Achievable Diversity-Multiplexing-Delay Tradeoff for ARQ Cooperative Broadcast Channels

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Abstract—Cooperative broadcast aims to deliver common messages to all receiver nodes in the wireless network by utilizing cooperation between them. This can have many important applications like broadcasting control messages and cellular phone-based teleconference/game among a group of people. In this paper, we will investigate the cooperative broadcast from the diversity-multiplexing-delay (D-M-D) tradeoff point of view, which has become a popular tool in analyzing multiple-input multiple-output (MIMO) and cooperative protocols. The major contribution is that we derive the optimal D-M-D tradeoff curves for arbitrary number of receive nodes and arbitrary maximum number of automatic retransmission request (ARQ) rounds. Interestingly, it is shown that the achievable diversity gain is always restricted by two special situations. Moreover, we have also compared our results with the previous results on the achievable diversity-multiplexing (D-M) tradeoff of cooperative broadcast channels where no delay is taken into consideration.

I. INTRODUCTION

Cooperative communication [1], [2] has been extensively investigated recently. This approach utilizes the broadcast nature of the wireless medium and improves the system efficiency by providing additional spatial diversity through node cooperations. But if nodes operate in half-duplex mode, part of the radio resource needs to be allocated for the transmission between the relays and the destinations, which considerably reduces the system throughput. However, if feedback is allowed, e.g., automatic retransmission request (ARQ), this will not be a problem because the relay will not transmit if the destination has successfully received the message. On the other hand, cooperative broadcast where common messages are transmitted to all the destinations in the network has also attracted research interest recently [3]-[5]. It has many applications like broadcasting control messages and cellular phone-based teleconference.

In this paper, we are going to investigate ARQ cooperative broadcast channels from optimal diversity-multiplexing-delay (D-M-D) tradeoff point of view. Diversity gain and multiplexing gain are two benefits that can be provided by multiple-input multiple-output (MIMO) systems. However, to enhance diversity gain, certain amount of multiplexing gain needs to be sacrificed. [6] is the pioneer in this area due to

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Due to the similarity between MIMO channels and cooperative channels, D-M and D-M-D tradeoffs have also been proven as a powerful tool in analyzing cooperative communication protocols. In [1], [2], and [9], D-M tradeoffs are used to compare performances of various cooperative protocols. [10] and [11] investigate the optimal, i.e. maximum achievable, D-M tradeoffs for amplify and forward (AF) strategies and decode and forward (DF) strategies in different network scenarios. [11] also proposes a dynamic decode and forward (DDF) protocol which achieves the genie-aided tradeoff upper bound (i.e. the cut-set bound) in low multiplexing gain region. While in high multiplexing gain region, [12] shows that higher diversity gain than DDF protocol can be obtained by varying the multiplexing gain in different time slots. For AF strategies, [13] illustrates that the genie-aided tradeoff upper bound can be achieved by a slotted AF scheme when the network size goes to infinity. What is more, [14] studies the full-duplex relay case from cut-set bound point of view. [15] calculates the optimal D-M-D tradeoff for DF protocols with single relay, and actually it can be regarded as a special case of our results which will be discussed in section IV.

In this work, optimal D-M-D tradeoff for cooperative broadcast channels with half-duplex single-antenna nodes is investigated. Given the number of receive nodes, maximum ARQ rounds (the term ARQ round will be specified in the next section) and multiplexing gain, we will derive the achievable diversity gain by proving that at low multiplexing gain region,
the diversity gain is determined by the situation where none of the receive nodes can decode the message, and at high multiplexing region, it is determined by the situation where all expect one node can decode the message right before the last ARQ round. The underlying reasons and the physical meaning of the results will be interpreted by dividing the diversity gain into node diversity gain and ARQ diversity gain. What is more, the comparison between our result and the DDF protocol will also be performed to verify the advantages of ARQ protocols.

The paper is organized as follows. Section II describes the system model of the cooperative broadcast protocol and the background of D-M and D-M-D tradeoff analysis. Section III derives the D-M-D tradeoff curves for cooperative broadcast system, while the discussions and comparisons of the results are carried out in Section IV. Finally, Section V gives the conclusions.

