Multi-Tone Transmissions over Two-User Cognitive Radio Channel with Weak Interference

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Abstract—A transmitter with cognitive capability can sense talk between the other transmitter-receiver pairs. When this transmitter knows full or partial message of the others, it can choose an efficient strategy to access the transmission medium. This is referred to as cognitive radio channel. This work aims to investigate multi-tone transmission over two-user cognitive radio channels where cross-talk interference is weak. Cognitive transmitter (Tx1) is assumed to have full knowledge of message that is sent by the other transmitter (Tx2) to its corresponding receiver (Rx2). Channel capacity is carefully analyzed for frequency-selective scenarios. Efficient power-allocation strategies at Tx1 are investigated for various wireless environments. It is shown that Tx1 can find an efficient resource-accessing strategy if the channel gain of Tx1-Rx1 (the corresponding receiver link) is larger than the channel gain of Tx2-Rx2 link. In this case, the cognitive transmitter Tx1 can offer better performance by employing equal power allocation approach. Otherwise, it is not worthy for Tx1 to access transmission medium of Tx2-Rx2 link.

I. INTRODUCTION

Cross-talk channel is one of fundamental problems in network information theory. Talk between one transmitter and its corresponding receiver interferes with the other transmitter-receiver pairs. This is referred to as interference channel that is often an user competition environment [1]-[2]. On the other hand, each transmitter sends its message in broadcasting fashion. Therefore, a transmitter with cognitive capability can sense message sent by other transmitters. When a cognitive transmitter has full (or more practically partial) knowledge of the other transmitters, it can establish an efficient strategy to access the transmission medium. This is referred to as cognitive radio channel whose achievable rate has been analyzed in [3]. Capacity theorem of cognitive radio channel has been carefully investigated in literatures (e.g., [4]-[5]).

High-data-rate communication systems often operate in frequency-selective environments. Multi-tone transmissions such as orthogonal frequency-division multiplexing (OFDM) turn out to be promising techniques to overcome channel frequency selectivity [6]. Therefore, investigation of multi-tone transmissions over cognitive radio channel has wide implications. Key issues this work seeks to address are:

1) : To analyze capacity of two-user cognitive radio channel in frequency-selective scenarios. In this work, capacity analysis is carried out only for the case with weak interference. This is because interference is usually weak in wireless networks. The case with strong interference is the next step of our work.

2) : To develop power allocation strategies for the cognitive transmitter. For this issue, two key questions need clear answers.

Q1. For what conditions, the cognitive transmitter should access the transmission medium that is being used by the other transmitter-receiver pair.

Q2. If the cognitive transmitter decides to access the transmission medium, how to perform efficient power-allocation in various wireless environments?

Results presented in this work are based on two assumptions:

A1. The cognitive transmitter has full knowledge of message communicated between the other transmitter-receiver pair.

A2. The cognitive transmitter perfectly knows channel gains for all links.

These two assumptions are of course not practical. However, it is reasonable for carrying out theoretical investigation to find the fundamental performance limit in information theory viewpoint. As a next step, these assumptions will be extended to more practical scenarios such as consideration of partial knowledge of the other transmitter-receiver pair and imperfect channel knowledge. Numerical results (based on theoretical channel model ¹) show that the cognitive transmitter can find an efficient resource-accessing strategy if the channel between itself and the corresponding receiver is better than that between the other transmitter-receiver pair. In this case, the cognitive transmitter Tx1 can offer better performance by employing equal power allocation (EPA) approach.

