ANALYSIS OF AN INDUSTRIAL COMPONENT COMMONALITY PROBLEM

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Abstract

We discuss a case study of an industrial production-marketing coordination problem involving component commonality. For the product line considered, the strategic goal of the company is to move from the current low volume market to a high volume market. The marketing department believes that this can be achieved by substantially lowering the end products’ prices. However, this requires a product redesign to lower production costs in order to maintain profit margins. The redesign decision involves grouping end products into families. All products within one family use the same version of some components. This paper fits in the stream of recent literature on component commonality where the focus has shifted from inventory cost savings to production and development cost savings. Further, we consider both costs and revenues, leading to a profit maximization approach. The price elasticity of demand determines the relationship between the price level and number of units sold. Consequently, we integrate information from different functional areas such as production, marketing and accounting. We formulate the problem as a Net-Present-Value investment decision. We propose a mixed integer nonlinear optimization.
model to find the optimal commonality decision. The recommendation based on our analysis has been implemented in the company. In addition, the application allows us to experimentally validate some claims made in the literature and obtain managerial insights into the trade-offs.

1. Introduction

We study a real life component commonality problem for a large international manufacturing company, a major player in the industrial power tools industry. This study relates to one specific machine type and product line, called the T-range. The machines range from small units for domestic use to very large models for industrial applications. The kilowatt (kW) size is the main indicator of the power and size of the machine. The company launched a project to develop a completely new and cheaper design for the T-range. The goal of the project was to boost sales volumes and profits of the product line while at the same time increasing the quality and benefits for the customer. This can be achieved by lowering market prices which requires a decrease of production costs to maintain profit margin. The redesign will lead to the required reduction in total manufacturing costs.

Currently, this T-range contains 8 models, which we label T1 to T8 with increasing kW size. The T-range is split up into families, which are defined as a group of end products sharing the same component. There are three families: (1) T1 to T4, (2) T5 and T6, and (3) T7 and T8. The simplified Bill of Material (BOM) for the current range is given in Figure 1. The frame (F) is the housing and all mechanical and electrical appliances that form a machine. The stage (S) is an essential component for the product. The frame and stage are common components for each family and are always dimensioned to fit the requirements
of the machine with the highest performance in the family. For example, all the models in
the first family use the same stage (S4) and frame (F4) which are specifically dimensioned
for T4, the highest kW machine in that family. The engine (E) is specific for each model.
The redesign project includes a review of the actual range split of the families and a
proposal for a new split. The component commonality decision must therefore determine
which end products will use the same stage and frame in the redesigned product line.

![Diagram of product line](image)

**Figure 1.** Simplified BOM for the current split.

The main elements and relationships of this problem are visualized in Figure 2. The details
will be explained in Section 2. The key cost factors that drive the commonality decision
are the unit production costs (a) and the development costs (b). The component
commonality decision has opposite effects on these two cost factors, resulting in a trade-off. It is the company’s practice to determine the price (d) by adding a fixed mark-up (c) to
the costs (a). However, cost minimization alone is not relevant here. The number of units
sold (f) is not fixed, but depends on the price (d) through the price elasticity (e). Taking
both costs and revenues into account leads to a profit maximization approach and
therefore the design and pricing issues must be considered simultaneously. We formulate
this problem as an investment decision. The optimization of the component commonality
decision weighs the investment costs, i.e. the development costs for the common
components (b), against the future extra profit (g) to calculate the Net Present Value (h) of the project.

\[
\begin{array}{ccc}
\text{Unit Production Costs (a)} & \text{Development Costs (b)} \\
\text{Fixed Mark Up (c)} & \text{Extra Profit (g)} \\
\text{Sales Price (d)} & \text{Price Elasticity (e)} \\
\text{Units Sold (f)} & \text{MAX NPV (h)}
\end{array}
\]

