for a given set of \( E_c(k,b_c,b_r) \)

and \( E_r(k,b_c,b_r) \) values. We solve the optimization problem in

two ways: 1) first to calculate the theoretical upper bound that we
call the Duplex Shannon Capacity, and then 2) to calculate a tighter

implementation-specific upper bound that is subject to additional

constraints such as the bit-cap constraint and specific coding gains.

In order to solve the optimization problem, the integer-programming

algorithm requires the pair of \( (E_r,E_c) \) values for each pair of \( \theta_0, b_r \); \( \in \{0 \ldots b_{\text{max}} \} \). There are two different approaches needed to calculate

the set of \( (E_r,E_c) \) values to obtain the Duplex Shannon Capacity and
to obtain the implementation specific bound. For each of the two
cases we use the measured values of the CNR (\( M \)) and ENR (\( \alpha \)).

The Duplex Shannon Capacity: To calculate the theoretical

upper bound, which we refer to as the Duplex Shannon Capacity for

bi-directional transmission through the DSL channel, we calculate

the pair of \( E_c \) and \( E_r \) values by solving the two simultaneous

equations (16), where the \( S_c \) and \( S_r \) would be expressed in terms of

the power values as

\[
E_c(\cdot,\cdot) = S_c \cdot \left( \frac{E_r M_r}{1 + E_r \alpha_c} \right)
\]

subject to the constraint,

\[
v(k,b_c,b_r) = 0 \quad \text{if } E_c(\cdot,\cdot) > P_{\text{mask}}(\cdot,\cdot)
\]

for all, \( k = 1 \ldots N \), \( 0 \leq b_c, b_r \leq b_{\text{max}} \).

The Implementation-Specific Capacity: Alternatively, we

solve the same two simultaneous equations (16) to obtain the

estimates of \( E_c \) and \( E_r \) for any given values of \( S_c \) and \( S_r \), for each pair of \( b_c \) and \( b_r \) values. We define the ‘given’ SNR at the receiver as the SNR of the received signal that is required for the desired operating

BER and for the QAM constellation size that is to be received/demodulated. These SNR values may be pre-stored as a function of BER and the number of bits, \( S(BER,b) \) for the coder/decoder that is implemented. Then, we evaluate the values of \( E_c \) and \( E_r \) by selecting the SNR values from the table for each pair of \( b_c \) and \( b_r \).

Depending upon whether the Duplex Shannon Capacity or the

Implementation-specific Capacity is to be calculated, the selected \( S_c \)

and \( S_r \) as described above are used in equation (17). Then the
calculated set of the values of $E_c$ and $E_r$ is used to solve the ILP problem defined by (12)−(15).

$$x = \begin{bmatrix} E_c \\ E_r \end{bmatrix} = A^{-1}y, \quad y = \begin{bmatrix} S_r \\ S_c \end{bmatrix} A = \begin{bmatrix} M_r & -S_r \alpha_c \\ -S_c \alpha_c & M_c \end{bmatrix}$$ (17)

V. SIMULATION RESULTS

We used the branch-and-bound algorithm for solving ILPs based on linear-programming relaxation to obtain the two capacities defined above. Then we compared the capacities with a practical ‘Greedy’ algorithm [5] expected to be published shortly, to obtain the implementation-specific data rates.

The simulation parameters consisted of channel transfer functions that correspond to seven (7) loops of 26-AWG twisted copper pairs (3−21 kft in steps of 3kft), noise PSDs that corresponds to –140 dBm/Hz (referred to as WGN) and also to 10-ISDNs, 10-HDSLs in the same binder and 4-TIs in adjacent binders (referred to as Mixed). Using the SNR tables that incorporate coding gains that correspond to Turbo Trellis Coded modulation, we calculated the data rates for specific implementations by solving (16) for $E_c$ and $E_r$. All simulation results are based on the ERLE value of $10^3$.

TABLE I tabulates the sum of the US and DS bit rates for the seven loops. The Shannon Capacity for FO duplex operation forms the upper bound of the data rates. The implementation-specific upper bound was obtained by using $b_{max}=8$ bits, which is also the bit-cap limit in the implementation of the bit-loading algorithm. A comparison of the tight upper bound and the results of the bit-loading algorithm shows that the “Greedy” algorithm achieves rates very close to the optimum achieved by the ILP. The difference between the ILP row and the corresponding Duplex Capacity corresponds to the improvement that could be achieved by improving the coding gains. The difference between the ILP row and the “Greedy” algorithm row corresponds to the improvement that could be achieved in the algorithm. Data rates on the short loops increase when the bit-cap limit $b_{max}$ is increased from 8 to 10, because the transmission power is well under the PSD mask on the shorter loops, so the QAM-constellation size can be increased from $2^8$ to $2^{10}$, without exceeding the PSD mask.