II. TWO-USER MULTI-TONE COGNITIVE RADIO CHANNEL WITH WEAK INTERFERENCE

Consider a two-user multi-tone cognitive radio channel (as depicted in Fig.1) where information is transmitted over \(N\) parallel flat sub-channels. Specifically, Tx1 sends information \(X_{1,n}\) to Rx1 with power \(P_{1,n}\) at rate \(R_{1,n}\), while Tx2 sends information \(X_{2,n}\) with power \(P_{2,n}\) at rate \(R_{2,n}\) (the subscript \(n\) denotes sub-channel index). As depicted in Fig.1, message \(W_{1,n}\) and \(W_{2,n}\) are both known at Decoder 1 (Tx1). A splitting

¹This is often used for numerical investigation in information theory, e.g., [7]-[8].
code is employed and has two encoding functions
\[ X_{1,n} = f_1(W_{1,n}, W_{2,n}), \]
\[ X_{2,n} = f_2(W_{2,n}). \]

At nth sub-channel, Tx1 devotes \((1 - \alpha_n)\) fraction of its power \(P_{1,n}\) to the transmission of \(W_{2,n}\), while Tx2 devotes its entire power \(P_{2,n}\) to this effort. The receive signals at receivers are expressible as
\[ Y_{1,n} = a_{11,n}X_{1,n} + a_{21,n}X_{2,n} + V_{1,n}, \]
\[ Y_{2,n} = a_{12,n}X_{1,n} + a_{22,n}X_{2,n} + V_{2,n}, \]
where \(a_{ij,n}\) denotes the channel gain for the \(n\)-th sub-channel between the \(i\)-th transmitter and the \(j\)-th receiver, and \(V_{i,n}\) for Gaussian noise with zero mean and variance \(N_o\). The interference considered here is weak interference that is defined in [8]
\[ I(X_{2,n}; Y_{1,n} | X_{1,n}) < I(X_{2,n}; Y_{2,n} | X_{1,n}), \]
\[ I(X_{1,n}; Y_{2,n} | X_{2,n}) < I(X_{1,n}; Y_{1,n} | X_{2,n}), \]
where \(I(;;)\) denotes the mutual information between two random variables. By defining a coefficient \(H_{ij,n} = (a_{ij,n}^2)/(N_o)\), capacity of the system model (3) is given by Wu in [9]
\[ R_{1,n} \leq C \left( \frac{\alpha_n P_{1,n} H_{11,n}}{N_o} \right), \]
\[ R_{2,n} \leq C \left( \frac{(1 - \alpha_n)P_{1,n} H_{12,n} + P_{2,n} H_{22,n}}{N_o + \alpha_n P_{1,n} H_{12,n}} \right)^2, \]

where \(C(x) = \frac{1}{2} \log_2(1 + x)\). Based on the above system description, we will analyze the capacity and power allocation for this two-user multi-tone cognitive radio channel in frequency-selective scenarios in Section III.

III. POWER ALLOCATION FOR TWO-USER MULTI-TONE COGNITIVE RADIO CHANNEL

It is well recognized that, for single-link parallel transmissions, channel capacity can be achieved by employing optimum power allocation. This motivates us to first analyze the capacity of two-user cognitive radio channel in frequency-selective scenarios, and then investigate power allocation schemes for the cognitive radio channel.

A. Capacity of Two-user Cognitive Radio Channel in Frequency-Selective Scenario

The results (6) and (7) show that the capacity of cognitive radio channel depends on the parameter \(\alpha_n\). However, no result has been reported so far on an appropriate setup of \(\alpha_n\). First of all, the cognitive transmitter Tx1 should not cause rate penalty for Tx2-Rx2 link. Hence, the upper bound in (7) equals to
\[ C((P_{2,n} H_{22,n})/(N_o)). \]
Resolving this equation leads to the result \(0 < \alpha_n < \xi_{1,n}\) with
\[ \xi_{1,n} = \frac{1}{P_{2,n} H_{22,n}} \left( \sqrt{P_{1,n} H_{12,n} P_{2,n} H_{22,n} + N_o} + N_o^2 - N_o \right)^2 \]
\[ \frac{1}{P_{1,n} H_{12,n} (P_{2,n} H_{22,n} + N_o)^2}. \]