**Figure 2.** Key elements and relationships in the product range split problem

Our paper has several contributions. First, we present an industrial component commonality problem with several new aspects. Our real life case supports the recent shift in focus in the literature on component commonality from the savings in inventory costs to the impact on production and development costs, as these costs have a much bigger impact. Furthermore, cost and revenue sides are integrated, leading to a profit maximization approach. The price sensitivity provides the link between the price level and sales. This component commonality problem requires a cross-functional approach, integrating marketing, production and accounting information within an investment analysis framework. Further, our industrial problem considers two different components simultaneously. Second, we develop a mathematical programming model for this problem. This formulation is flexible, allowing us to model a variety of industrial constraints. In an extension of our model, we explore the problem of product line pricing by treating the selling price as a decision variable. Third, our industrial case allows us to experimentally validate earlier claims made in the literature and we also perform a series of experiments to obtain general insights into the problem.
The paper is organized as follows. In section 2 we give a detailed analysis of the case problem and relate it to the relevant literature. All the factors mentioned in Figure 2 will be discussed in detail here. Section 3 describes the general mathematical formulation. In section 4 we extend the model with additional industrial constraints. Finally, results and managerial insights for our specific case are discussed in section 5 and 6.

2. Problem Analysis and Literature Review

2.1. Cross-Functional Analysis

The design of families and common components is not a stand-alone decision. It affects the entire value chain of the company. Eynan and Rosenblatt (1996) remark: “The decision on whether to employ component commonality is usually made at the design stage. In general, the component commonality decision affects, and is part of, other major functions of the business such as purchasing, production and marketing.” In order to make a good decision, information from different departments is needed. Recently, some papers (Krishnan et al. 1999, Ramdas and Sawhney 2001, Desai et al. 2001) introduced marketing aspects in component commonality problems. In our case, the production decision concerns the redesign and range split and the marketing department must provide price sensitivity information and determine the price level. Further, data on costs and the cost of capital are estimated in close cooperation with the accounting department.

2.2. Cost Drivers: Development and Production Costs

The two main costs drivers in our problem are the development costs and the production costs. The component commonality decision has an opposite effect on these two costs, so
we need to find the right trade-off. Development costs for the new frames and stages include the costs for the new design, engineering, production of a prototype and testing. This development will take approximately 3 years. An estimate of the development costs of the frames and stages specifically dimensioned for each of the new T models was given by the engineering department and is increasing with the kW size. Because models within one family make use of the same frame and stage, only the frame and stage for the highest kW size in each family needs to be developed. The total development cost for the complete range is therefore the sum of the development costs for the common component in each family. The unit production cost consists of four different components: the cost of the stage, the cost of the frame, the cost of the engine and the other costs, including the wages, logistics and overhead. The cost of the engine is model dependent because each T-model has a different engine. The other costs are family dependent, as each model within the same family uses the same stage and frame. The four cost elements have different weights in the total unit production cost: stage (30%), frame (45%), engine (10%) and other costs (15%). These percentages are approximate averages over all the models. Because the family dependent costs constitute the largest part of the total cost (i.e. approximately 90%) it follows that the unit production cost for models within one family does not vary much, but varies considerably across families. The use of more commonality has a positive effect on the total development costs, as we have to develop fewer components in total. The disadvantage of using the same component within a family is the larger unit production cost for the component if it is used for a product with a lower performance than for which the component was originally designed. In other words, the component is too expensive for some of the machines in which it is used. This deficiency is referred to in the literature as excess capability or excess functionality.
In previous studies (see e.g. Collier 1982, McClain et al. 1984, Baker 1985, Baker et al. 1986, Gerchak et al. 1988, Eynan and Rosenblatt 1996, Hillier 2000, 2002, Ma et al. 2002) the benefits of component commonality were almost solely associated with a decrease in inventory, safety stock and order costs due to the risk pooling effect. This industrial case study validates the importance of the development costs and unit production costs on the component commonality decision. The people at the company felt that the inventory costs were not significant compared to the development costs. Demand over time is supposed to be known and the company operates in a Make-to-Order environment. Therefore, we do not consider the effect of component commonality on inventory cost. Thonemann and Brandeau (2000) observe as well that these potential inventory savings are small in comparison to the potential savings in complexity costs. Fisher et al. (1999) identified the development costs as one of the key drivers of component commonality. Perera et al. (1999) associate the benefits of common parts at the level of development cost with avoiding duplicate R&D costs. Dogramaci (1979), Krishnan et al. (1999), Ramdas and Sawhney (2001) and Ramdas et al. (2003) developed models for component commonality problems where the development costs are also a major element of the total cost function. Component commonality has two opposing effects on the unit production cost (Rutenberg 1979, Fisher et al. 1999, Perera et al. 1999). We already discussed the excess functionality, but commonality can also lead to a decrease in production costs through economies of scale. Our company asserted that economies of scale are not significant for the problem which motivated this study. Design engineers felt that the major cost reduction would result from the new design and range split. Possible economies of scale would be relatively small. There are two justifications for this assumption. First, the procurement costs constitute a considerable part of the unit production cost and in this case they are unaffected by the total volume. A similar argument is presented by
Thonemann and Brandeau (2000). Second, the firm will operate mainly in the flat part of the experience curve because of the large time horizon. This is especially true because the machine is part of their core business in which they have decades of manufacturing experience. A final cost driver that is sometimes considered is the system cost (Fisher et al. 1999) or complexity cost (Thonemann and Brandeau 2000), which include those costs that are directly related to the number of different components. No such costs were identified in our case. An excellent discussion of the cost effects of component commonality can be found in Labro (2004).

