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AN APPLICATION OF FUZZY SOFT SET IN MEDICAL DIAGNOSIS USING FUZZY ARITHMETIC OPERATIONS ON FUZZY NUMBER

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ABSTRACT

The concept of soft set is one of the recent topics developed for dealing with the uncertainties present in most of our real life situations. The parameterization tools of soft set theory enhance the flexibility of its applications to different problems. Here we apply fuzzy soft set technology through the well known Sanchez's approach for medical diagnosis using arithmetic operations on fuzzy numbers and exhibit the technique with a hypothetical case study.

Key words: *Soft set, fuzzy soft set, fuzzy number, defuzzification*

INTRODUCTION

A number of real life problems in engineering, medical sciences, social sciences economics etc. involve imprecise data and their solution involves the use of mathematical principles based on uncertainty and imprecision. Such uncertainties are being dealt with the help of topics like probability, fuzzy set theory, intuitionistic fuzzy set, vague set, theory of interval mathematics, rough set theory etc. However, Molodtsov [1] has shown that each of the above topics has some inherent difficulties due to the inadequacy of their parameterization tools. Then he initiated a different concept called soft set theory as a new mathematical tool for dealing with uncertainties which is free from the limitations of the above topics. Soft set theory has a rich potential for applications in several directions, few of which had been explained by Molodtsov in his pioneer work [1] and by Maji *et al* [2].

In this paper we study Sanchez's [3] method for medical diagnosis using the notion of fuzzy soft set together with arithmetic operations on fuzzy number and exhibit the technique with a hypothetical case study.

Preliminaries

In this section we present a brief summary of the definition and results of the soft sets and fuzzy soft sets which are useful for subsequent discussion.

Definition 1:

Suppose U is an initial universe set and E is a set of parameters. Let $P(U)$ denote the power set of U and $A \subset E$. Then a pair (F, A) is called a soft set over U, where F is mapping from A into $P(U)$.

Definition 2:

Suppose U be a universal set, E a set of parameters and $A \subset E$. Also suppose $\square(U)$ denote the set of all fuzzy subsets of U. Then a pair (F, A) is called a fuzzy soft set over U, where F is mapping from A into $\square(U)$.

Definition 3:

For two fuzzy soft sets (F, A) and (G, B) over a common universe U, we have

(i) $(F, A) \tilde{\subset} (G, B)$, if $A \subset B$ and $\forall e \in A,\ F(e)$ is a fuzzy subset of $G(e)$

i.e., (F, A) is a fuzzy soft subset of (G, B).

(ii) (F, A) is said a fuzzy soft superset of (G, B), denoted by $(F, A) \tilde{\supset} (G, B)$ if (G, B) is a fuzzy soft subset of (F, A).

(iii) $(F, A) = (G, B)$, if $(F, A) \tilde{\subset} (G, B)$ and $(G, B) \tilde{\subset} (F, A)$.

Definition 4:

The complement of fuzzy soft set (F, A) denoted by $(F, A)^c$, is defined by $(F, A)^c = (F^c, \neg A)$, where $F^c : \neg A \to \square U$ is a mapping given by $F^c(a) =$ fuzzy complement of $F(\neg a), \forall a \in \neg A)$.

Definition 5:

A fuzzy soft set (F, A) is said to be absolute fuzzy soft set over U, denoted by $\tilde{A}$, if $F(e) = U, \forall e \in A$.

Definition 6:

($AND(\wedge)$ operation of two fuzzy soft sets), $(F, A) \wedge (G, B) = (H, A \times B)$ where $H(\alpha, \beta) = F(\alpha) \tilde{\cap} G(\beta), \forall \alpha \in A$ and $\forall \beta \in B$.

Definition 7:

($OR(\vee)$ operation of two fuzzy soft sets), $(F, A) \vee (G, B) = (O, A \times B)$ where

$O(\alpha,\beta) = F(\alpha) \tilde{\cup} G(\beta), \forall \alpha \in A$ and $\forall \beta \in B$.

Example 1:

Let $U = \{h_1, h_2, h_3\}$ be the set of three houses and $E = \{\cos tly(e_1), cheap(e_2), beautiful(e_3)\}$ be the set of parameters.