The notation used in this paper is as follows. The superscripts $H$ and $T$ denote the Hermitian transpose and the transpose, respectively. $\otimes$ is the Kronecker product, $I_T$ is an $T \times T$ identity matrix and $\overline{A}$ is the complement of a set $A$. We use $(x)^+$ to denote function $\max\{x,0\}$ and $\doteq$ to denote exponential equality, i.e., $f(x) \doteq x^a$ means that $a = \lim (\log f(x)/\log x)$. And $\leq$ and $\geq$ carry similar meanings.

II. SYSTEM MODEL AND BACKGROUND

A. Protocol description

The cooperative broadcast protocol considered in this paper is illustrated in Fig. 1 where one single antenna source node (node 0) is broadcasting common messages to $K$ single antenna receive nodes (node 1 to node $K$). The nodes are assumed to be half-duplex, which means that they will start transmission only after they have decoded the message. The source node broadcasts a message in the first block $l=1$. If a node decode the message successfully (may not correctly), it will send an ACK (ACKnowledgment) message back and cooperatively broadcasts the message with the source node using suitable space-time code in the following blocks. If a node cannot decode the message, a NACK (Negative ACKnowledgment) message will be sent back. We refer to the transmission of a message as an ARQ process and the successive transmissions of coded versions of the same message as “ARQ rounds”. One ARQ process continues until all receive nodes can decode the message or the maximum number of ARQ rounds $l=L$ is reached. Then the source node will broadcast a new message in the next block. The ACK and NACK messages are assumed to be sent back without any error and delay and the radio resource occupied by these messages is ignored.

B. System and channel model

The channels between any two nodes $\{h_{ik}; i=0,...,K; k=1,...,K\}$ are i.i.d. (Independent Identically Distributed) zero-mean complex Gaussian random variables with unit variance and keep constant during one ARQ process. Without loss of generality, we also assume node $i$ decodes the message not later than node $k$ if $i < k$. Thus the received signal $y_{k,l}$ at the $k$-th receive node in the $l$-th block of an ARQ process can be expressed as:

$$y_{k,l} = \sum_{j=0}^{l-1} \sqrt{P} h_{ijk} x_{i,j} + n_{k,j}$$

where $x_{i,j}$ is the signal transmitted by the $i$-th node at $l$-th block, $n_{k,j}$ is the receive noise and $y_{k,l}$ and $n_{k,j}$ are all $T \times 1$ vectors, where $T$ is the block length. $\rho$ is the SNR (Signal to Noise Ratio), which is defined as $P_r/\sigma_n^2$, where $P_r$ is the transmit power of each node and $\sigma_n^2$ is the receive noise power. $\Gamma^{(l-1)}$ is the number of nodes that have successfully decoded the message until the $(l-1)$-th ARQ round. We further define $\Gamma^{(L)} = K - 1$, $\Gamma^{(0)} = 0$ and $\Gamma^{(-1)} = -1$, which will be used in the next section.

The decoder at each node is allowed to combine received signals over all the $l$ previous rounds. For the simplicity of presentation, after $l$ rounds, the overall received signal at the $k$-th node can be written as:

$$y_k^{(l)} = \sqrt{\rho} H_k^{(l)} x^{(l)} + n_k^{(l)}$$

where $x^{(l)} = \left[ x_{0,1}^T, x_{0,2}^T, x_{1,2}^T, ..., x_{l-1,2}^T, ..., x_{l-1,l-1}^T \right]^T$ ,

$$y_k^{(l)} = \left[ y_{k,1}^T, ..., y_{k,l}^T \right]^T$$

and $n_k^{(l)} = \left[ n_{k,1}^T, ..., n_{k,l}^T \right]^T$. The channel matrix $H_k^{(l)}$ is an $lT \times \sum_{j=0}^{l-1}(l-j)T$ matrix which is given by:

$$H_k^{(l)} = \begin{bmatrix} h_{0k} & h_{0k} & ... & h_{0k} & h_{1k} & ... & h_{1k} & ... & h_{l-k} & ... & h_{l-k} \\ h_{0k} & h_{0k} & ... & h_{0k} & h_{1k} & ... & h_{1k} & ... & h_{l-k} & ... & h_{l-k} \\ ... & ... & ... & ... & ... & ... & ... & ... & ... & ... & ... \\ h_{0k} & h_{0k} & ... & h_{0k} & h_{1k} & ... & h_{1k} & ... & h_{l-k} & ... & h_{l-k} \end{bmatrix} \otimes I_T$$