2.3. The Pricing Decision

The current pricing practice of the company is based on two major principles. Firstly, a markup pricing strategy is used. Adding a mark-up to the costs is a common pricing strategy (Kotler et al. 1999). In a comprehensive survey of recent industrial pricing practices (Noble and Gruca 1999), cost-plus pricing is actually the most often cited strategy. Secondly, it is desirable for marketing reasons that the selling price over the range is proportional to the kW size. Krishnan et al. (1999) also observed that for technology-based industrial products, such as in our case, it is reasonable to assume that the selling price of the end product is a function of the performance level because of customer expectations.

In section 2.2 we explained how the commonality decision determines the costs. In this section we will explain how these costs will determine the sales price through fixed mark up. More specifically, the company goes through the following three steps in order to determine the prices for the entire product range. First, given a specific range split, the total unit cost of each model can be determined, by adding the specific cost components.
Second, an ordinary least squares (OLS) regression is performed on the new unit cost with the kW size as the dependent variable. This gives a linearized new unit cost. Third, a mark-up is added to this linearized unit cost. The gross margin is the selling price minus the actual unit cost. This price structure now has the two characteristics that the company wants: 1) it is based on a markup and 2) the selling prices are proportional in the performance. Note that because of this markup, the price depends on the commonality decision and hence is not determined independently from this.

![Figure 3](image_url)

**Figure 3.** Cost structure of a new possible range.

Figure 3 depicts this pricing structure. We consider a split proposal where the first family contains the T1 and T2 models, the second family contains the T3, T4 and T5 machines and the last family includes the three largest models. The first curve, labeled ‘Cost’, depicts the new unit cost of the models, given this specific range split decision. Observe that, as explained earlier, the largest gaps are between the families whereas within a family
the cost does not vary much. The second curve, labeled ‘Linearized Cost’, is the OLS regression and the third curve, labeled ‘Selling Price’, is the new selling price.

2.4. Revenue Modeling and Profit Optimization

In our investment analysis problem, the impact of the component commonality decision on the revenue is a key element and the only relevant objective is profit maximization. The total profit is determined by the unit cost, the selling price and the number of units sold. The design determines the cost and the price determines the sales volume. In order to optimize profit, however, the design and pricing issues must be taken into account simultaneously (See Figure 2). Therefore, an important input for this study is the price elasticity of demand to determine the effect of lowering the prices on the number of units sold. Based on a quantitative model (which is confidential and beyond the scope of the paper) and additional market research data, the marketing department of the company estimated the price sensitivity for the T models.

The focus of component commonality problems has mainly been on cost minimization (e.g. Fisher et al. 1999, Thonemann and Brandeau 2000). Recently, some papers appeared that consider profit maximization. Krishnan et al. (1999) note that obtaining reliable information constitutes a major problem. However, they recognize that both cost information and demand information is needed. In the same line, Ramdas and Sawhney (2001) consider both revenue implications at the product level and cost implications at the component level. The modular design problem of Chakravarty and Balakrishnan (2001) also includes market demand considerations and a profit maximization approach. Desai et al. (2001) explicitly consider the trade-off between manufacturing cost reduction and the
opportunity to extract price premiums through product differentiation. Commonality may

2.5. The Investment Framework and Time Frame

The most accurate way of evaluating this project is the net present value (NPV) method. When evaluating this project, we must look at incremental profits, compared to the current profits. The development costs are then weighed against the extra profits over several years, taking the time value of money into account. We use the cost of capital \( r \) of the firm as discount rate. It takes an expected three years to develop the components. The development cost is spent equally over this period. In year 4, the new products are introduced in the market and revenue is taken into account for 5 years from then on. Demand is estimated over the next five years. The choice of the time frame depends on the duration of the life cycle. Management must decide on the appropriate length.

