Fuzzy Sliding-Mode Controllers with Applications

Q. P. Ha, Member, IEEE, Q. H. Nguyen, D. C. Rye, Member, IEEE, and H. F. Durrant-Whyte

Abstract—This paper concerns the design of robust control systems using sliding-mode control that incorporates a fuzzy tuning technique. The control law superposes equivalent control, switching control, and fuzzy control. An equivalent control law is first designed using pole placement. Switching control is then added to guarantee that the state reaches the sliding mode in the presence of parameter and disturbance uncertainties. Fuzzy tuning schemes are employed to improve control performance and to reduce chattering in the sliding mode. The practical application of fuzzy logic is proposed here as a computational-intelligence approach to engineering problems associated with sliding-mode controllers. The proposed method can have a number of industrial applications including the joint control of a hydraulically actuated mini-excavator as presented in this paper. The control hardware is described together with simulated and experimental results. High performance and attenuated chatter are achieved. The results obtained verify the validity of the proposed control approach to dynamic systems characterized by severe uncertainties.

Index Terms—Electrohydraulic servo, fuzzy tuning, robotic excavation, robustness, sliding-mode control.

I. INTRODUCTION

VARIABLE-STRUCTURE systems with a sliding mode were discussed first in the Soviet literature [1], [2], and have been widely developed in recent years. Comprehensive surveys of variable-structure control can be found in [3], [4]. The salient advantage of sliding-mode control (SMC) derives from the property of robustness to structured and unstructured uncertainties once the system enters the sliding mode. Note, however, that system robustness is not assured until the sliding mode is reached. The main drawback of SMC is “chattering” which can excite undesirable high-frequency dynamics. Moreover, problems relating to simplification of design procedures, control performance enhancement in the reaching mode, and chattering alleviation remain to be explored fully [5]. Several methods of chattering reduction have been reported. One approach [6] places a boundary layer around the switching surface such that the relay control is replaced by a saturation function. Another method [7] replaces a max–min-type control by a unit vector function. These approaches, however, provide no guarantee of convergence to the sliding mode and involve a tradeoff between chattering and robustness. Continuous SMC, as proposed in [8], can exponentially drive the system state to a chattering-free sliding mode but tends to produce conservative designs. Reduced chattering may be achieved without sacrificing robust performance by combining the attractive features of fuzzy control with SMC [9]–[11]. Fuzzy logic, first proposed by Zadeh [12], has proven to be a potent tool for controlling ill-defined or parameter-variant plants. By encapsulating heuristic engineering rules a fuzzy logic controller can cope well with severe uncertainties, although a heavy computational burden may arise with some implementations. Fuzzy schemes with explicit expressions for tuning can avoid this problem [13].

The control methodology proposed here is a computational-intelligence approach to some of the engineering problems associated with sliding-mode controllers. This paper discusses the design of a sliding-mode controller incorporating fuzzy tuning techniques to achieve reduced chatter and system robustness against parameter uncertainty, load disturbance, and nonlinearities. The proposed method can be applied to a number of industrial applications. In this paper, the method is applied to the control of a hydraulically actuated mini-excavator. Simulated and experimental results obtained verify the validity of the proposed method in terms of smooth sliding control and robust performance in the presence of parameter and load variations, and of nonlinearities.

II. CONTROL DESIGN

A. Problem Formulation

Consider a linear system of the form

$$\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) + w(x, t)$$

(1)

where \(x(t) \in R^n\) is the state vector, \(u(t) \in R^m\) is the control vector, and \(w(x, t) \in R^q\) is the vector of disturbances and unmodeled dynamics; \(A \in R^{n \times n}\) and \(B \in R^{n \times m}\) are the nominal system constant matrices with \(\text{rank}(B) = m < n\); and \(\Delta A\) and \(\Delta B\) are uncertainties. The following assumptions are made.