The channel state information is assumed to be only available at the receiver. So the capacity of the channel is
\[ C_k^{(i)}(\rho) = \log \det \left( I_T + \rho H_k^{(i)} H_k^{(i) H} \right) \]  

Let \( R(\rho) \) be the data rate per channel use. An outage is defined as the event that the capacity of the channel does not support a target data rate

\[ O_k^{(i)} = \{ H : C_k^{(i)}(\rho) < IT \cdot R(\rho) \} \]  

C. D-M and D-M-D tradeoff

A family of codes is said to achieve diversity gain of \( d \) and spatial multiplexing gain of \( r \) if

\[ r = \lim_{\rho \to \infty} \frac{R(\rho)}{\log \rho} \quad \text{and} \quad d = \lim_{\rho \to \infty} \frac{\log P_e(\rho)}{\log \rho} \]

where \( P_e(\rho) \) is the average error probability of the codes. The optimal D-M tradeoff yields the maximum diversity gain \( d \) for every value of \( r \), and is denoted as \( d(r) \). The major conclusion of [6] is that the optimal diversity gain of coherent block-fading MIMO channels with \( M \) transmit antennas, \( N \) receive antennas and multiplexing gain \( r \), is given by:

\[ d_{DM}(r) = f_{DM}(r) \]

where \( f_{DM}(r) \) is a piece-wise linear function joining the points \((k, (M - k)(N - k))\) for \( k = 0, \ldots, \min\{M, N\}\), given block length \( T \geq M + N - 1 \).

If ARQ is allowed, higher diversity gain can be achieved given the multiplexing gain, which is called ARQ diversity gain and is characterized by the following optimal D-M-D tradeoff [8]:

\[ d_{DM-D}(r) = f_{DM-D}(r/L) \]

for \( LT \geq M + N - 1 \) and \( 0 \leq r \leq \min\{M, N\}\), where \( L \) is the maximum ARQ rounds. Note that it is achieved given the channels are static during one ARQ process. However, in this situation, we need to redefine the multiplexing gain \( r \) and the average error probability \( P_e(\rho) \). Let \( A_l \) denote the event that an ACK is fed back at round \( l \) and \( R_l(\rho) = r_l \log \rho \) denote the data rate in the first block. Then the effective multiplexing gain is defined as

\[ r_e = \lim_{\rho \to \infty} \left( R_l(\rho) \left\{ 1 + \sum_{i=l}^{K} \Pr(\bar{A}_i, \ldots, \bar{A}_l) \right\} \log \rho \right) \]

Let \( E_i \) denote the decoding outcome is not correct with \( l \) received blocks. The overall average error probability is given by:

\[ P_e(\rho) = \sum_{l=1}^{L} \Pr(E_1, A_1, \ldots, A_{l-1}, A_l) + \Pr(E, A_1, \ldots, A_{L-1}) \]

which means that the error occurs either when the receiver decodes erroneously or when it cannot decode at the end of \( L \) ARQ rounds.

Furthermore, we will use the following facts from [11]. Let \( \alpha_{ik} \) denote the exponential order of \( 1/|h_{ik}|^2 \), i.e.,

\[ \alpha_{ik} = -\lim_{\rho \to \infty} \log \left( |h_{ik}|^2 \right) / \log(\rho) \]

We note that \( \alpha_{ik} \) is distributed as:

\[ p(\alpha_{ik}) = \lim_{\rho \to \infty} \rho^{-\alpha_{ik}} \exp(-\rho) \]

which, at high SNR region, becomes

\[ p(\alpha_{ik}) = \begin{cases} 0 & \text{for } \alpha_{ik} < 0 \\ \rho^{-\min\{\alpha_{i0} + \alpha_{i1} + \ldots + \alpha_{ik}\}} & \text{for } \alpha_{ik} \geq 0 \end{cases} \]