3. An Initial Model

Essentially, the product range split problem as described here is an assortment problem. In such a problem we have a set of sizes of a product with associated demands. For practical reasons, we cannot make all the different sizes and we have to choose a subset of sizes which will be produced. Demand for a size which is not produced can be met by a larger size of the chosen subset. Such a substitution involves a substitution cost (Pentico 1976). In our case we have such an assortment problem at the component or subassembly level. Next we develop a general mathematical programming model for the optimization of the product range split with one type of common component in which the NPV is maximized. In a next section, the company-specific constraints are discussed. First, we introduce some notation:
Sets:

\( A \) : set of all end products, ordered according to performance requirement, index \( i \).

Parameters:

\( dc_i \) : development cost of the common component dimensioned for model \( i \), \( \forall i \in A \),

\( cc_i \) : unit production cost of the common component dimensioned for model \( i \), \( \forall i \in A \),

\( cm_i \) : model specific unit production costs for model \( i \), \( \forall i \in A \),

\( m_i \) : percentage mark-up for model \( i \) to calculate the selling price, \( \forall i \in A \),

\( e_i \) : the price elasticity of model \( i \), \( \forall i \in A \),

\( osp_i \) : old selling price for model \( i \), \( \forall i \in A \),

\( ox_i \) : old number of units sold of model \( i \), \( \forall i \in A \),

\( to_i \) : total old yearly profit for model \( i \), \( \forall i \in A \),

\( q \) : estimated time for development,

\( l \) : number of years considered for revenue generation.

Variables:

\( z_{ik} = 1 \) if model \( i \) uses the common component which is dimensioned for model \( k \),

\( = 0 \) otherwise, \( \forall i, k \in A, k \geq i \),

\( nc_i \) : new unit production cost for model \( i \), \( \forall i \in A \),

\( nsp_i \) : new selling price for model \( i \), \( \forall i \in A \),

\( nx_i \) : new number of units sold for model \( i \), \( \forall i \in A \),

\( tdc \) : total development costs for the components,

\( te_i \) : total yearly extra profit for model \( i \), \( \forall i \in A \),

\( NPV \) : net present value of the project.
The optimization model is given next:

1. maximize net present value;

   \[ \text{MAX } NPV \]  

subject to:

2. determine the split;

   \[ \sum_{k=i}^{i+l} z_{ik} = 1 \quad \forall i \in A \]  

   \[ z_{ik} \leq z_{i+1,k} \quad \forall i, k \in A, k > i \]  

3. calculate the total development costs for the common components;

   \[ tdc = \sum_{i \in A} dc_i z_{ii} \]  

4. calculate the new unit production cost;

   \[ nc_i = \sum_{k=i}^{i+l} (cc_k) z_{ik} + cm_i \quad \forall i \in A \]  

5. calculate new selling prices;

   \[ nspl_i = (1 + m_i) nc_i \quad \forall i \in A \]  

6. calculate new number of units sold via the price elasticity;

   \[ nx_i = ox_i (1 + \varepsilon_i \frac{nspl_i - osp_i}{osp_i}) \quad \forall i \in A \]  

7. calculate the total yearly extra profit

   \[ te_i = nx_i (nspl_i - nc_i) - to_i \quad \forall i \in A \]  

8. calculate NPV;

   \[ NPV = \sum_{i=1}^{A} \left( - \frac{tdc}{(q)(1+r)^i} \right) + \sum_{i \in A} \sum_{i+l \in A} \frac{te_i}{(1+r)^i} \]  

9. integrality and non-negativity constraints;