Secondly, we need to maximize the transmission rate for Tx1-Rx1 link. Considering the interference (due to Tx2) as noise, capacity of Tx1-Rx1 link for the \(n\)-th sub-channel is
\[ C_{1,n} |_{\alpha_n = \xi_{1,n}} \]
\[ = C \left( \frac{\xi_{1,n} P_{1,n} H_{11,n}}{N_o + \left( (1 - \xi_{1,n})P_{1,n} H_{11,n} + \sqrt{P_{2,n} H_{21,n}} \right)^2} \right). \]

Considering the interference (due to Tx2) as message, this interference is removable at Rx1 for the condition
\[ I(X_{2,n}; Y_{1,n} | X_{1,n}) \geq I(X_{2,n}; Y_{2,n} | X_{1,n}), \]
This inequality leads to the result \(0 < \alpha_n < \xi_{2,n}\) with
\[ \xi_{2,n} = 1 - \frac{(\sqrt{P_{2,n} H_{22,n}} - \sqrt{P_{2,n} H_{21,n}})^2}{P_{1,n} H_{11,n}}. \]

In this case, capacity of Tx1-Rx1 link for the \(n\)-th sub-channel is given by
\[ C_{1,n} |_{\alpha_n = \xi_{2,n}} \]
\[ = C \left( \frac{\xi_{2,n} P_{1,n} H_{11,n} + (1 - \xi_{2,n})P_{1,n} H_{11,n} + \sqrt{P_{2,n} H_{21,n}}^2}{N_o} \right). \]

If \(\xi_{2,n} \geq \xi_{1,n}\), then (9) is the maximum transmission rate of Tx1 with \(\alpha_n = \xi_{1,n}\). Otherwise, the maximum transmission rate is attained at such a \(\alpha_n\), i.e.,
\[ \hat{\alpha}_n = \arg \max_{\alpha_n} \left( C_{1,n} |_{\alpha_n = \xi_{1,n}}, C_{1,n} |_{\alpha_n = \xi_{2,n}} \right). \]

B. Power Allocation Schemes

Power allocation scheme for two-user multi-tone cognitive radio channel could be separately solved between each user. The power allocation scheme for non-cognitive transmitter Tx2 can be done independently of the cognitive transmitter Tx1. Here, we consider two power-allocation schemes for Tx1, i.e., water-filling (WF) approach and EPA approach.
1) WF approach: The objective of WF approach for two-user cognitive radio channel is to maximize the rate for the cognitive transmitter $Tx_1$ subject to total power constraint $P_1$ and secure there is no rate penalty caused at $Tx_2$. This can be mathematically modeled as

$$
\min \left( \sum_{n=1}^{N} (-R_{1,n} + \lambda_1 \left( \sum_{n=1}^{N} P_{1,n} - P_1 \right)) \right),
$$

where $\lambda_1$ stands for Lagrange multiply and

$$
\sum_{n=1}^{N} P_{1,n} = P_1.
$$

It can be found that both the capacity region (6)-(7) and the power constraint are convex [10]. Moreover, a strictly feasible point always exists, i.e., there exists $P_{1,n}$ and $P_{1,n}(n = 1, \ldots, N)$ such that (6)-(7) maintains strict inequalities. Hence, Slater’s condition is fulfilled, and the strong duality holds [10]. In this case, the above dual problem can be separately considered for each sub-channel. In other words, the problem (14) that contains $2N$ variables can be decomposed into $N$ convex problem (2 variables) that can be processed in parallel. Then, the dual problem (14) can be simplified into the following optimization problem

$$
\min_{n} (-R_{1,n} + \lambda_1 P_{1,n}).
$$

Once $\lambda_n$ has been determined, the extreme value of (16) can be calculated. This WF approach for two-user cognitive radio channel with weak interference channel can be implemented as followings:

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Initialization:
For all $n$, let $P_{1,n} = 0$;
$P_{2,n}$ has been given according to $Tx_2$ power allocation scheme;