Thus, for i.i.d. random variables \( \{\alpha_{ik} ; i = 0, \ldots, K - 1; k = 1, \ldots, K\} \), the probability that they belong to a set \( O \) can be characterized by:

\[ \int_O \prod_{ik} p(\alpha_{ik}) \]  

III. TRADEOFF CURVES

In this section, we derive the optimal D-M-D tradeoff curve for ARQ cooperative broadcast channels.

Theorem: the optimal diversity gain of slow fading coherent broadcast channels with one source node, \( K \) receive nodes, effective multiplexing gain \( r_e \) and maximum ARQ rounds \( L \geq 2 \) is given by:

\[ d_{e^{(L)}(r_e)} = \begin{cases} K - \left( \frac{1}{L} + \frac{K - 1}{L - 1} \right) r_e & 0 \leq r_e \leq r^* \\ (2K - 1) - \frac{K(L - 1)}{L - 2} r_e & r^* < r_e \leq 1 \end{cases} \]

with the block length \( LT \geq K(L - 1) + 1 \) and

\[ r^* = \begin{cases} 1 & L \geq 3 \\ 1 & L = 2 \end{cases} \]

Proof: similar to (5), the overall error probability of cooperative broadcast channels can be written as:

\[ P_e(\rho) = \sum_{l=1}^{L} \Pr\left( \bigcup_{l=1}^{K} \{E_l^{(1)}, A_l^{(1)}, \ldots, A_l^{(1)}, A_l^{(1)}\} \right) \]

\[ + \Pr\left( \bigcup_{l=1}^{K} \{E_l^{(1)}, A_l^{(1)}, \ldots, A_l^{(1)}\} \right) \]

where \( E_k^{(1)} \) is the event that the \( k \)-th node makes a decoding error at the \( l \)-th round and \( A_l^{(1)} \) is the event that an ACK message is sent back by the \( k \)-th node at the \( l \)-th round. Similar to [8], by utilizing a proper decoder, we can prove that the first term in (9) is exponentially less than the second term, and thus is neglectable when \( \rho \to \infty \). For the details, please refer to the Appendix I in [8].

As a result, we only need to calculate the second term in (9), which can be expressed as

\[ \Pr\left( \bigcup_{l=1}^{K} \{E_l^{(1)}, A_l^{(1)}, \ldots, A_l^{(1)}\} \right) \]

\[ \Pr\left( \bigcup_{l=1}^{K} \{E_l^{(1)}, A_l^{(1)}, \ldots, A_l^{(1)}\} \right) \]
\[
\sum_{\Gamma^{(i)}} \cdots \sum_{\Gamma^{(i-1)}} \Pr\left( \varepsilon^{(L)}_{k}, A^{(L-1)}_{k}, \cdots, A^{(L-1)}_{k} \mid \Gamma^{(i)}, \ldots, \Gamma^{(i-1)} \right) \Pr\left( \Gamma^{(i-2)} \mid \Gamma^{(i)}, \ldots, \Gamma^{(i-1)} \right) \cdots \Pr\left( \Gamma^{(0)} \right).
\]