Consider two fuzzy soft sets (F, A) and (G, B), where $A = \{e_1, e_2\}$ and $B = \{e_1, e_2, e_3\} \subset E$ given by $(F, A) = \{F(e_1) = \{(h_1,.6),(h_2,.4),(h_3,.3)\}, F(e_2) = \{(h_1,.6),(h_2,.7),(h_3,.5)\}\}$

and $(G, B) =$

$\{G(e_1) = \{(h_1,.6),(h_2,.4),(h_3,.3)\}, G(e_2) = \{(h_1,.6),(h_2,.7),(h_3,.5)\}, G(e_3) = \{(h_1,.2),(h_2,.4),(h_3,.5)\}\}$
then

(i) $(F, A)^c = F(\neg e_1) = \{(h_1,.4),(h_2,.6),(h_3,.7)\}, F(\neg e_2) = \{(h_1,.7),(h_2,.3),(h_3,.5)\}$

(ii) $(F, A) \tilde{\subseteq} (G, B)$

Some propositions

Let (F, A) and (G, B) be two fuzzy soft sets over a common universe U, then

(i) $(F, A) \tilde{\cup} (F, A) = (F, A)$.	(Idempotent)
(ii) $(F, A) \tilde{\cap} (F, A) = (F, A)$.	(Idempotent)
(iii) $(F, A) \tilde{\cup} \phi = \phi$.	(Identity)
(iv) $(F, A) \tilde{\cap} \phi = \phi$.	(Identity)
(v) $(F, A) \tilde{\cup} \tilde{A} = \tilde{A}$. where $\tilde{A}$ is an absolute fuzzy soft set.	(Identity)
(vi) $(F, A) \tilde{\cap} \tilde{A} = \tilde{A}$.	(Identity)
(vii) $((F, A) \tilde{\cup} (G, B))^C = (F, A)^C \tilde{\cup} (G, B)^C$.	(De Morgan's law)
(viii) $((F, A) \tilde{\cap} (G, B))^C = (F, A)^C \tilde{\cap} (G, B)^C$.	(De Morgan's law)
(ix) $((F, A) \vee (G, B))^C = (F, A)^C \wedge (G, B)^C$.	(De Morgan's law)
(x) $((F, A) \wedge (G, B))^C = (F, A)^C \vee (G, B)^C$.	(De Morgan's law)

Arithmetic operations on fuzzy numbers

Definition 1:

A fuzzy set A on the universe of discourse $\mathbb{R}$ (the set of all real numbers) is convex if and only if for a_1, a_2 in U $f_A(\lambda a_1 + (1-\lambda)a_2) \geq \min\{f_A(a_1), f_A(a_2)\}$ where $\lambda \in [0,1]$.

Definition 2:

A fuzzy set A on the universe of discourse U is called a normal fuzzy set if $\exists a_i \in U$ such that $f_A(a_i) = 1$.

Definition 3:

A fuzzy number is a fuzzy set defined on the universe of discourse $\mathbb{R}$ which is both convex and normal.

A fuzzy number A on the universe of discourse $\mathbb{R}$ may be characterized by a triangular distribution function parameterized by a triplet (*a, b, c*). The membership function of the fuzzy number A is defined as

$$f_A(u) = \begin{cases} 0, & u < a \\ \dfrac{u-a}{b-a}, & a \leq u \leq b \\ \dfrac{c-u}{c-b}, & b \leq u \leq c \\ 0, & u > c \end{cases}$$

Let A and B be two triangular fuzzy numbers parameterized by the triplet $b_1 = (a_1, b_1, c_1) \quad and \quad b_2 = (a_2, b_2, c_2)$ respectively.

Then addition and multiplication of A and B as given in [4] are

$$\begin{aligned} A \oplus B &= \tilde{a}_2 \oplus \tilde{b}_2 \\ &= (a_1, a_2, a_3) \oplus (b_1, b_2, b_3) \\ &= (a_1 + b_1, a_2 + b_2, a_3 + b_3) \end{aligned}$$

and

$$\begin{aligned} A \otimes B &= \tilde{a}_2 \otimes \tilde{b}_2 \\ &= (a_1, a_2, a_3) \otimes (b_1, b_2, b_3) \\ &= (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3) \end{aligned}$$

For example, if $A = (7,8,9) \quad and \quad B = (2,3,4)$ are two triangular fuzzy numbers, then

$$A \oplus B = (7,8,9) \oplus (2,3,4) = (9,11,13) = \tilde{11}$$

$$A \otimes B = (7,8,9) \otimes (2,3,4) = (14,24,36) = \tilde{24}.$$

Similarly the addition and multiplication of two trapezoidal fuzzy numbers $A = (a_1, a_2, a_3, a_4)$ *and* $B = (b_1, b_2, b_3, b_4)$ are given as

$$A \oplus B = (a_1, a_2, a_3, a_4) \oplus (b_1, b_2, b_3, b_4) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

$$A \otimes B = (a_1, a_2, a_3, a_4) \otimes (b_1, b_2, b_3, b_4) = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4)$$