Loading cognitive transmitter
$\Delta p_1 = P_1/Q$;
Let $V_n = 0$ (power allocation indicator);
$c_{1,n} = 0$;
Repeat the following $Q$ times;
Let $c_{1,n|\alpha = \xi_{1,n}} = 0$ and $c_{1,n|\alpha = \xi_{2,n}} = 0$;
Apply $\Delta p_{1,n}$ and $P_{2,n}$ into (8) and (11) to obtain $\xi_{1,n}$ and $\xi_{2,n}$;
If $\xi_{2,n} \geq \xi_{1,n}$, apply $\Delta p_1$ and $P_{2,n}$ into (9) to obtain $c_{\text{temp}}^{(1)}$
Otherwise, $c_{\text{temp}}^{(1)} = \max\left( c_{1,n|\alpha = \xi_{1,n}}, c_{1,n|\alpha = \xi_{2,n}} \right)$;
Let $\hat{n} = \arg\max_{n} \left( c_{1,n}^{(\text{temp})} \right)$;
$V_{\hat{n}} = V_{\hat{n}} + 1$;
End;
$P_{1,n} = V_{\hat{n}} \Delta p_1$;

2) EPA approach: For EPA, $P_{1,n}$ is given as below

$$
P_{1,n} = \frac{P_1}{N},
$$

where $N$ is the total number of sub-channels.

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IV. Numerical Results

Numerical results were used to visually illustrate the performance of proposed power allocation approach in terms of physical-layer throughput (bits/Hz/Sec). To protect the weak interference assumption, we consider two theoretical frequency-selective cross-talk channel models as depicted in Fig. 2-(a) and Fig. 6-(a), respectively. In Fig. 2-(a) (Channel A) and Fig. 6-(a) (Channel B), the channel gain of $Tx_1$-$Rx_1$ and $Tx_2$-$Rx_2$ links (i.e., $a_{11}, a_{22}$) is always larger than the channel gain of $Tx_1$-$Rx_2$ and $Tx_2$-$Rx_1$ links (i.e., $a_{12}, a_{21}$). In fact this kind of theoretical model of frequency-selective channel is widely used in information theory (e.g.,[7]). Further in Fig. 2-(a) (Channel A), the channel gain of $Tx_1$-$Rx_1$ link (i.e., $a_{11}$) is always larger than the channel gain of $Tx_2$-$Rx_2$ link (i.e., $a_{22}$). In Fig. 6-(a) (Channel B), the channel gain of $Tx_1$-$Rx_1$ link (i.e., $a_{11}$) is always smaller than the channel gain of $Tx_2$-$Rx_2$ link (i.e., $a_{22}$). In numerical analysis, we let the power constraint for both user to be identical. The SNR was defined as transmit power to noise ratio. The baseline for comparison is the throughput performance in the scenario that the same transmit power has been cost at $Tx_1$ with non-cognitive capability.

Fig. 2-(b) illustrated the throughput performance for Channel A in three different scenarios. The solid lines denote performance for both transmitters employing WF approach. It is shown that there is no rate penalty for $Tx_2$-$Rx_2$ link, but $Tx_1$-$Rx_1$ offers slightly better performance than baseline. The dash lines denote performance for both user using EPA approach. It is observed that although $Tx_2$-$Rx_2$ link suffers small rate penalty, the performance of $Tx_1$-$Rx_1$ link has been significantly improved with increasing SNR. The dash-dot lines denote the $Tx_1$ employed the WF approach, but the $Tx_2$ employed the EPA approach. It is shown that throughput of $Tx_1$-$Rx_1$ offers similar performance when WF approach are employed at both $Tx_1$ and $Tx_2$. These results indicate that if the channel gain of $Tx_1$-$Rx_1$ link is better than the channel gain of $Tx_2$-$Rx_2$ link, the throughput can be improved by employing cognitive transmitter. Also, when EPA approach are employed at both $Tx_1$ and $Tx_2$, the throughput can be significantly improved.