A1) The uncertainties are continuous matrix functions of a vector of uncertain parameters \(p \in P \subset R^q\)

$$\Delta A = \Delta A(p) \quad \Delta B = \Delta B(p).$$

(2)

A2) There exist matrices \(D(p) \in R^{m \times n}, E(p) \in R^{m \times m}\), and vector \(v(x, t) \in R^n\) such that the following matching conditions are satisfied:

$$\Delta A = BD(p) \quad \text{and} \quad \max_{1 \leq i \leq m} |D_{ij}(p)| \leq \delta_i,$$

$$i = 1, 2, \ldots, n \quad \forall p \in P \quad (3a)$$

$$\Delta B = BE(p) \quad \text{and} \quad E(p) = \text{diag}(E_{ij}), \quad \max_{1 \leq i, j \leq m} |E_{ij}| \leq \varepsilon < 1 \quad \forall p \in P \quad (3b)$$

Manuscript received August 17, 1998; revised October 2, 2000. Abstract published on the Internet November 15, 2000. This work was supported by the Australian Research Council, NS Komatsu Pty. Ltd., and the Co-operative Research Centre for Mining Technology and Equipment. The work of Q. P. Ha was supported by a research grant from The University of Technology, Sydney, Australia.

Q. P. Ha was with the Australian Centre for Field Robotics, The University of Sydney, Sydney, NSW 2006, Australia. He is now with the Faculty of Engineering, The University of Technology, Sydney, NSW 2007, Australia.

Q. H. Nguyen, D. C. Rye, and H. F. Durrant-Whyte are with the Australian Centre for Field Robotics, The University of Sydney, Sydney, NSW 2006, Australia.

Publisher Item Identifier S 0278-0046(01)01117-0.
\[ w(x, t) = Bv(x, t) \quad \text{and} \quad |v_j(x, t)| \leq v_j \quad \forall x, \forall t, \quad j = 1, 2, \ldots, m. \tag{3c} \]

Remark 1: The uncertainties \( \Delta A \) and \( \Delta B \) represent perturbations in system parameters and disturbances that are unknown but bounded. The constraint imposed on \( \varepsilon \) is to ensure that the level of the uncertainty \( \Delta B \) is not so large that the direction of the control action can be reversed.

The objective is to design a controller that provides robust performance in the presence of uncertainties (2) within given bounds \( \delta_j (j = 1, 2, \ldots, n) \), and \( v_j (j = 1, 2, \ldots, m) \). A switching function of dimension \( m \) is to be chosen

\[ s(x) = Cx = [s_1(x), s_2(x), \ldots, s_m(x)]^T \tag{4} \]

where \( C = [c_1^T, c_2^T, \ldots, c_m^T]^T \) is an \( m \times n \) constant matrix and \( s_j(x) \) describes a constituent linear surface

\[ S_j = \{ x \mid s_j(x) = c_j^T x = 0 \}, \quad j = 1, 2, \ldots, m. \tag{5} \]

The eventual switching surface, \( S \), of dimension \( (n - m) \) is the intersection [4] of all component surfaces (5)

\[ S = \{ x \mid s(x) = Cx = 0 \} = S_1 \cap S_2 \cap \cdots \cap S_m. \tag{6} \]

The sliding mode is the constrained motion of states along trajectories on the sliding surface \( S \). A sliding-mode controller must ensure that for any initial state \( x_0 = x(t_0) \) the state \( x(t) \) first reaches \( S \), in finite time \( t_r \), and then remains on \( S \) for all \( t > t_r > t_0 \).

B. Equivalent Control

A necessary condition for the state trajectory to remain on the sliding surface \( S \) is \( \delta(x) = 0 \). For the nominal system where \( \Delta A = 0, \Delta B = 0 \) and \( w(x, t) = 0 \), the state trajectory remains on the switching surface \( S \) if

\[ C(Ax + Bu_E) = 0 \tag{7} \]

where \( u_E(t) \) is the equivalent control input vector. Linear feedback is proposed for \( u_E(t) \) to assign the desired dynamics to the closed-loop system

\[ u(t) = -K_Ex(t) \tag{8} \]

where the equivalent control gain \( K_E \) can be obtained from a pole-placement technique. By substituting (8) into (7), \( K_E \) may be expressed as

\[ K_E = (CB)^{-1}CA \tag{9} \]

where the existence of the inverse of \( CB \) is a necessary condition for the existence of the sliding surface. From (9), the matrix \( C \) of the switching function (4) can be obtained as a solution to the following equation:

\[ (A - BK_E)^T C^T = 0. \tag{10} \]

Remark 2: Note that the closed-loop system eigenstructure of the matrix \( A - BK_E \) includes \( m \) eigenvalues at the origin of the complex plane generated from the conditions \( s_j(x) = c_j^T x = 0, j = 1, 2, \ldots, m \). A solution of (10) is not unique since there are only \( n - m \) independent equations with \( m \times n \) variables for the entries of \( C \).