Using the results from Section III.C in [6], we have
\[
\Pr\left( \varepsilon^{(L)}_{k}, A^{(L-1)}_{k}, \cdots, A^{(L-1)}_{k} \mid \Gamma^{(i)}, \ldots, \Gamma^{(i-1)} \right) = \Pr\left( \varepsilon^{(L)}_{k} \mid \Gamma^{(i)}, \ldots, \Gamma^{(i-1)} \right)
\]
under the condition that \( LT \geq K(L-1)+1 \). Note that the outage event \( O^{(L)}_{k} \) is defined in (3) and here it is equal to
\[
O^{(L)}_{k} = \left\{ \alpha_{ok}, \alpha_{ik}, \ldots, \alpha_{(l-1)k} \mid \left( l-l \alpha_{ok} \right)^{+}, \left( l-l \alpha_{ok} - l(1-\alpha_{ok}) \right)^{+}, \ldots, \left( l-l \alpha_{ok} - (l-1)\alpha_{ok} \right)^{+}, \ldots, \left( l-l(1-\alpha_{ok}) \right)^{+} \right\} < r_{1},
\]
where \( r_{1} \) is the multiplexing gain in the first block as discussed in (4). Together with (6) and (7), we can get
\[
\Pr\left( O^{(L)}_{k} \mid \Gamma^{(i)}, \ldots, \Gamma^{(i-1)} \right) = \int_{\mathcal{O}^{(L)}_{k}} \rho^{\left\{ a_{ok}+a_{ik}+\cdots+a_{(l-1)k} \right\}} d\alpha_{ok} d\alpha_{ik} \ldots d\alpha_{(l-1)k} \]
\[
= \rho^{\left\{ a_{ok}+a_{ik}+\cdots+a_{(l-1)k} \right\}}.
\]

This is a linear optimization problem with linear constraints, so the optimal value is achieved at vertices of the region \( O^{(L)}_{k} \). Careful examination reveals that
\[
\Pr\left( O^{(L)}_{k} \mid \Gamma^{(i)}, \ldots, \Gamma^{(i-1)} \right) = \rho^{-f(l,r_{1})}
\]

Where
\[
f(l,r) = \min_{j\leq l} \left( \Gamma^{(i-k)} + 1 - \frac{\Gamma^{(i-j)} - \Gamma^{(i-j-1)}}{j} r \right).
\]

For the term \( \Pr\left( \Gamma^{(i)} \mid \Gamma^{(i)}, \ldots, \Gamma^{(i-1)} \right) \) in (10), it can be interpreted as the probability that, among \( K-1-l \) nodes that have not decoded the message until \( (l-1) \)-th round, \( \Gamma^{(i)} - \Gamma^{(i-1)} \) nodes successfully decode at the \( j \)-th round, while \( K-1-\Gamma^{(i)} \) still not. (Note that the \( K \)-th node is assumed to be the last one that decodes the message). So it is equal to
\[
\Pr\left( \Gamma^{(i)} \mid \Gamma^{(i)}, \ldots, \Gamma^{(i-1)} \right) = \left\{ \begin{array}{ll}
K-1-\Gamma^{(i-1)} & \text{if } j = i \\
\Gamma^{(i)} - \Gamma^{(i-1)} & \text{if } j > 1
\end{array} \right.
\]
\[
\left[ \Pr\left( A^{(i)}_{k} \mid \bar{A}^{(i)}_{k}, \ldots, \bar{A}^{(i-1)}_{k}, \Gamma^{(i)}, \ldots, \Gamma^{(i-1)} \right) \right]^{K-1-\Gamma^{(i)}}
\]
\[
\left[ \Pr\left( \bar{A}^{(i)}_{k} \mid A^{(i)}_{k}, \ldots, A^{(i-1)}_{k}, \Gamma^{(i)}, \ldots, \Gamma^{(i-1)} \right) \right]^{\Gamma^{(i)} - \Gamma^{(i-1)}}
\]

Following the same approach discussed in Appendix I in [8], we can get
\[
\Pr\left( \bar{A}^{(i)}_{k}, \ldots, \bar{A}^{(i)}_{k} \mid \Gamma^{(i)}, \ldots, \Gamma^{(i-1)} \right) = \Pr\left( O^{(i)}_{k} \mid \Gamma^{(i)}, \ldots, \Gamma^{(i-1)} \right) \pm \rho^{-f(l,r_{1})}
\]

Combine (9)-(14), we have
\[
d_{c}^{L}\left( L,r_{1} \right) = \min_{\Gamma^{(i)}, \ldots, \Gamma^{(i-1)}} \left\{ f(L,r_{1}) + \sum_{l=1}^{K-1} \left( \Gamma^{(i)} - \Gamma^{(i-1)} \right) f(l,r_{1}) \right\}
\]

where
\[
g(L,r_{1}) = \left( f(L,r_{1}) + \sum_{l=1}^{K-1} \left( \Gamma^{(i)} - \Gamma^{(i-1)} \right) f(l,r_{1}) \right)
\]

In the following, sometimes we will just use \( g\left( \Gamma^{(i)}, \ldots, \Gamma^{(i-1)} \right) \) to denote the event \( \left( \Gamma^{(i)}, \Gamma^{(i-1)}, \ldots, \Gamma^{(i-L)} \right) \).