Next we present a defuzzification method of trapezoidal fuzzy number. Take a trapezoidal fuzzy number parameterized by a quadruplet (p, q, r, s) as shown in the figure 1. Then the defuzzification value *t* of the fuzzy number is calculated from the figure as follows:

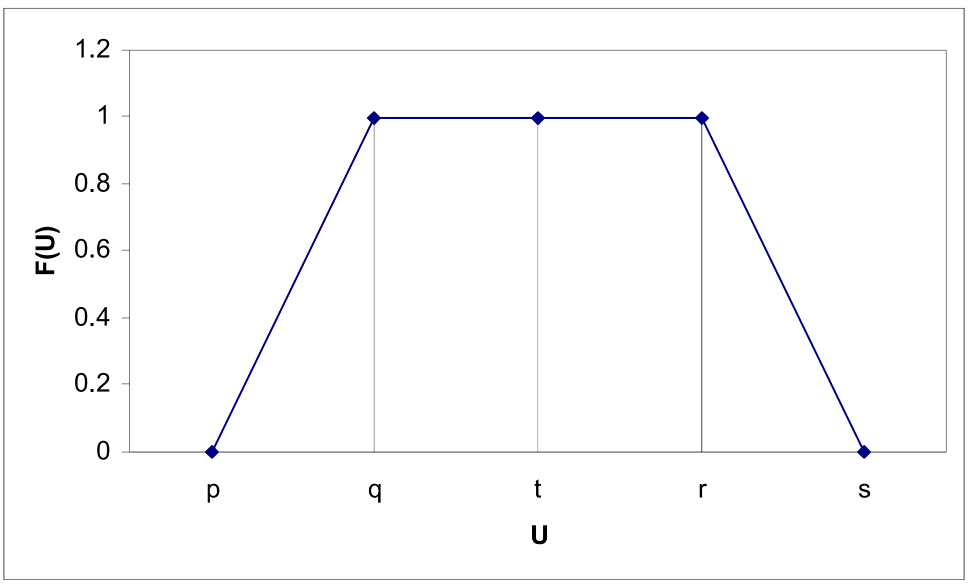

Figure 1: A trapezoidal fuzzy number M = (p, q, r, s)

$$(t-q)(1)+\frac{1}{2}(q-p)(1)=(r-t)(1)+\frac{1}{2}(s-r)(1)$$

$$\Rightarrow (t-q)+\frac{1}{2}(q-p)=(r-t)+\frac{1}{2}(s-r)$$

$$\Rightarrow (t-q)-(r-t)=\frac{1}{2}(s-r)-\frac{1}{2}(q-p)$$

$$\Rightarrow 2t=\frac{s-r-q+p}{2}+q+r$$

$$\Rightarrow 2t=\frac{p+q+r+s}{2}$$

$$\Rightarrow t=\frac{p+q+r+s}{4}$$

Similarly the defuzzification value *e* of a triangular fuzzy number (*a*, *b, c*) is equal to

$$e=\frac{a+b+b+c}{4}\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

METHODOLOGY AND ALGORITHM

In this section we present an algorithm for medical diagnosis using fuzzy arithmetic operations. Assume that there is a set of *m* patients, $P=\{p_1, p_{2,}\ p_3, \ldots\ldots., p_m\}$ with a set of *n* symptoms $S=\{s_1, s_2, s_3, \ldots.., s_n\}$ related to a set of *k* diseases $D=\{d_1, d_2, d_3, \ldots.., d_k\}$. We apply fuzzy soft set technology to develop a technique through Sanchez's method to diagnose which patient is suffering from what disease.

For this, construct a fuzzy soft set (*F, P)* over *S* where *F* is a mapping $F: P \to \square S$. This fuzzy soft set gives a relation matrix *Q*, called patient-symptom matrix, where the entries are fuzzy numbers $\tilde{p}$ parameterized by a triplet (*p*-1*, p, p*+1).Then construct another fuzzy soft set (*G, S*) over *D*, where *G* is a mapping $G: S \to \square D$. This fuzzy soft set gives a relation matrix (weighted matrix), say *R*, called symptom-disease matrix, where each element denote the weight of the symptoms for a certain disease. These elements are also taken as triangular fuzzy numbers.