Remark 3: In the single-input case \( m = 1 \), \( C^T = [c_1^T, c_2^T, \ldots, c_n^T]^T \) can be calculated with \( c_n \) chosen to be unity and \( k_E^T \) obtained from assigning \( n - 1 \) sliding eigenvalues and one eigenvalue at the origin of the complex plane [13].

C. Switching Control

If the initial state \( x_0 \) is not on the sliding surface \( S \), or there is a deviation of the representative point from \( S \) due to parameter variations and/or disturbances, the controller must be designed such that it can drive the system state \( x(t) \) to the sliding mode \( s(x) = 0 \). The system trajectory under the condition that the state will move toward and reach the sliding surface is called the reaching mode or reaching phase. By choosing the Lyapunov function candidate

\[ V(x, t) = 0.5s^T s \tag{11} \]

a reaching condition is given by

\[ \dot{V}(x, t) = s^T \dot{s} < 0, \quad s \neq 0. \tag{12} \]

To satisfy the reaching condition, \( u_E(t) \) is augmented by a switching control term, \( u_S(t) \in R^m \), to be determined.

Theorem 1: Consider the linear system (1) with uncertainties satisfying assumptions A1) and A2). If the sliding-mode controller is designed such that

\[ u = u_E + u_S = -K_E x + u_S \tag{13} \]

with the equivalent gain \( K_E \) given in (9), and the switching control vector \( u_S \) whose entries are given by

\[ u_{S,j} = \left[ \frac{v_j}{1 - \varepsilon} + \sum_{i=1}^{n} \delta_i [K_{E,j}][x_i] \right] \text{sgn}(\varphi_j(x)), \quad j = 1, 2, \ldots, m \tag{14} \]

where \( K_{E,j} \) is the element of the \( j \)th row and \( i \)th column of \( K_E \)

\[ \varphi_j(x) = s^T C b_j \tag{15} \]

and \( b_j \in R^n \) is the \( j \)th column of \( B \), then the state vector \( x(t) \) asymptotically converges to zero.

Proof: Let us define the positive-definite Lyapunov function (11).

From (1), (3), (7), and (13), we have

\[ \dot{V} = s^T \dot{s} = s^T C[(A + \Delta A)x + (B + \Delta B) \quad \text{with the equivalent gain } K_E \text{ given in (9), and the switching control vector } u_S \text{ whose entries are given by} \]

\[ u_{S,j} = \left[ \frac{v_j}{1 - \varepsilon} + \sum_{i=1}^{n} \delta_i [K_{E,j}][x_i] \right] \text{sgn}(\varphi_j(x)), \quad j = 1, 2, \ldots, m \tag{14} \]

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\[ u_{S,j} = \left[ \frac{v_j}{1 - \varepsilon} + \sum_{i=1}^{n} \delta_i [K_{E,j}][x_i] \right] \text{sgn}(\varphi_j(x)), \quad j = 1, 2, \ldots, m \tag{14} \]

where \( K_{E,j} \) is the element of the \( j \)th row and \( i \)th column of \( K_E \)

\[ \varphi_j(x) = s^T C b_j \tag{15} \]

and \( b_j \in R^n \) is the \( j \)th column of \( B \), then the state vector \( x(t) \) asymptotically converges to zero.
parameter vector $\mathbf{p} \in \mathcal{P}$. Consider first the case $\varphi_j(\mathbf{x}) > 0$. From the matching condition, we have

$$\sum_{i=1}^{n}(D_{ji} - E_{ji}K_{E,ji})x_i + (1 + E_{ji})u_{S,j} + v_j < 0$$

$$\forall \mathbf{p} \in \mathcal{P}, \quad j = 1,2,\ldots,m$$

if

$$u_{S,j} < \frac{-v_j + \sum_{i=1}^{n}(D_{ji} - E_{ji}K_{E,ji})x_i}{1 + E_{ji}}$$

$$\forall \mathbf{p} \in \mathcal{P}, \quad j = 1,2,\ldots,m.$$ 

Thus, in view of the constraints given in (3), for $\varphi_j(\mathbf{x}) > 0$, (12) will hold if we choose

$$u_{S,j} = \frac{-v_j + \sum_{i=1}^{n}(D_{ji} - E_{ji}K_{E,ji})x_i}{1 + E_{ji}}$$