   \[ z_{ik} \in \{0,1\}, nx_i \geq 0 \quad \forall i, k \in A, k \geq i \]
The objective function (1) maximizes the NPV of the project. Constraints (2) and (3) define the component commonality decision. Every machine uses exactly one version of the common component (2). If $k$ is the highest performance machine of the family of machine $i$ then all the models from $i$ up to $k$ must use the same common component optimally dimensioned for machine $k$ (3). The remaining constraints (4) to (9) are definition constraints used to construct the objective function and all the variables in these constraints depend on our main variable $z_{ik}$. The calculation of the development costs is done in constraint (4). We only develop the components for the highest performance model in each family. If the variable $z_{ii}$ equals one then we develop the common component optimally dimensioned for model $i$. The new unit production cost of model $i$ is calculated in constraint (5) as the sum of the cost of the common component, this is the family dependent cost, and the other costs, which are the model specific costs. The new selling price is determined as the new unit cost plus a percent gross profit (6). The new number of units sold is calculated via the price sensitivity (7). The price elasticity is defined as the percentage change in sales that results from a 1 percentage change in price and for regular goods the price elasticity is negative. Constraint (8) calculates the total yearly extra profit for each model. With these elements and taking into account the correct timing of the cash flows, we can calculate the NPV of the project (9). The integrality constraints are imposed in (10). Also the number of units sold must be positive. Observe that in constraint (8) there is a multiplication of two decision variables. Consequently we have a mixed integer nonlinear program.
4. Further Industrial Considerations

During the study many specific industrial considerations emerged and we had to extend our basic model to include all real life issues. A good choice of the variables in the basic model made it possible to easily implement new constraints. We will discuss these industrial considerations without too much detail. We have incorporated the following additional engineering and marketing issues into our model:

- We consider two common component types simultaneously, namely the stage and frame. As a result we introduce a new set of range split variables for the second common component. We also model the requirement that there can only be a stage split if there is a frame split for that model, but not necessarily vice versa.

- Design requirements were modeled such as a minimum and maximum number of members in a family.

- An extra requirement was included, limiting the ratio of the highest power to the lowest power within a family.

- We investigate possible product line extensions to check whether it would be beneficial to include some new machines. These machines were either optional, meaning that we have to decide whether or not to include them as a new model, or fixed, meaning that we only had to decide to which family they would belong.

- We also implemented the requirement that the price should be proportional to the performance level, as discussed previously. This can be taken into account in our model, as the slope and intercept of the Ordinary Least Square regression equation can be computed with a simple formula and included in the optimization model.

5. Implementation and results

This research is an extension of previous studies on the use of common components done
by the consultancy firm OM Partners (Belgium) at other subdivisions of the company in 1990 and 1994. This is the third time that a similar problem was studied for the company, and after this project, a new study for a different product line was undertaken. This illustrates the relevance and value added of the study and indicates that this is a generic problem for the manufacturer. The new critical element in this study is the integration of production and engineering issues with marketing aspects and the profit maximization approach. The company recognized from the outset that this was a cross-functional problem and hence the project team consisted of people from both engineering and marketing. The team was then extended with the three authors. During several meetings many topics were discussed such as the relevant costs, the definition of families, the objective, the estimation of key input figures and technical constraints. Regular feedback allowed us to further extend the model until all of the relevant considerations were taken into account. The preliminary analyses also provided valuable feedback to the company. For example, sensitivity analysis revealed that the NPV was very sensitive to the estimated price elasticity. Therefore, the marketing department decided to obtain more accurate information by actually performing a real-life market test in a rather isolated market. The analyses also provided valuable general insights into the effects of component commonality on the many trade-offs that were inherent to this problem. One of the benefits is a better understanding of the relevant costs and the interdependencies between the key elements of the decision.

We have programmed and solved our real life case using LINGO 3.0 (Schrage 1998). This specialized software is capable of optimizing mixed integer nonlinear problems. The optimization problem for the manufacturer was solved within 52 seconds on a Pentium III 750 Mhz PC under Windows NT. The optimal design is depicted as the first proposal in
Figure 4. The range is extended with a new machine (TN) at the lower end of the performance level. There are three families and the project yields a positive NPV. We have also done a further analysis evaluating other alternative splits. It is then interesting to know by how much these solutions deviate from the optimal solution. In Figure 5 we set the NPV of the optimal solution to 100% and compare this with the NPV of other alternatives. It is clear that proposals for a split into 4 families are inferior to the splits into three families. The main reason for this was the large development cost for an extra family. Alternative 2 is an example of a split where there are 4 families for the frame, but only 3 families for the stage.