Thus the general form of Q and R are

$$Q = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_m \end{array} \begin{array}{c} \begin{array}{cccccc} s_1 & s_2 & s_3 & \cdots & s_n \end{array} \\ \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} & \cdots & \tilde{a}_{2n} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} & \cdots & \tilde{a}_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \tilde{a}_{m3} & \cdots & \tilde{a}_{mn} \end{bmatrix} \end{array} \text{ and } R = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ \vdots \\ s_n \end{array} \begin{array}{c} \begin{array}{cccccc} d_1 & d_2 & d_3 & \cdots & d_k \end{array} \\ \begin{bmatrix} \tilde{b}_{11} & \tilde{b}_{12} & \tilde{b}_{13} & \cdots & \tilde{b}_{1k} \\ \tilde{b}_{21} & \tilde{b}_{22} & \tilde{b}_{23} & \cdots & \tilde{b}_{2k} \\ \tilde{b}_{31} & \tilde{b}_{32} & \tilde{b}_{33} & \cdots & \tilde{b}_{3k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{b}_{n1} & \tilde{b}_{n2} & \tilde{b}_{n3} & \cdots & \tilde{b}_{nk} \end{bmatrix} \end{array}$$

Now performing the transformation operation $Q \otimes R$ we get the patient-diagnosis matrix D^* as follows:

$$D^* = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_m \end{array} \begin{array}{c} \begin{array}{cccccc} d_1 & d_2 & d_3 & \cdots & d_k \end{array} \\ \begin{bmatrix} \tilde{c}_{11} & \tilde{c}_{12} & \tilde{c}_{13} & \cdots & \tilde{c}_{1k} \\ \tilde{c}_{21} & \tilde{c}_{22} & \tilde{c}_{23} & \cdots & \tilde{c}_{2k} \\ \tilde{c}_{31} & \tilde{c}_{32} & \tilde{c}_{33} & \cdots & \tilde{c}_{3k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{c}_{m1} & \tilde{c}_{m2} & \tilde{c}_{m3} & \cdots & \tilde{c}_{mk} \end{bmatrix} \end{array}, \text{ where } c_{il} = \sum_{j=1}^{n} a_{ij} b_{jl}.$$

Then defuzzifying each element of the above matrix by (1) we get the crisp diagnosis matrix as

$$D^{**} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_m \end{array} \begin{array}{c} \begin{array}{cccccc} d_1 & d_2 & d_3 & \cdots & d_k \end{array} \\ \begin{bmatrix} v_{11} & v_{12} & v_{13} & \cdots & v_{1k} \\ v_{21} & v_{22} & v_{23} & \cdots & v_{2k} \\ v_{31} & v_{32} & v_{33} & \cdots & v_{3k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ v_{m1} & v_{m2} & v_{m3} & \cdots & v_{mk} \end{bmatrix} \end{array}$$

Now if $\max_{1\le l\le k} v_{il} = v_{is}$ then we conclude that the patient p_i is suffering from disease d_s. In case $\max_{1\le l\le k} v_{il}$ occurs for more than one value of *l*, $1 \le l \le m$ then we can reassess the symptoms to break the tie.

Algorithm

Step I: Input the soft set (*F, P)* to obtain the patient-symptom matrix *Q*.

Step II: Input the soft set (*G, S)* to obtain the symptom-disease matrix *R*.

Step III: Perform the transformation operation $Q \otimes R$ to get the patient diagnosis matrix D^*.

Step IV: Defuzzify all the elements of the matrix D^* by (1) to obtain the matrix D^{**}.

Step V: Find s for which $v_{is} = \max_{l} v_{il}$.

Then we conclude that the patient p_i is suffering from disease d_s. In case $\max_{1\le l\le k} v_{il}$ occurs for more than one value of *l*, then we can reassess the symptoms to break the tie.

Case Study

Suppose there are three patients John, George and Albert in a hospital with symptoms temperature, headache, cough and stomach problem. Let the possible diseases relating to the above symptoms be viral fever, typhoid and malaria. Now take $P = \{p_1, p_2 p_3\}$ as the universal set where p_1, p_2 and p_3 represents patients John, George and Albert respectively. Next consider the set $S = \{s_1, s_2, s_3, s_4\}$ as universal set where s_1, s_2, s_3, s_4 represents symptoms temperature, headache, cough and stomach problem respectively and the set $D = \{d_1, d_2, d_3\}$, where d_1, d_2 and d_3 represent the diseases viral fever, typhoid and malaria respectively.