In order to demonstrate the advantage of the duplex operation in presence of the realistic echo canceller performance Fig. 4) we compare the Duplex Shannon Capacity to the optimal simplex FD capacity that can be obtained from (1). In a “zipper-like” FD simplex operation, half the bandwidth would be allocated to US and the remaining half to the DS transmission directions, if approximately equal data rates are desired. Hence, the theoretical Shannon capacity with FD operation is half the Shannon capacity that could potentially be obtained in FO duplex operation. If the echo canceller performance were to be unacceptable, i.e., $\varepsilon_{ck} = 1$ and $\varepsilon_{kr} = 1$ then the optimal solution would choose $\{E_{ck} = 0, E_{kr} \geq 0\}$ or $\{E_{kr} = 0, E_{ck} \geq 0\}$ such that the capacity would degenerate to the FD simplex capacity by implementing a ‘Zipper-like’ solution [3]. With better ERLE values, the duplex operation would result in a capacity curve that would be bounded by these two extremes. Therefore, if the modem design has the flexibility to use the FO operation, it would always result in data capacity that is greater than or equal to the capacity that is possible with the FD simplex operation.

A comparison of the last two shaded rows in the table shows that duplex operation can provide significantly higher data rates than simplex operation for most (<15 kft) loop lengths. On 15 kft loops, the duplex rate is marginally higher than simplex rates. On longer loops the signal attenuation is so high that, should FO operation be used, the residual echo would dominate the weak received signal. Therefore, the optimal solution would consist of allocating disjoint frequency bands to the US and DS signals, which is the FD simplex operation. Therefore, the curves for duplex and simplex capacities coalesce for the long loops.

VI. SUMMARY AND CONCLUSIONS

(1) The primary contribution of this paper is the definition of the bi-directional Duplex Shannon Capacity of a DSL channel on which the CO and CP transmitters communicate under fixed power budgets and PSD masks, and the US and DS signals are permitted to overlap in frequency. We define the Duplex Shannon Capacity as the maximum bi-directional information transfer rate that is theoretically possible during frequency-overlapped echo-cancelled duplex transmission. (2) The second contribution consists of the identification, formulation, and execution of an ILP-based approach for calculating the Duplex Shannon and Implementation-specific data capacities of the channel. (3) We compare the Duplex Shannon Capacity with the performance of the conventional frequency-division, or simplex transmission capacity of the channel to demonstrate the significant increase in data capacity that can be obtained by the implementation of echo cancellation. (4) We also compare the Duplex Shannon and Implementation Capacities with the data rates obtained by duplex water-filling based on a ‘Greedy’ bit-loading algorithm. The results show that the data rate obtained by the ‘Greedy’ [5] algorithm is very close to the optimum data rate. A future area of research consists of designing an algorithm of acceptable complexity that would generate the optimum or a provably near-optimum bit-loading profile.

REFERENCES

### TABLE I: COMPARISON OF DATA RATES OBTAINED BY THE ALGORITHM WITH ILP RATES AND SHANNON CAPACITIES

<table>
<thead>
<tr>
<th>Noise PSD=Mixed&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Loop Length (kft)</th>
<th>US+DS Date rates (Mbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>bitcap=8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;Greedy&quot;: BER=10^-5</td>
<td>16.000</td>
<td>13.416</td>
</tr>
<tr>
<td>ILP: BER=10^-5</td>
<td>16.000</td>
<td>13.660</td>
</tr>
<tr>
<td>Duplex Capacity</td>
<td>16.000</td>
<td>15.016</td>
</tr>
<tr>
<td>bitcap=10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;Greedy&quot;: BER=10^-5</td>
<td>20.000</td>
<td>14.584</td>
</tr>
<tr>
<td>ILP: BER=10^-5</td>
<td>20.000</td>
<td>14.996</td>
</tr>
<tr>
<td>Duplex Capacity</td>
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<td>17.236</td>
</tr>
<tr>
<td>Duplex Shannon Capacity</td>
<td>29.860</td>
<td>18.408</td>
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<tr>
<td>Simplex Shannon Capacity</td>
<td>16.956</td>
<td>11.224</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Noise PSD=WGN&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Loop Length (kft)</th>
<th>US+DS Date rates (Mbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>bitcap=8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Algorithm: BER=10^-5</td>
<td>16.000</td>
<td>13.984</td>
</tr>
<tr>
<td>Duplex Capacity</td>
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<tr>
<td>bitcap=10</td>
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<tr>
<td>&quot;Greedy&quot;: BER=10^-5</td>
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<td>15.344</td>
</tr>
<tr>
<td>Duplex Capacity</td>
<td>20.000</td>
<td>17.648</td>
</tr>
</tbody>
</table>

<sup>a</sup> Mixed corresponds to 10-ISDNs, 10-HDSLs, and 4-T1s in adjacent binder.

<sup>b</sup> WGN corresponds to White Gaussian Noise with PSD= –140 dBm/Hz.

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**Fig. 1:** Duplex allocation is a coupled problem

**Fig. 2:** Capacity of one frequency bin for ERLE=10^-7 as $E_c$ & $E_r$ are varied

**Fig. 3:** Capacity of one frequency bin for ERLE=10^-4 as $E_c$ & $E_r$ are varied

**Fig. 4:** Comparison of duplex and simplex Shannon capacities