To explain this phenomena, we also plot in Fig.3, Fig.4 and Fig.5 the power and rate allocation at transmitters for each sub-channel. The SNR was chose as 12 dB. In Fig.3-(a), the square mark denotes the power for $Tx_2$. The round mark denotes the power of $Tx_1$. The star mark denotes the power that $Tx_1$ devoted to its own message. In Fig.3-(b), the round mark denotes the rate of $Tx_1$, square mark the rate of $Tx_2$. When WF approach was employed at both transmitters, it is shown in Fig.3-(a), $Tx_1$ cannot load large power to the sub-channels that have the large channel gain. This helps to avoid the rate penalty for $Tx_2$-$Rx_2$ link, and secure the interference can be recovered at the $Rx_1$. In Fig.4-(a), as EPA approach has been employed at both transmitters, $Tx_1$ could occupy the sub-channel with large channel gain. The power $Tx_1$ devoted to its own message $W_1$, i.e., $\alpha P_1$, was increased. Hence, in
Fig. 4-(b), the rate of Tx1 has been significantly increased. In Fig. 5-(a), WF approach was employed at Tx1, and EPA approach at Tx2. Again Tx1 could load large power to the sub-channel with larger channel gain. Therefore in Fig. 5-(b), the performance is similar as when WF approach are employed at both transmitters.

The throughput performance over Channel B was plotted in Fig. 6-(b). It is observed that performance of Tx1-Rx1 link is not outperform the baseline. EPA approach employed at Tx1 still offers the best performance, when it is also employed at Tx2. To explain this phenomena, we plot the power and rate allocation for both transmitter at each sub-channel when SNR was chosen as 12 dB in Fig. 7, Fig. 8, and Fig. 9. It is observed in Fig. 7-(a), large power has been allocated at Tx2 for the sub-channels have better gain. To avoid rate penalty, Tx1 has to reduce the rate particularly for those sub-channels. In Fig. 8-(a), as power was equally allocated for both transmitter, the rate of Tx1 has been slightly increased, shown in Fig. 8-(b). But the power that Tx1 devoted to its own message $W_1$, i.e., $a_1P_1$, is less to compare with it in Fig. 4-(a). In Fig. 9-(a), EPA approach was employed at Tx2, and WF approach was employed at Tx1. It offers very similar performance when WF allocation approach are employed at both transmitters, as shown in Fig. 7-(a).

V. CONCLUSION

In this paper, we have investigated multi-tone transmission over two-user cognitive radio channel. Assuming the cognitive transmitter had full knowledge of messages communicated between the other transmitter-receiver pair, channel capacity has been carefully analyzed for channel frequency-selective scenarios. Moreover, we have investigated efficient power-allocation schemes for the cognitive transmitter. It has been found that the cognitive transmitter could have an efficient resource-accessing strategy under a certain channel condition. The EPA approach outperformed the WF approach when the cognitive transmitter was allowed to access the transmission medium.

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REFERENCES

Fig. 4: (a) Power allocation Vs. frequency tones for each user, SNR=12 dB; (b) Rate allocation Vs. frequency tones for each user, SNR=12 dB;

Fig. 5: (a) Power allocation Vs. frequency tones for each user, SNR=12 dB; (b) Rate allocation Vs. frequency tones for each user, SNR=12 dB;

Fig. 6: (a) Two-user weak interference frequency-selective channel B; (b) Throughput performance Vs Transmit SNR for each user (Channel B)

Fig. 7: (a) Power allocation Vs. frequency tones for each user, SNR=12 dB; (b) Rate allocation Vs. frequency tones for each user, SNR=12 dB;

Fig. 8: (a) Power allocation Vs. frequency tones for each user, SNR=12 dB; (b) Rate allocation Vs. frequency tones for each user, SNR=12 dB;

Fig. 9: (a) Power allocation Vs. frequency tones for each user, SNR=12 dB; (b) Rate allocation Vs. frequency tones for each user, SNR=12 dB;