Reduced Complexity Sphere Decoder for Spatial Modulation Detection Receivers

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Abstract—In this paper a novel detection algorithm for spatial modulation (SM) based on sphere decoder (SD) tree search idea is proposed. The aim is to reduce the receiver complexity of the existing optimal decoder while maintaining an optimum performance. The algorithm performs a maximum likelihood (ML) search, only over those points that lie inside a sphere, centered at the received signal, of given radius. It is shown with the aid of analytical derivations, that for a SNR (signal-to-noise ratio) between 2 dB and 18 dB at least 45% and up to 85% reduction in the number of complex operations can be achieved with a close to optimal bit-error-ratio (BER) performance.

Index Terms—Spatial modulation, Sphere decoder, MIMO.

I. INTRODUCTION

To cope with the demand for indoor wireless access to bandwidth-intensive applications such as the Internet multimedia streaming applications (Voice over IP (VoIP), streaming video and music, gaming, and network attached storage (NAS)), there is a need for increasing data throughput of current networks [1]. The maximum data rate of most wireless local area networks (WLANs) based on the IEEE 802.11 set of standards (802.11a/b/g) typically ranges from 2 Mbps up to 54 Mbps net bit rate (excluding the physical layer protocol overhead). The IEEE 802.11n amendment is proposed to significantly improve network throughput over previous standards. The increase in the maximum raw physical net bit rate is achieved by introducing the multiple-input multiple-output (MIMO) techniques [1], [2]. A data rate of 600 Mbps can be achieved for four parallel streams at 40 MHz channel bandwidth. However, implementing four parallel streams demands high computational power, which corresponds to long processing time and high power consumption. Therefore, complexity reduction algorithms for spatial multiplexing MIMO systems, such as sphere decoder [3]–[7, and references therein], are proposed to alleviate this problem.

The SD algorithm avoids an exhaustive search by examining only those points that lie inside a sphere with radius C. The performance of the SD algorithm is closely tied to the choice of the initial radius. The chosen radius should be large enough so that the sphere contains the solution. However, the larger the radius is, the longer the search takes, which increases the complexity. On the other hand, a small radius may cause the algorithm to fail finding any point inside the sphere.

In this paper, the SD tree search structure is adopted to reduce the complexity of the optimum ML decoder of SM [8]–[11]. In SM, multiple antennas exist at the transmitter, but only one of them transmits at a time, to avoid interchannel interference (ICI) at the receiver input. The active antenna transmits a symbol from the complex signal constellation diagram. The receiver first determines via an additional antenna detector which of the antennas has sent information (digital information is encoded into the antenna constellation). Therefore, there is information transmission at this stage. In a second step, conventional data detection in the complex signal space is carried out. The receiver applies the optimum decoder [11] to estimate the complex symbol and the spatial symbol, and uses the two estimations to retrieve the original data bit sequence. It is shown that the complexity of the optimum receiver increases linearly with the number of transmit antennas. This is unlike other spatial multiplexing MIMO techniques applying ML detection where the complexity increases exponentially with the number of transmit antennas.

The existing SD algorithms in literature can be applied to SM by adding a zero as a constellation point. This, however, does not consider the basic and fundamental principle of SM, that at any giving time, only one antenna is active. Therefore, the complexity of such a system increases exponentially with the number of transmit antennas. In addition, the Euclidean distances between constellation points decrease by considering the zero as a constellation point, which significantly degrades system performance. Thereafter, a modified SD algorithm based on tree search structure that is tailored to SM is presented. It is shown with the aid of analytical derivations, that a reduction of 45% and up to 85% in the number of complex operations can be achieved by using the proposed SM-SD algorithm, while maintaining an almost optimum performance combined with a complexity that increases linearly, and not exponentially, with the number of transmit antennas.

The remainder of this paper is organised as follows: Section II introduces SM system with the optimum ML decoder. In Section III, the proposed SM-SD algorithm for SM is presented. Section IV presents analytical calculations for the complexity of SM-SD and the initial radius selection method. Simulation results are presented in Section V, and the paper is concluded in section VI.

