Symmetry and Fourier Descriptor: A Hybrid Feature For NURBS based B-Rep Models Retrieval

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Abstract
As the number of models in 3D databases grows, an efficient 3D models indexing mechanism and a similarity measure to ease model retrieval are necessary. In this paper, we present a query-by-model framework for NURBS based B-Rep models retrieval that combines partial symmetry of the object and the Fourier shape descriptor of canonical 2D projections of the 3D models. In fact, most objects are composed by similar parts up to an isometry. By detecting the dominant partial symmetry of a given NURBS based B-Rep model, we define two canonical planes from which the Fourier descriptors are extracted to measure the similarity among 3D models.

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1. Introduction
Parametric surfaces, in particular Non-Uniform Rational B-Spline (NURBS), provide a powerful tool for the academic and industrial communities concerned with the design and analysis of objects [DB99]. NURBS based Boundary Representations (NURBS based B-Reps) are industrial standards and are widely used in different domains such as molecular chemistry [BLMP97], 3D geographical information systems [CSM03] and mechanical components design [CH06]. With the explosion of 3D applications in numerous fields of science, the number of 3D models is growing rapidly. Reusing models can help users avoiding to “reinvent the wheel” and increasing the efficiency of industrial product development. Therefore, 3D repositories require algorithms and techniques that are efficient and robust in the back-up organization to index and search the 3D models.

Recently, sketch based approach has become a new trend in 3D shape retrieval where the contours of 3D shapes are extracted and converted into 2D images supplying sketched features to the indexing and searching process [LLJ*10, ERB*12]. This approach provides a friendly interface that allows the users to sketch out a simple 2D shape as a query. The significant success in these works attracted our interests. However, there exists two major limitations that affect the retrieval results. First, they do not define a canonical view that can best describe the overall shapes of similar objects. That is, 2D images are generated from multiple views in [LLJ*10] or are chosen from a set of potential views in [ERB*12]. It does not ensure that the views taken from the similar 3D objects are the same. This can limit the accuracy of the retrieval results in a free sketching context. Second, these algorithms depend on the quality of the representation that it uses to generate contours. Poor models pose a significant problem for contours extraction techniques.

Generally, similarities within a 3D model is a common phenomenon both in natural and in synthetic objects [MPWC12]. Many objects are composed by similar parts up
to a rotation, a translation or a symmetry. Geometric redundancy is an essential property that 3D designers commonly use in their conceptions. Therefore, the similarity detection within a 3D model can be an important preprocessing step to extract model features that are used to compare the similarity among various models in a database. To clarify this idea, consider now the three NURBS based B-Rep models in Figure 1, we show models in two categories: instrument and car. In each sub-figure, the real model is displayed at the top and the contour of its corresponding projection on a global symmetric plane is at the bottom. We can see that the projection on the symmetric plane is a canonical 2D representation that can help us distinguishing shapes between different models. In fact, by applying the D1 distance proposed by Osada et al. [OFCD02], we estimate the shape distribution of each model by evaluating the distances between the samples on its contour to its barycenter. The histograms in Figure 2 shows that the two car models have a similar shape and have a quite different one compared to the guitar.

From this idea, we propose a feature that combines the symmetric property and a 2D shape descriptor of NURBS based B-Rep models. Also, we propose a query-by-model retrieval framework that can be divided into two phases. First, for each of available models, a preprocessing phase estimates its features by detecting its dominant partial symmetry and evaluates the shape descriptors of the silhouettes extracted from the canonical 2D representations. These features are then stored together with the corresponding models. Second, when a model is queried to find other models similar to its shape, it is evaluated the shape descriptor based on its partial symmetry. This shape descriptor is compared to all available shape descriptors to retrieve the desired models. In the context of our work, we suppose that all available 3D models in our database are closed form and have a dominant partial symmetry.

2. Related work

Conceptually, a typical 3D shape retrieval framework consists of a database with an index structure created offline and an online query engine [TV08]. It requires a definition of feature representing the 3D models to ease the similarity comparison between models. In the last decade, several techniques proposed different definitions on 3D shape features; they can be classified in the following approaches: feature based, graph based and geometry based [IIL∗10, BKS∗05, TV08]. For B-Rep models, the nature of their geometrical structures is a good starting point for direct matching. In an early work, El-Mehalawi et al. [EMMO3] proposed an attributed graph approach for retrieving similar designs in a database of mechanical components. In this graph, the nodes represent model surfaces and the links connecting nodes represent the common edge of the corresponding surfaces. Therefore, the model matching problem is reduced to a graph comparison problem. However, graph comparison is NP complete: the comparison becomes costly and inefficient when the number of available models grows or the B-Rep models structures are large and complicated. Chu et al. [CH06] proposed a search scheme which considers form-feature, topological and geometric information. They apply a characteristic that combines the topology graph and the shape distribution to measure the similarity among 3D mechanical components. This scheme seems not to be suitable for our work because the B-Rep objects in a model do not have adjacency information (the structure of these models will be presented in section 3.1). Li et al. [LST∗12] propose a combination of the components annotations and the rotational symmetry within B-Rep CAD mechanical models to support partial retrieval. This is a good reference as they use similarity information of a B-Rep model to enhance the search. But our models in this work are more general than their models: they only consider models composed by primitive surfaces.