Similarly, when $\varphi_j(\mathbf{x}) < 0$, we obtain

$$\sum_{i=1}^{n}(D_{ji} - E_{ji}K_{E,ji})x_i + (1 + E_{ji})u_{S,j} + v_j > 0$$

$$\forall \mathbf{p} \in \mathcal{P}, \quad j = 1,2,\ldots,m$$

if

$$u_{S,j} > \frac{-v_j + \sum_{i=1}^{n}(D_{ji} - E_{ji}K_{E,ji})x_i}{1 + E_{ji}}$$

$$\forall \mathbf{p} \in \mathcal{P}, \quad j = 1,2,\ldots,m.$$ 

In this case, (12) will hold if we choose

$$u_{S,j} = \frac{-v_j + \sum_{i=1}^{n}(D_{ji} - E_{ji}K_{E,ji})x_i}{1 + E_{ji}}$$

In summary, $u_{S,j}$ can be written in the form of (14). This concludes the proof.

Remark 4: If the plant parameters are known, then $\delta \approx 0$ and $\varepsilon \approx 0$, and the switching control signal can be decreased to

$$u_{S,j} = -v_j \text{sgn}(\varphi_j(\mathbf{x})), \quad j = 1,2,\ldots,m$$

Remark 5: If the influence of external disturbances is negligible ($v_j = 0$) then the switching control signal can be chosen as

$$u_{S,j} = -\sum_{i=1}^{n}\frac{\delta_i + \varepsilon|K_{E,ji}|}{1 - \varepsilon}x_i \text{sgn}(\varphi_j(\mathbf{x})x_i),$$

$$j = 1,2,\ldots,m$$

or in the form

$$u_S(t) = -K_Sx(t)$$

where the switching gain $K_S$ is calculated by

$$K_{S,j} = \frac{\delta_j + \varepsilon|K_{E,ji}|}{1 - \varepsilon} \text{sgn}(\varphi_j(\mathbf{x})x_i),$$

In the single-input case ($m = 1$), (19) becomes

$$k_{S,j} = \frac{\delta_j + \varepsilon|K_{E,ji}|}{1 - \varepsilon} \text{sgn}(sc^Tbx_i)$$

the result given in [13].

D. Fuzzy Control

An additional control signal $u_F$ is introduced to accelerate the reaching phase and to reduce chattering while maintaining sliding behavior. The fuzzy control term $u_{F,j}$ is proposed to have the following form:

$$u_{F,j} = -\gamma_j\varphi_j(\mathbf{x}), \quad j = 1,2,\ldots,m$$

where $\gamma_j$ is a weighting factor. The component $u_{F,j}$ will continuously be adjusted by the use of fuzzy logic, depending on both $s_j$ in (4) and $\delta_j$ or the change of $s_j$ and $\Delta s_j$ [14]. From empirical knowledge of the design of sliding-mode controllers, a large switching gain will force the state trajectories to approach the sliding surface $s_j = 0$ rapidly, but at the same time, tend to excite chattering. Thus, when the state trajectories are far from the sliding surface, that is, when the value of $|s_j|$ is large, the switching gain should be correspondingly increased and vice versa. Furthermore, when the state trajectories deviate from the sliding surface ($s_j,\Delta s_j > 0$), if $|\Delta s_j|$ is large the switching gain should be increased to force the trajectories back, and vice versa. When the state trajectories are approaching the sliding surface ($s_j,\Delta s_j < 0$), if $|\Delta s_j|$ is large the switching gain should be decreased in order to reduce chattering, and vice versa. Using fuzzy labels large and small for $|s_j|$ and $|\Delta s_j|$, the membership functions