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II. SPATIAL MODULATION WITH ML DECODER

The system model of SM with the mapping table to antenna indices and binary phase shift keying (BPSK) symbols is depicted in Fig. 1. A MIMO system consisting of four transmit antennas $N_t = 4$ and four receive antennas $N_r = 4$, for illustration purposes\(^1\), is considered.

In Fig. 1, $q(n)$ is the incoming binary data to be transmitted over the MIMO channel. In SM, each $m = \log_2(MN_t)$ bits, where $M$ is the QAM constellation size, are transmitted at a particular time instance. The matrix $x(n)$ is created by grouping each $m$ bits from $q(n)$ as the column vectors of $x(n)$. The matrix $x(n)$ is then mapped to another matrix $s(n)$ according to the mapping table as shown in Fig. 1. Each column vector in $s(n)$ contains the data to be transmitted at a particular time instance over the MIMO channel. Since, however, only one element in each column vector of $s(n)$ is different from zero; only one antenna will be active at a time instance.

For instance, an example is shown in Fig. 1 for two time instances. The incoming data sequence $q(n) = [1 0 1 0 1 0]$ is mapped to

$$s(n) = \begin{pmatrix}
0 & 0 \\
-1 & 0 \\
0 & +1 \\
(\text{n=1}) & (\text{n=2})
\end{pmatrix}.$$  \hspace{1cm} (1)

In the first time instance, the second antenna will be active and transmitting the BPSK symbol $s = -1$. All other antennas at this particular time instance will be off. In the second time instance, all transmit antennas are off except antenna three which will be transmitting the symbol $s = +1$. Hence, an overall increase in spectral efficiency by $\log_2(N_t)$ as compared to single input single output (SISO) system is achieved.

\(^{1}\) For different number of antennas and different constellation diagrams of SM, the reader is kindly requested to refer to [8] for more details.

The transmitted signal experiences an $N_r$-dim additive white Gaussian noise (AWGN). The channel is assumed to be flat fading channel with an independent and identically distributed (i.i.d.) entries according to $CN(0, \sigma_n^2)$. The received signal at a specific time instance is given by,

$$y = Hs + v$$ \hspace{1cm} (2)

where $H$ is an $N_r \times N_t$ MIMO channel matrix, and $v$ is an AWGN vector $\sim CN(0, \sigma_n^2)$.

At the receiver, the optimum SM decoder is considered to estimate the complex symbol $\hat{s}$ and the spatial symbol $\hat{\ell}$ as follows \cite{11}:

$$\begin{bmatrix} \hat{\ell}, \hat{s} \end{bmatrix} = \arg \max_{\ell,s} p_y(y|s,H) \hspace{1cm} (3)$$

$$= \arg\min_{\ell,s} ||g_\ell||^2_F - 2\Re\{y^Hg_\ell\},$$ \hspace{1cm} (4)

where $g_\ell = h_\ell s$ is the received vector when transmitting the symbol $s$ from antenna index $\ell$ where $1 \leq \ell \leq N_t$ and $s \in \{M\}$, $h_\ell$ is the channel vector containing the channel path gains from transmit antenna $\ell$ to all receive antennas, and $\Re$ is the real part of a complex number. In addition, $\cdot^H$ denotes the Hermitian of a vector or a matrix, $\| \cdot \|_F$ is the Frobenius norm of a vector/matrix and

$$p_y(y|s,H) = \pi^{-N_t}(-\|y-Hs\|_F^2).$$ \hspace{1cm} (5)

The evaluation of (4) is computationally expensive as it needs to be evaluated for all possible antennas and modulation symbols. This requires $(4N_r-1)$ complex operations evaluated $N_t M$ times, i.e.,

$$\psi_{opt} = N_t M (4N_r - 1)$$ \hspace{1cm} (6)

From (6) it can be seen, and as stated before, that the receiver complexity increases linearly with the number of transmit antennas.