**Figure 4.** Optimal solution and comparison of different alternative splits.

Based on the above analyses and in cooperation with the project team, we made a recommendation to the management of the company. Our proposal was accepted and implemented. From that moment we were no longer involved in the project as the next step was the development of the new common parts, which was a task for the engineering department. About three and a half years after this study, the company introduced the
redesigned product range. The new range is currently advertised on the corporate website and the information about the size of the frames for the models corresponds exactly with our proposal for the range split. The company proudly announced that the new range offers increased customer value and choice. Indeed, the higher output value for a lower price and the introduction of several new variants at the low end according to our proposal realized those objectives. Because the aim of the redesign was to move into a high volume market, the project was of strategic importance to the company. Therefore, we were not allowed to disclose many of the specific details about the company, the analyses and recommendation. Yet, the paper provides a good insight into the key elements of the problem.

Krishnan et al. (1999) observe that the treatment of price as a decision variable in the model would be an interesting research topic. In the previous model, a cost-based pricing method was used, according to the company practice. However, the company could exploit the relationship between the price level and units sold and choose an optimal selling price. As such, the total extra profit resulting from the product range split decision could be increased. It is possible to include this product line pricing decision in a new model by removing the constraints (6) in which the new selling prices are calculated. As a result, the new selling price can be chosen freely. Applying the price optimization model gives a slightly different optimal range split, which is the same as Alternative 1 in Figure 5. However, optimizing the prices for the initial optimal proposal, results in a NPV that deviates only slightly from this new optimal solution. The NPV increased by making optimal use of the demand relationship. However, this analysis was not part of the original project as the company insisted on continuing their initial pricing strategy.
6. Managerial Insights

In the product range split problem, we must find the optimal split of a product range into families, where all the models in the same family share the same version of a component. A similar problem has also been studied by Fisher et al. (1999). They state that this as a relevant problem for products with the following properties:

1. the models in the range make use of the same type of component,
2. the models can be ranked by performance requirement of the common component,
3. the common components are downward compatible and have no substantial effect on the overall product quality or the quality as perceived by the customer,
4. the cost of the common component is increasing in the performance requirement.

Brake systems in cars or components for computers are good examples of products with such properties. Note that these properties also apply to our problem. Fisher et al. (1999) proposed a cost minimizing analytical model to determine the optimal number of common components for brake systems in the car industry. The authors expect the basic insights, derived from their analytical model, to be valid for other products as well if they satisfy the above properties. Next we empirically investigate whether the basic insights obtained by Fisher et al. are also valid for our company specific problem. This is interesting as we are working with real life data and as such this analysis has practical relevance. For our tests we have dropped the design requirements on the minimum and maximum number of splits, otherwise the number of splits is too limited.

We first investigate the relationship between development costs ($dc$) and the optimal number of common components ($N^*$). The proposed relationship from Fisher et al. is: $N^* \sim \sqrt{\frac{1}{dc}}$. In this experiment, we vary the development costs between 10% and 250% of
their actual value and calculate the resulting optimal number of common components with our model. The results of this analysis are depicted in the first graph of Figure 5. On the x-axis is the parameter that we changed, namely the development cost, given as a percentage of the original value. On the y-axis is the optimal number of common components. We observe a pattern that is consistent with the proposed relationship. If the development costs are very low, then it makes sense to develop a component for each individual machine, and as such we avoid the problem of excess functionality. For higher development costs it is optimal to include just a few common components. We also find that, when the solution has more than three splits, the model puts the extra split first at the higher kW level products. This is quite intuitive, because there exists a larger kW difference between the highest machines and it is exactly this difference that determines the cost of excess functionality.

![Figure 5](image.png)

**Figure 5.** The influence of development costs and total sales on the number of splits

A second basic insight from Fisher et al. is that the optimal number of common parts is proportional to the square root of the total sales \( V \): \( N^* \sim \sqrt{V} \). As the number of units sold is a variable in our model, we vary the old number of units sold, which are the basis to calculate the new sales, as can be seen from equation (7). We solve our model with
different old sales volumes, ranging from 10% to 1000% of their original value and observe the effect on the optimal number of common components. The results are depicted in the second graph of Figure 5. We remark again that our model is consistent with the proposed relation. The relationship is also intuitive, as larger sales give a higher total profit, which can compensate for the costs of developing more specific components.