Symmetry detection for 3D meshes is a well known subject with several approaches as geometric hashing, transformation space voting, planar reflective symmetry transform, graph based [MPWC12]. Recently, Cuillére et al. [CFS∗11] have presented a method to detect the similarity and the dissimilarity between NURBS based B-Rep models based on the inertia tensor and the control points net of NURBS surfaces. But they do not estimate the transformation between similar surfaces and they consider only the underlying surface but not the edges of B-Rep object. Li [LI11] also proposes an algorithm to detect the symmetry between primitive surfaces within a B-Rep model. In this paper, we propose an algorithm to identify partial symmetries within a NURBS based B-Rep model. This algorithm is independent from the parameterization of underlying NURBS surfaces. Using the dominant partial symmetry and the shape descriptor proposed in [LLJ∗10, ERB∗12], we derive a hybrid feature to support the shape retrieval.

This paper is organized as follow: section 3 introduces our algorithm for the symmetry detection within a NURBS based B-Rep model, section 4 shows our 3D retrieval framework, we demonstrate our retrieval approach with some experiments in section 5, we finish with the conclusion and some futur works in section 6.
3. Symmetry Detection

In this section, we present our algorithm for detecting the symmetry within a NURBS based B-Rep model. Our algorithm does not only identify automatically repeated faces in the model but also estimates the isometry between them. That is, if two patches $P_i^j, P_j^i$ are similar, there exists an isometry $T_i^j$ so that:

$$ P_j^i = T_i^j(P_i^j). \quad (1) $$

We consider isometries as compositions of the canonical isometries, i.e. rotations, translations and reflections. They may be direct or indirect. Also, depending on the nature of their linear parts, we characterize the plane symmetries among isometries.

3.1. Structure of NURBS based B-Rep Models

B-Rep (Boundary Representation) is a method for describing the 3D objects by representing each object as a composition of two parts: geometrical and topological entities. Geometrical entities are points, curves and surfaces. These entities are abstracted into topological entities as vertices, edges and faces. Vertices are simply 3D points lying on the surfaces, an edge is a piece of curve bounded by two vertices, and a face is a portion of a surface bounded by multiple edges. In addition, there are two other topological entities: loops and trims. Depending on the loop type (i.e., outer or inner), a loop defines the way to keep or to cut off bounded portions within a surface. A loop is also composed of multiple trims that are directly linked to the edges and which forms a closed boundary.

In our work, a B-Rep object consists of multiple faces and a B-Rep model is composed of multiple B-Rep objects. For instance, the plane model in Figure 3 is composed of 3 B-Rep objects plotted in different colors (figure 3a). Similarly, the plane tail B-Rep object (figure 3b) contains multiple faces. The face in figure 3c is bounded by edges (green) and vertices (red). In addition, the underlying surfaces of B-Rep faces are NURBS surfaces. By definition, a tensor product NURBS surface $S$ of bi-degree $(p,q)$ associated to two knots vectors $u = \{u_0, \ldots, u_n\}$ and $v = \{v_0, \ldots, v_m\}$ and a set of control points $C = \{P_{ij}\}$ $i \in [0, n-p], j \in [0, m-q]$ weighted by $w_{ij} \in \mathbb{R}$, is defined by the following equation:

$$ S(u,v) = \frac{\sum_{i=0}^{n-q} \sum_{j=0}^{m-p} N_i,p(u)N_j,q(v)w_{ij}P_{ij}}{\sum_{i=0}^{n-q} \sum_{j=0}^{m-p} N_i,p(u)N_j,q(v)w_{ij}}. \quad (2) $$

In other words, a B-Rep model contains numerous faces. For detecting symmetry in a model, we first seek symmetric faces within this model. We take advantage of the loop closed form within B-Rep faces, we propose a method for detecting similarity between faces based on the vertices that bound these faces. Moreover, loops in a face must not intersect each others to maintain the validity of the trimmed face. Generally, a face has an outer loop that may contain embedded inner loops. In this work, we only take into account vertices lying on the outer loop of the B-Rep faces.

To ease the notation for the next sections, we denote $M = \{F_i\}, i \in [0,n_F]$, the NURBS based B-Rep model that composed of $n_F$ faces. Each face is defined by $F_i = \{S_i^i, V_i^i\}$, where $S_i$ is the NURBS surface used by this face and $V_i = \{v_{i0}, \ldots, v_{in}\}$ is the set of $n$ vertices that bound $F_i$. We call the vertices $V_i$ the corners of face $F_i$.

3.2. Algorithm overview

Before entering into the algorithm details, let us introduce some definitions based on the entities of B-Rep objects. Let $F_i = \{S_i^i, V_i^i\}$ and $F_i = \{S_i^i, V_i^i\}$ two separated faces with the same number of corners (we release this constraint in 3.2.1).

Definition 3.1 $F_i^i$ and $F_i^j$ are said topologically similar up to a transformation $T_i^j$ iff $T_i^j$ maps $V_i^i$ to $V_i^j$:

$$ V_i^j = T_i^j(V_i^i). \quad (3) $$

Definition 3.2 $F_i^i$ and $F_i^j$ are said geometrically similar up to a transformation $T_i^j$ iff $T_i^j$ maps $S_i^i$ to $S_i^j$:

$$ S_i^j = T_i^j(S_i^i). \quad (4) $$

Note that, if $F_i^j$ and $F_i^j$ are topologically similar, we cannot conclude that they are geometrically similar. Similarly, they can be geometrically similar without being topologically similar.

Definition 3.3 $F_i^i$ and $F_i^j$ are said similar up to a transformation $T_i^j$ iff they are topological and geometrically similar up to this transformation.

These definitions are the key idea for the overall of our algorithm. Given a B-Rep model $M = \{F_i\}, i \in [0,n_F]$, the algorithm tries to get as more similar faces pairs as possible. It is divided in several steps:

Step 1. Match couples of faces that are topologically similar, i.e. there exists an isometry between the corners of two faces. Let $\Lambda = \{P_k\}$ be the set of pairs of similar faces where $P_k = \{F_k^i, F_k^j\}$ (section 3.2.1).