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**Fig. 1.** Spatial modulation system model and the mapping table to antenna indices and BPSK symbols. At each time instance, three bits are transmitted. Two are encoded in the antenna index and one in the BPSK symbol.
III. SM with SD Tree Structure

The considered SM-SD algorithm in this paper is a modified version of the SD algorithm presented in [3], by adopting the tree search structure, as shown in Fig. 2. The SM-SD performs a ML search only on paths that lead to points \((\tilde{\ell}, \tilde{s})\) with an error, less than or equal to the sphere radius \(C\). In this paper, the initial radius of the SM-SD algorithm is adjusted according to the noise level assuming the knowledge of the SNR at the receiver side known from previous received data. Then the radius is adapted when a point is found inside the sphere by the Euclidean distance of that point. The initial sphere radius considered in SM-SD is a function of the noise variance as given in [12],

\[
C^2 = 2\alpha N_r \sigma_n^2, \tag{7}
\]

where \(\sigma_n^2\) is the noise variance and \(\alpha\) is a constant chosen to maximise the probability of having the transmitted point inside the sphere. Depending on the SNR value, a major reduction in the number of calculated paths can be achieved.

The full procedure for SM-SD algorithm is explained in what follows. Let \(S\) be a set containing all possible transmit antennas and spatial symbol points, \(S = \{ (\ell, s) : \ell \in [1, \ldots, N_r], s \in \{ M \} \}\), \(\delta\) the Euclidean distance error, \(\psi\) the depth of the search on each path \((\ell, s)\). Then, the algorithm can be formulated as follows:

1) for \((\ell, s) \in S\)
   a) for \(i = 1 : N_r\)
      i) \(\delta(\ell, s) + = |y_i - H(\ell, s)s|^2\)
      ii) if \(\delta(\ell, s) \geq C^2\) then go to 1
      iii) \(\psi(\ell, s) + = 1\)
   b) \(C^2 = \delta(\ell, s)\)
2) \(S_\psi = \arg(\psi(\ell, s) = \max(\psi)) \)
3) \([\tilde{\ell}, \tilde{s}] = \arg \min_{(\ell, s) \in S_\psi} (\delta(\ell, s))\)

SM-SD algorithm search the paths leading to each point \((\ell, s)\) as long as it is still inside the sphere as depicted in Fig. 2. Whenever a point is found to be inside the sphere, the radius, \(C\), is updated with the Euclidean distance of that point. The path with the minimum Euclidean distance is considered to be the solution. A significant advantage of the proposed SM-SD algorithm is that it avoids the problem of having no points inside the sphere, which is a major problem of the conventional SD algorithms. SM-SD algorithm selects the path with the minimum Euclidean distance even if all the points were outside the sphere.

IV. Complexity Analysis and Initial Radius Selection Method

The ML receiver for SM, given in (3), can be re-written as,

\[
[\hat{\ell}_{ml}, \hat{s}_{ml}] = \arg \min_{\ell, s} \sum_{i=1}^{N_r} |y_i - H(i, \ell)s|^2 \tag{8}
\]

let

\[
z_i(\ell, s) = y_i - H(i, \ell)s \tag{9}
\]

and,

\[
y_i = H(i, \ell)t + v \tag{10}
\]

where \(s_t\) is the transmitted symbol at a particular time instant from antenna index \(\ell\), and \(v \sim \mathcal{C}\mathcal{N}(0, \sigma_n^2)\). Then,

\[
z_i(\ell, s) = v + H(i, \ell)t - H(i, \ell)s \tag{11}
\]

from (11), the probability density function (PDF) for \(z_i(\ell, s)\) is

\[
f_z(z_i(\ell, s)|s_t, \ell_t, \mathbf{H}, \sigma_n^2) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(z_i(\ell, s) - \mu_i(\ell, s))^2}{2\sigma_n^2}} \tag{12}
\]

where \(\mu_i(\ell, s)\) is

\[
\mu_i(\ell, s) = H(i, \ell)s_t - H(i, \ell)s \tag{13}
\]