Now we need to find out the value of \( \left( \Gamma^{(i)}, \Gamma^{(i-1)}, \ldots, \Gamma^{(i-L)} \right) \) that minimize (15). Note that given all the other variables fixed, \( g\left( \Gamma^{(i)}, \ldots, \Gamma^{(i-1)} \right) \) is a linear function of anyone of the variables \( \left( \Gamma^{(i)}, \Gamma^{(i-1)}, \ldots, \Gamma^{(i-L)} \right) \). As a result, the optimal value can only be achieved at the vertices and edges of the region
\[
\Omega = \left\{ \left( \Gamma^{(i)}, \Gamma^{(i-1)} \right) \mid 0 \leq \Gamma^{(i)} \leq \Gamma^{(i-1)} \leq \cdot \cdot \cdot \leq \Gamma^{(i-L-1)} \leq K-1 \right\}.
\]

There are \( L \) vertices in this region, which are denoted as
\[
v_{1} = (0,0,\ldots,0)
v_{2} = (0,0,\ldots,0,K-1)
\]
\[
\cdots
\]
\[
v_{L} = (K-1,K-1,\ldots,K-1).
\]

As a result, there are \( \left( L \atop 2 \right) \) edges. Let \( e_{ij} \) denote the edge that connects \( v_{i} \) and \( v_{j} \).

- If \( |j-i| = 1 \), \( g\left( e_{ij} \right) \) is just a linear function of \( \Gamma^{(i-1)} \). The minimum value is achieved at the vertices.
- If \( |j-i| > 1 \), \( g\left( e_{ij} \right) \) is a quadratic function of \( \Gamma^{(i-1)}, \ldots, \Gamma^{(i-j+1)} \) on edge \( e_{ij} \), where \( \Gamma^{(i-j+1)} = \Gamma^{(i-j+1)} + t \). Then it is not difficult to find out that \( g^{\prime\prime}(t) \leq 0 \), i.e., \( g(t) \) is concave function on
0 \leq t \leq K-1$. Consequently, the minimum value is also achieved at vertices.

As a result, we only need to compare the value of \( g(v_1), \ldots, g(v_L) \). By considering the case \( K \geq L \), \( L > K \geq L/2 \) and \( L/2 > K \), it is tedious but not difficult to show that the minimum value is always achieved by \( g(v_1) \) or \( g(v_2) \). Thus, we can obtain (8) in terms of \( r_1 \) from (15).

Finally, from the definition (4), it is apparent that \( r_e = r_1 \) with \( \rho \to \infty \). And this completes our proof.

IV. DISCUSSIONS AND COMPARISONS

In this section, we discuss the underlying meaning of the optimal D-M-D tradeoff curve obtained from the previous section and compare it with that of DDF cooperative broadcast protocol.

It can be observed from (8) that, when the multiplexing rate \( r_1 \) is small, the diversity gain is restricted by \( g(v_1) \), i.e., the case in which none of the nodes can decode the message for all ARQ rounds. The reason is straightforward. If there is one more node decoding the message, it will broadcast in the next round and thus provide \( 1 - r_e \) more diversity gain, which we refer as node diversity gain, to all the other nodes. In the case \( g(v_1) \), the occurrence of the outage event only requires the \( K \) channels to be in outage and is the least one in the region \( \Omega \). Note that \( g(v_1) \), the case in which all the other nodes except the last node (node \( K \)) decode the message at the first round, also only requires \( K \) channels to be in outage. However, it will provide more ARQ diversity gain which will be discussed in the following paragraph, so it is still dominated by the case \( g(v_1) \).