$$\mu_{s,j,\text{large}} = 1 - \exp\left(-\frac{|s_j|}{\sigma_{s,j}}\right)$$

$$\mu_{s,j,\text{small}} = \exp\left(-\frac{|s_j|}{\sigma_{s,j}}\right)$$

$$\mu_{\Delta s,j,\text{large}} = 1 - \exp\left(-\frac{|\Delta s_j|}{\sigma_{\Delta s,j}}\right)$$

$$\mu_{\Delta s,j,\text{small}} = \exp\left(-\frac{|\Delta s_j|}{\sigma_{\Delta s,j}}\right)$$

can be defined. Here, $\sigma_{s,j}$ and $\sigma_{\Delta s,j}$ are some positive constants.

The following fuzzy rules are proposed based on this reasoning:

1) if $|s_j|$ is large, then $\gamma_j$ is large;
2) if $|s_j|$ is small, then $\gamma_j$ is small;
3) if $s_j,\Delta s_j > 0$ and $|\Delta s_j|$ is large, then $\gamma_j$ is large;
4) if $s_j,\Delta s_j > 0$ and $|\Delta s_j|$ is small, then $\gamma_j$ is small;
5) if $s_j,\Delta s_j < 0$ and $|\Delta s_j|$ is large, then $\gamma_j$ is small;
6) if $s_j,\Delta s_j < 0$ and $|\Delta s_j|$ is small, then $\gamma_j$ is large.

Singletons are defined as membership functions for $\gamma_j$

$$\mu_{\gamma,j,\text{small}} = \begin{cases} 1, & \gamma_j = 0 \\ 0, & \gamma_j \neq 0 \end{cases}$$

$$\mu_{\gamma,j,\text{large}} = \begin{cases} 1, & \gamma_j = \gamma_m,j \\ 0, & \gamma_j \neq \gamma_m,j \end{cases}$$

(23)
where $\gamma_{m,j}$ is some positive scaling factor. Using the max–min defuzzification method for the fuzzy schemes above we obtain [13]

$$
\gamma_j = \begin{cases} 
\gamma_{m,j} \left(1 - \exp \left(-\frac{|s_j|}{\sigma_{s_j}}\right)\right) \left(1 - \exp \left(-\frac{|\Delta s_j|}{\sigma_{\Delta s_j}}\right)\right), & s_j \Delta s_j > 0 \\
\gamma_{m,j} \left(1 - \exp \left(-\frac{|s_j|}{\sigma_{s_j}}\right)\right) \exp \left(-\frac{|\Delta s_j|}{\sigma_{\Delta s_j}}\right), & s_j \Delta s_j < 0
\end{cases}
$$

(24)

where $\gamma_{m,j}$, $\sigma_{s_j}$, and $\sigma_{\Delta s_j}$ are tuning parameters.

**Remark 6:** If the change of $\Delta s_j$ is negligible, employing only tuning schemes 1) and 2) results in a simple expression for $\gamma_j$:

$$
\gamma_j = \gamma_{m,j} \left(1 - \exp \left(-\frac{|s_j|}{\sigma_{s_j}}\right)\right).
$$

(25)

Note that by the expressions (24) and (25), the $j$th control action $u_j$ is tuned according to the corresponding sliding function $s_j$. In some cases, $s_j$ may approach zero while the representative point is still far from the sliding surface because $s_j \neq 0$ are sufficiently large. The fuzzy control $u_{F,j}$ may be more efficient if it is tuned according to $\varphi_j = s^T C \mathbf{D} \mathbf{j}$ as defined in (15). Neglecting the change of $\Delta s_j$, the following rule base is proposed:

7) if $\varphi_j$ is positive large, then $u_{F,j}$ is negative large;  
8) if $\varphi_j$ is positive small, then $u_{F,j}$ is negative small;  
9) if $\varphi_j$ is negative large, then $u_{F,j}$ is positive large;  
10) if $\varphi_j$ is negative small, then $u_{F,j}$ is positive small.