The SM-SD performs a ML search only on paths that lead to points \((\ell, s)\) with an error, less or equal to \(C\). In other words, the SM-SD algorithm calculates

\[
\gamma_k(\ell, s) = \sum_{i=1}^{k} |y_i - H(i, \ell)s|^2 \tag{14}
\]

\[
= \sum_{i=1}^{k} |z_i(\ell, s)|^2 = \frac{\sigma_n^2}{2} \kappa_k(\ell, s) \tag{15}
\]

if \(\gamma_k(\ell, s) \leq C^2\), it continues to the next level \(k\), where \(k = [1, \ldots, N_r]\) and

\[
\kappa_k(\ell, s) = \sum_{i=1}^{k} \left(\frac{z_i(\ell, s)}{\sigma_n/\sqrt{2}}\right)^2 \tag{16}
\]

Hence, the probability of having a point \((\ell, s)\) at a level \(k\) inside the sphere giving a radius \(C\) is,

\[
p_k(\ell, s, C) = \Pr \left( \gamma_k(\ell, s) \leq C^2 \right) \left( s_t, \ell_t, \mathbf{H}, \sigma_n^2 \right) \]

\[
= \Pr \left( \kappa_k(\ell, s) \leq \left(\frac{C}{\sigma_n/\sqrt{2}}\right)^2 \right) \left( s_t, \ell_t, \mathbf{H}, \sigma_n^2 \right) \tag{17}
\]
From (16), \( \kappa_k(\ell, s) \) is a squared summation of non zero mean normal distributed random variables, with variance equal to one. Thereby, the distribution of the random variable \( \kappa_k(\ell, s) \) is a non central chi-squared random variable with \( 2k \) degree of freedom, and the non-central parameter \( \lambda_k(\ell, s) \) equals to [13]

\[
\lambda_k(\ell, s) = \sum_{i=1}^{k} \frac{|\mu_i(\ell, s)|^2}{\sigma_n^2/2}.
\]

Consequently, the probability of having a point \((\ell, s)\) at a level \( k \) inside the sphere giving a radius \( C \) is

\[
p_k(\ell, s, C) = F \left( \left( \frac{C}{\sigma_n/\sqrt{2}} \right)^2, 2k, \lambda_k(\ell, s) \right)
\]

where \( F(\cdot, \cdot, \cdot) \) is the CDF (cumulative distribution function) of the random variable \( \kappa \). A closed form expression for (19) is not available. However, a solution can be obtained as shown in [13, (2.1-124)] by noting that the degree of freedom is always a multiple of 2. Therefore, (19) can be expressed in terms of the generalised Marcum’s \( Q \) function as follows,

\[
p_k(\ell, s, C) = 1 - Q_k \left( \frac{\lambda_k(\ell, s)}{\sigma_n/\sqrt{2}} \right)
\]

From (20), the total number of Euclidean distance equations completed by SM-SD in each path \((\ell, s)\) is given by,

\[
\xi_{\ell, s} = \sum_{k=1}^{N_r} p_k(\ell, s, C) = \sum_{k=1}^{N_r} \left( 1 - Q_k \left( \frac{\lambda_k(\ell, s)}{\sigma_n/\sqrt{2}} \right) \right)
\]

and the total number of Euclidean distance equations completed by the SM-SD algorithm is

\[
\xi = \sum_{\ell=1}^{N_r} \sum_{s \in M-QAM} \xi_{\ell, s}
\]

The derived number of Euclidean distance equations in (22) assumes that, initially, the algorithm knows if the point is inside or outside the sphere. However, this is not true and a correction factor is needed to consider the initial calculated equations. To account for this, the number of Euclidean distance equations is modified as follows,

\[
\xi_{\text{SM-SD}} = \begin{cases} 
\xi + N_t M & \xi \leq N_t M (N_r - 1) \\
\xi & \text{otherwise}
\end{cases}
\]

where \( N_t M \) is the initial number of calculated Euclidean distance equations.