Suppose,

$F(s_1) = \{p_1 / \widetilde{8}, p_2 / \widetilde{7}, p_3 / \widetilde{4}\}$, $F(s_2) = \{p_1 / \widetilde{4}, p_2 / \widetilde{3}, p_3 / \widetilde{5}\}, F(s_3) = \{p_1 / \widetilde{6}, p_2 / \widetilde{4}.p_3 / \widetilde{4}\}$ and $F(s_4) = \{p_1 / \widetilde{3}.p_2 / \widetilde{6}, p_3 / \widetilde{7}\}$. Then the fuzzy soft set (F, S) is a parameterized family of all fuzzy sets over *S* and gives a collection of approximate description of the patient-symptoms in the hospital. This fuzzy soft set (F, P) represents the relation matrix (patient-symptom matrix) *Q* and is given by

$$Q = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cccc} s_1 & s_2 & s_3 & s_4 \end{array} \\ \begin{bmatrix} \tilde{8} & \tilde{4} & \tilde{6} & \tilde{3} \\ \tilde{7} & \tilde{3} & \tilde{4} & \tilde{6} \\ \tilde{4} & \tilde{5} & \tilde{4} & \tilde{7} \end{bmatrix} \end{array}$$

Next suppose that $G(s_1) = \{d_1/\tilde{9}, d_2/\tilde{6}, d_3/\tilde{1}\}$, $G(s_2) = \{d_1/\tilde{4}, d_2/\tilde{5}, d_3/\tilde{6}\}$, $G(s_3) = \{d_1/\tilde{5}, d_2/\tilde{2}, d_3/\tilde{5}\}$, $G(s_4) = \{p_1/\tilde{2}, d_2/\tilde{8}, d_3/\tilde{8}\}$ so that the fuzzy soft set (G, S) is a parameterized family $\{G(s_1), G(s_2), G(s_3), G(s_4)\}$ of all fuzzy sets over the set S where $G : S \rightarrow \square D$ and is determined from expert medical documentation. Thus the fuzzy soft set (G, S) gives an approximate description of the three diseases and their symptoms. This soft set is represented by a relation matrix (symptom-disease matrix) R and is given by

$$R = \begin{array}{c} \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{bmatrix} \tilde{9} & \tilde{6} & \tilde{1} \\ \tilde{4} & \tilde{5} & \tilde{6} \\ \tilde{5} & \tilde{2} & \tilde{5} \\ \tilde{2} & \tilde{8} & \tilde{8} \end{bmatrix} \end{array}$$

Then performing the transformation operation $Q \otimes R$ we get the patient- diagnosis matrix D^* as

$$D^* = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{bmatrix} \tilde{124} & \tilde{104} & \tilde{86} \\ \tilde{107} & \tilde{113} & \tilde{93} \\ \tilde{90} & \tilde{113} & \tilde{110} \end{bmatrix} \end{array}$$

Where, $\tilde{124} = (87,124,164)$, $\tilde{107} = (71,107,151)$, $\tilde{90} = (54,90,134)$

$\tilde{104} = (66,104,150)$, $\tilde{113} = (76,113,158)$

Now defuzzifying the above matrix, we get

$$D^{**} = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{bmatrix} 124.75 & 106 & 88 \\ 109 & 115 & 95 \\ 92 & 115 & 112 \end{bmatrix} \end{array}$$

It is clear from the above matrix that patient p_1 is suffering from disease d_1 and patients p_2 and p_3 both are suffering from disease d_2.

REFERENCES

1. **Molodtsov D. 1999.** Soft set theory-first result, *Computers and Mathematics with Applications*, 37: pp 19-31
2. **Maji P. K., Biswas R. and Roy A. R. 2001.** Fuzzy Soft Set, *The Journal of Fuzzy Mathematics,* 9(3): pp 677-692.
3. **Sanchez E. 1979.** Inverse of fuzzy relations, Application to possibility distributions and medical diagnosis, *Fuzzy Sets and Systems*, 2(1): pp 75-86.
4. **Kaufmann A. and Gupta M. M. 1991.** *Introduction to Fuzzy Arithmetic Theory and Applications.* Van Nostrand Reinhold, New York.

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