Next, to obtain more managerial insights, we also report further experiments on randomly generated data. We look at the impact of four parameters on the number of common components. These four factors are: price elasticity, the development costs, the excess functionality cost and the width of the performance range. For these experiments we use the basic model, as described in (1)-(10). As such, the results are not influenced by the company specific constraints. There are eight models in the range. To determine the performance requirement $p_i$, i.e. the kW size, of the models in the range, we randomly draw 8 uniformly distributed numbers from the performance range $[1,P]$. For the base case we generate 5 problems with different performance requirements and we report on the average number of common components over these five instances. The other parameters in the base case are determined as follows. The unit cost of the common component is determined by the performance requirement $p_i$ through the following linear relationship: $cc_i = 200 + 3p_i$, which reflects the actual relationship in the study. The excess functionality cost is altered by changing the slope in this cost equation. A larger slope indicates that there is more excess functionality between subsequent models. In the base case, we set the model specific unit production cost ($cm_i$) equal to the unit production cost of the common component ($cc_i$). The old street price is set at $2*(cm_i + cc_i)$. We use a constant mark-up of 50% to calculate the new street prices. Each model in the range has a constant yearly demand of 100 units. Just like in the case, we use a time horizon of 3 years
for development \((q)\) and 5 years for revenue generation \((l)\). The discount rate is 10%. The remainder of the parameter values for the base case can be found in Table 1. In the experiments we change the parameters one at a time to explore the effect on the optimal number of common components \(N^*\). The details about the parameter ranges can be also found in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Base case</th>
<th>Range</th>
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<tbody>
<tr>
<td>Price Elasticity ((\varepsilon))</td>
<td>10%</td>
<td>{5, 10, 20, 40}</td>
</tr>
<tr>
<td>Development cost ((dc_i))</td>
<td>(1000 * cc_i)</td>
<td>{10<em>cc_i, 100</em>cc_i, 1000<em>cc_i, 10000</em>cc_i}</td>
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<tr>
<td>Excess functionality (EF)</td>
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<td>{0, 1, 2, 3, 4, 5, 10, 20, 50}</td>
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<tr>
<td>Maximum Performance (P)</td>
<td>100</td>
<td>{50, 100, 250, 500}</td>
</tr>
</tbody>
</table>

*Table 1. Parameters and range for experiments*

The results of our experiments are summarized in Table 2. Our experiments indicate that a higher price elasticity will lead to less component commonality, i.e. more components, when the price is lowered. By changing the price elasticity, we actually influence the number of units sold and this leads to increased profits. This extra profit can cover the development cost of more common components. The magnitude of the development costs also has a clear effect on the number of common components. If the development costs are very high, we just want to develop a few common components, whereas if this cost is low, then it is justified to make a component specific for each model. This is confirmed by our experiments. An increasing excess functionality cost will intuitively lead to more components, as the cost of using an overdimensioned component increases. Again our experiments confirmed this. Finally, the performance range has an effect on the optimal number of common components through the excess functionality cost. If the performance gaps between subsequent models are high, then the excess functionality cost is also high,
justifying the use of more components. These experiments were done using LINGO 3.0 on a Pentium III 750 Mhz PC under Windows NT. All of the problems are solved within a few minutes.

<table>
<thead>
<tr>
<th>Price Elasticity</th>
<th>Development Costs</th>
<th>Excess Functionality</th>
<th>Maximum Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>ε</td>
<td>N*</td>
<td>DC</td>
<td>N*</td>
</tr>
<tr>
<td>5</td>
<td>1.4</td>
<td>10*CC</td>
<td>7.8</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>100*CC</td>
<td>4.4</td>
</tr>
<tr>
<td>20</td>
<td>2.6</td>
<td>1,000*CC</td>
<td>2</td>
</tr>
<tr>
<td>40</td>
<td>3.2</td>
<td>10,000*CC</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Table 2. Effect on the number of common components for several parameters

7. Conclusion

In this paper we describe a real life product range split problem, which is essentially an assortment problem and a component commonality problem. We propose a mixed integer nonlinear optimization model for this problem. The component commonality decision affects both the production costs and development costs. The key feature of our analysis is the incorporation of the demand relationship, where the volume sold is price dependent. Integration of both costs and revenues is necessary for the investment analysis because the criterion is profit maximization. Within the investment analysis framework we optimize our product range split decision to investigate if it is beneficial to undertake the planned redesign. Our models were then applied to solve a real product range split problem at a large manufacturer, demonstrating that the formulation is capable of incorporating several industrial constraints. The recommendation we proposed, in cooperation with the
production and marketing department and based on our analysis, has been implemented at
the company and is currently advertised on the company’s website. Finally we performed
several computational experiments investigating the influence of several parameters,
providing managerial insights in the trade-offs of this problem.

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