The SM-SD updates \( C \) whenever a point is found inside the sphere, with the Euclidean distance of that point. Hence, \( C \) in (21), is updated as follows,

\[
C = \sum_{k=1}^{N_r} |v + \mu_k(\ell, s)|^2
\]

In summary, the calculation for the number of complex operations for the SM-SD algorithm is as follows,

1) for \( \ell = 1 : N_r \\
a) \text{for } s \in M-QAM \\
i) \text{calculate } \xi_{\ell, s} (21) \text{ and } p_{N_r}(\ell, s, C) (20) \\
ii) \text{if } \mu_{N_r}(\ell, s) > \zeta \text{ then } \text{ } \\
C = \sum_{k=1}^{N_r} |v + \mu_k(\ell, s)|^2
\]

2) calculate \( \xi (22) \)

3) calculate \( \xi_{\text{SM-SD}} (23) \)

where by numerical simulations the optimal value for \( \zeta \) is found to be 0.9.

Additionally, (14) needs 3 complex operations, yielding a total number of complex operations equals to,

\[
\psi_{\text{SM-SD}} = 3 \times \xi_{\text{SM-SD}}
\]

The value of \( \alpha \) for the initial radius in (7) is chosen to increase the probability of having the transmitted point \((\ell_t, s_t)\) inside the sphere. Hence,

\[
p_{N_r}(\ell_t, s_t, C) = F \left( \left( \frac{C}{\sigma_n/\sqrt{2}} \right)^2, 2N_r, \lambda_{N_r}(\ell_t, s_t) \right)
\]

\[
= F \left( 4\alpha N_r, 2N_r, \lambda_{N_r}(\ell_t, s_t) \right)
\]

\[
= 1 - \varepsilon
\]

The probability \( 1 - \varepsilon \) is set as a value close to 1. For \( \varepsilon = 10^{-6} \) and \( N_r = 4, 8 \), \( \alpha = 3, 2 \) respectively.

\section{Simulation Results}

In the following, Monte Carlo simulation results for at least \( 10^6 \) channel realisations are considered to compare the performance of SM with the optimum detection technique and the proposed SM-SD algorithm. In the analysis, two spectral efficiencies are considered, \( m = 4, 6 \) bits/symbol and \( (N_t = N_r = 8) \).

The BER results versus SNR for the two different spectral efficiencies are depicted in Fig. 3. An important observation is that SM-SD algorithm and the optimum detector have an identical performance. That is because the initial radius \( C \) for SM-SD is chosen so it would give a high probability of having the transmitted point inside the sphere, which is done by choosing \( \alpha \) that increases the probability in (26), where in the case of \( N_r = 8 \) and \( \varepsilon = 10^{-6} \), \( \alpha = 2 \).

The reduction of computational complexity,

\[
R = \frac{\psi_{\text{opt}} - \psi_{\text{SM-SD}}}{\psi_{\text{opt}}}
\]

is depicted in Fig. 4 and Fig. 5 for the two different spectral efficiencies. A significant reduction of at least 45% and up to 85% is reported. It can be observed that a higher reduction in computational complexity is achieved for high SNR values and/or low modulation orders. At high SNR the sphere radius is small, which leads to lower complexity. The higher complexity reduction for high modulation orders is mainly due to the increase of the number of calculations of the optimal decoder.
VI. SUMMARY AND CONCLUSIONS

The complexity of the optimum ML detector for SM is significantly reduced using the SD tree search applied in a novel fashion. The existing SD algorithms in literature are computationally more expensive if directly applied to SM and will result in performance degradation. The SM-SD algorithm exploits the fact that only a single antenna is active and performs a ML search only over the points that are inside the sphere of a given radius (depending on the SNR) and centered at the received point. The performance of the proposed detection algorithm is optimum and it yields in excess of 45% reduction in the number of complex operations.

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