Consider the case $\varphi_j > 0$. Choosing sigmoidal membership functions for $\varphi_j$

$$
\mu_{\varphi_j,\text{large}} = \tanh \left(\frac{\varphi_j}{\sigma_{\varphi_j}}\right), \quad \mu_{\varphi_j,\text{small}} = 1 - \mu_{\varphi_j,\text{large}}
$$

singleton for $u_{F,j}$

$$
\mu_{u_{F,j},\text{large}} = \begin{cases} 
1, & u_{F,j} = -u_{F,m,j} \\
0, & u_{F,j} \neq -u_{F,m,j}
\end{cases}
$$

$$
\mu_{u_{F,j},\text{small}} = \begin{cases} 
1, & u_{F,j} = 0 \\
0, & u_{F,j} \neq 0
\end{cases}
$$

and using rules 7) and 8), we obtain

$$
u_{F,j} = \frac{\mu_{\varphi_j,\text{large}} (-u_{F,m,j}) - \mu_{\varphi_j,\text{small}}(0)}{\mu_{\varphi_j,\text{large}} + \mu_{\varphi_j,\text{small}}} = -u_{F,m,j} \tanh \left(\frac{\varphi_j}{\sigma_{\varphi_j}}\right), \quad \varphi_j > 0
$$

(26)

where $u_{F,m,j}$ and $\sigma_{\varphi_j}$ are some positive constants. Similarly, when $\varphi_j < 0$, we can obtain the same expression for tuning $u_{F,j}$ as in (26). In summary, the fuzzy control $u_{F,j}$ can be written as follows:

$$
u_{F,j} = -u_{F,m,j} \tanh \left(\frac{\varphi_j}{\sigma_{\varphi_j}}\right).
$$

(27)

**Theorem 2:** Consider the linear system (1) with uncertainties satisfying the assumptions A1) and A2). If the robust sliding-mode controller is designed such that

$$
u = u_E + u_S + u_F
$$

(28)

with the equivalent control $u_E$ given in (8), the robust control vector $u_S$ given in (14), and the fuzzy control $u_F$ defined by (21) where $\gamma_j$ is given by (24) or (25), or defined by (27) and where $\varphi_j$ is given by (15); then, the state vector $\mathbf{x}(t)$ asymptotically converges to zero.

**Proof:** With the Lyapunov function candidate (11) and the control law (28) the first time derivative $\dot{V}$ is now obtained as

$$
\dot{V} = s^T CB\left[(D - EK_F)x + (I_m + E)(u_S + u_F) + v\right]
$$

$$
= \sum_{j=1}^{m} \varphi_j [f_j^T x + (1 + E_{jj})(u_{S,j} + u_{F,j}) + v_j].
$$

For $\varphi_j \neq 0$, condition (12) will hold if each term of the sum above is negative for any parameter vector $p \in P$ or if

$$
\varphi_j [f_j^T x + (1 + E_{jj})(u_{S,j} + u_{F,j}) + v_j] < 0 \quad \forall p \in P, \quad j = 1, 2, \ldots, m.
$$

If the fuzzy control is given by (21), the preceding expression becomes

$$
[\varphi_j [f_j^T x + (1 + E_{jj})(u_{S,j} + u_{F,j}) + v_j]] - (1 + E_{jj})\gamma_j \varphi_j^2
$$

which is negative $\forall p \in P, \quad j = 1, 2, \ldots, m$, because the term in square brackets is negative (by Theorem 1), $1 + E_{jj} > 0$ (by condition (3b)), and $\gamma_j > 0$ (according to (24) or (25)). If the fuzzy control is given by (27), then we also have

$$
[\varphi_j [f_j^T x + (1 + E_{jj})(u_{S,j} + u_{F,j})] - (1 + E_{jj})\varphi_j u_{F,m,j}]
$$

$$
\cdot \tanh \left(\frac{\varphi_j}{\sigma_{\varphi_j}}\right) < 0 \quad \forall p \in P, \quad j = 1, 2, \ldots, m
$$

because $\varphi_j \tanh(\varphi_j/\sigma_{\varphi_j}) > 0$ for $\varphi_j \neq 0$. Thus, $\dot{V} < 0$. This concludes the proof.