An interesting result in the theorem is that, when the multiplexing rate \( r_1 \) is large, the diversity gain is restricted by \( g(v_2) \), the case in which \( K-1 \) nodes decode successfully at the \((L-1)\)-th ARQ round. This phenomenon can be explained as follows. The ARQ cooperative broadcast protocol provides ARQ diversity gain in each round as the MIMO ARQ system [8]. When the multiplexing gain is large, this ARQ diversity gain might surpass the node diversity gain \( 1 - r_e \). In other words, from the system perspective, the chance that a node still cannot decode might be less than the chance that it decodes and provides an extra diversity gain \( 1 - r_e \) to all the other nodes. As a result, in the high multiplexing rate region, where the node diversity gain \( 1 - r_e \) is small, the probability that which point within region \( \Omega \) dominates should be carefully examined. And it is revealed that \( g(v_2) \) is the one that dominates. What is more, it is worth mentioning that the ARQ diversity gain is implemented by a form of incremental redundancy [16] and requires neither the nodes to transmit on a different frequency nor the channel to vary from blocks to blocks.

In Fig. 2, the D-M-D tradeoff curve is plotted for the \( L=3 \) and \( K=5 \) case and compared with the Dynamic Decode and Forward (DDF) cooperative broadcast protocol proposed in [11]. The DDF protocol assumes that all the transmission is accomplished within one block and no feedback from receiver to transmitter is allowed. As can be observed from the figure, the optimal diversity gain achieved by the ARQ cooperative broadcast protocol is much higher than that of DDF protocol at high multiplexing gain region, which means that allowing certain degrees of delay and feedback can significantly enhance the system performance.

D-M-D tradeoff curves achieved by different maximum ARQ rounds are illustrated in Fig. 3, where the number of receive nodes are fixed to be 5. Referring to the figure, the tradeoff curve becomes flatter as \( L \) increases. This means that larger maximum ARQ rounds can provide higher diversity gain without destroying multiplexing gain. Actually, as \( L \to \infty \), we can achieve full diversity gain and full multiplexing gain simultaneously. This can be explained by noticing that we are analyzing the high SNR region, where the occurrence of a NACK is rare and therefore, in most cases, the transmission succeeds in the first block. And the rare errors are corrected by the ARQ process, which increases the system diversity.

In Fig. 4, we show D-M-D tradeoff curves achieved by different numbers of receive nodes, with \( L=3 \). As can be anticipated, more nodes provide more diversity, especially in low multiplexing rate region. In this figure, the diversity gains at multiplexing gain \( r=1 \) are the same for different \( K \), but this is not true for all \( L \). Generally, when \( r=1 \), the diversity gain should also increase with \( L \).

Finally, our results are also the solutions to the general cooperative channels with arbitrary number of relays and ARQ rounds. In this sense, the result in [15] can be regarded as a special case with \( K=2 \). However, in the derivation for the \( L=3 \) case, it appears to have forgot to take the implicit condition \( 1 - \mu_{sd} \geq 0 \) (\( \mu_{sd} \) is defined in that paper) into the consideration.

V. CONCLUSIONS

Cooperative broadcast is likely to have a lot of applications in next generation wireless communication systems which motivate the investigations in this paper. We derive the optimal D-M-D tradeoff curves for ARQ cooperative broadcast channels. We show that the optimal diversity gain is restricted by the situation where none of the receive nodes can decode the message for all ARQ rounds at low multiplexing gain region, and by the situation where all except one receive node can decode the message right before the last ARQ round at high multiplexing gain region. Furthermore, our results are also compared with the previous results on D-M tradeoff in cooperative broadcast systems, which demonstrate the benefit of feedback and relaxed delay requirement. In future work, it would be interesting to consider the distributed space-time architecture that can achieve this optimal D-M-D tradeoff.
Figure 2. Comparison between DDF cooperative broadcast and ARQ cooperative broadcast.

Figure 3. D-M-D tradeoff curves achieved by 5 receive nodes and different numbers of maximum ARQ rounds.

Figure 4. D-M-D tradeoff curves achieved by maximum ARQ rounds equal to 3 and different numbers of receive nodes.

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