**Remark 7:** Close consideration of the fuzzy control (21) with respect to the tuning components (24) or (25) reveals that the fuzzy control magnitude increases with an increase of the sliding function magnitude $|s_j|$, but the sign of the fuzzy control component is opposite that of the sliding function component. The fuzzy control (27) also has the same influence to the overall control action but is sensitive not only to the sliding function $s_j$ but also to the sliding vector $\mathbf{s}$. Tuning the coefficients $\sigma_{s_j}$ and $\sigma_{\Delta s_j}$ or $\sigma_{s_j}$ depends generally on the magnitudes of the sliding functions, while the values $\gamma_{m,j}$ or $u_{F,m,j}$ depend on the saturation condition of the control input.

**III. Applications**

The proposed fuzzy SMC technique can find a number of industrial applications. A linear two-mass servo electric drive was reported [13] to have strong robustness to parameter and load variations, and to nonlinearities. The method can be applied to load frequency control in power generation. The dynamic model for load frequency control of a single-area power system can be linearized and represented by a fourth-order state...
equation. The use of a variable-structure controller for such a system was introduced in [15]. Using the proposed controller the system can exhibit chattering-reduced responses with high insensitivity to load changes and plant parameter variations even when considering governor backlash and generation-rate constraints, as indicated in [16]. For multi-input systems, a two-degrees-of-freedom robotic manipulator was considered in [17]. The tracking error responses and end-point trajectory verified robustness of the system with the proposed controller against external disturbances and parameter uncertainty with diminished chattering. SMC techniques have also been proposed for electrohydraulic servo control [18]–[20]. It is known that a tradeoff exists between robustness and chattering, whereas the design of a provident sliding-mode controller [19] for systems with severe uncertainties may be a formidable problem. The combination of variable structure control with proportional–integral control was proposed for a hydraulic motor system with enhanced performance as demonstrated by simulation [20]. The application of these techniques to hydraulic servo systems requires a number of practical tests to demonstrate their feasibility. In the following, the validity of our proposed method is verified through joint control of a hydraulically actuated mini-excavator. The control hardware is described, together with the results of simulated and physical experiments.

A. Experimental Machine

A hydraulic robotic excavator [21] is used for our experiments. Fig. 1 shows a 1.5-t Komatsu PC05-7 mini-excavator, which has been retrofitted with electrohydraulic servo valves, spool valve position and pressure transducers, 12-b absolute joint angle encoders, and Moog digital controllers. Additional ancillary equipment needed to support the servo valves is also fitted: an accumulator with unloading valve, solenoid check valves, and an air radiator. The machine has eight hydraulic axes: two rubber tracks, cab slew, arm swing, boom, arm, bucket, and a small blade for backfilling. Closed-loop control of all axes is achieved by four proprietary M2000 Programmable Servo Controllers (PSCs), digital controllers that are commanded and coordinated by an industrial IBM-compatible personal computer (PC). The four M2000 axis controllers are enabled by a current loop; each module controls two axes.

The PC communicates with the digital controllers through a control area network (CAN) bus, and issues track velocity commands, and position set points to the other axes. At this time, system motion commands are input via a joystick that is interfaced to the PC. Hardware for the control of each axis is organized as shown in Fig. 2.

Conventional proportional–integral–derivative (PID) controllers were initially implemented for each of the excavator’s axes, and were experimentally evaluated. The preliminary experiments, however, identified a need for more robust control strategies, particularly during the force-constrained motion of the excavator bucket in interactions with the soil. Nonlinear effects occurring during this interaction, and in the hydraulic system itself, complicate the control requirements. It is known that changes in oil viscosity, friction between the piston and cylinder, oil flow through the hydraulic servo valve, and variable loading will cause hydraulic control systems to suffer from highly nonlinear time-variant dynamics, load sensitivity, and parameter uncertainty. The proposed fuzzy SMC method appears to provide an appropriate solution to such problems.

B. Controller Design

The hydraulic actuators incorporated in the blade, boom swing, boom, arm, and bucket attachments of the excavator are axial hydraulic cylinders. The flow of hydraulic oil to the cylinder is regulated by a direct-drive servo valve with an electrically controlled closed loop that controls valve spool position. We assume that the piston is halfway along the cylinder, the sum of the pressures at the two compartments is equal to the supply pressure, and the cylinder is considered to be effectively as a double-rod one. The hydraulic system for one axis may then be described by the following state equation of the form (1):

$$\begin{bmatrix} \dot{c}_1(t) \\ \dot{c}_2(t) \\ \dot{c}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_1 & -a_2 & -a_3 \end{bmatrix} \begin{bmatrix} c_1(t) \\ c_2(t) \\ c_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix} (u + u_d) + \begin{bmatrix} 0 \\ 0 \\ f \end{bmatrix} \tag{29}$$

where $c_1(t) = x_1(t) - x_{1d}(t)$, $c_2(t) = \dot{x}_1(t) = \ddot{x}_1(t) - \dot{x}_{1d}(t)$, and $c_3(t) = \ddot{x}_1(t) = \dddot{x}_1(t) - \ddot{x}_{1d}(t)$ are, respectively, the
tracking errors, \( x_1 \) is the linear position of the piston within the hydraulic cylinder (m), \( x_d(t) \) is the desired position of the piston (m), \( u_d(t) = b^{-1}(\dot{\alpha}_3 \dot{x}_d + \dot{\alpha}_2 \ddot{x}_d + \dot{\alpha}_1 x_1) \) is the control in the steady state, and \( f(t) \) represents the force due to parameter variations, load, and other disturbances on the cylinder (N). Here, the dot (\( \cdot \)) notation is used for the time derivative whereas the circumflex (\( ^\wedge \)) denotes the nominal value of a parameter, assumed to be known. The matrix entries are

\[
\begin{align*}
a_1 &= \frac{4\beta}{A} \frac{C_{ip} G}{M} \\
a_2 &= \frac{4\beta}{L} \frac{A}{M} + \frac{4\beta}{A} \frac{C_{ip} B}{M} + \frac{G}{M} \\
a_3 &= \frac{4\beta C}{A} + \frac{B}{M} \\
b &= b\sqrt{1 - \frac{P_L \text{sgn}(\dot{x}_1)}{P_S}} \\
b &= \frac{4\beta C_d}{L} \sqrt{\frac{P_S}{M}} \\
P_L &= [M\dot{x}_1 + B\ddot{x}_1 + F_e \text{sgn}(\dot{x}_1) + G\dot{x}_1 + F_e(t)]/A
\end{align*}
\]

where

\[
\begin{align*}
P_S &\quad \text{supply pressure;} \\
M &\quad \text{moving mass;} \\
F_e &\quad \text{total external force;} \\
L &\quad \text{piston stroke length;} \\
A &\quad \text{piston area;} \\
\beta &\quad \text{oil bulk modulus;} \\
C_{ip} &\quad \text{a leakage coefficient;} \\
C_d &\quad \text{flow discharge coefficient for a sharp orifice opening area;} \\
\rho &\quad \text{oil density;} \\
K &\quad \text{a fixed gain.}
\end{align*}
\]

The friction is modeled as composed of a linear viscous term \( B\dot{x}_1 \), a Coulomb friction term \( F_e \text{sgn}(\dot{x}_1) \), and an instantaneous friction \( G\dot{x}_1 \) due to the tangential stiffness of the static contact. Assuming that there exist the bounds \( \delta_i > 0 \), \( i = 1, 2, 3 \), \( \varepsilon > 0 \), and \( v > 0 \) such that

\[
\begin{align*}
a_i &= \hat{a}_i + \hat{b} \Delta a_i, \quad |\Delta a_i| \leq \delta_i, \quad i = 1, 2, 3, \\
b &= \hat{b}(1 + \Delta b^*), \quad |\Delta b^*| \leq \varepsilon < 1, \\
f &= \hat{b} \Delta f^*, \quad |\Delta f^*| \leq v
\end{align*}
\]

the control objective is to design a chattering-attenuated sliding-mode controller that provides robust performance in the presence of uncertainties. By choosing \( C = [c_1 \ c_2 \ 1] \), a switching function of the form (4) can be written as

\[
s = c_3 + c_2 \varepsilon + c_1 \varepsilon
\]

where \( c_1 \) and \( c_2 \) are positive constants to be specified according to the desired eigenstructure \( \{\lambda | \lambda^2 + c_2 \lambda + c_1 = 0 \} \) of the sliding surface. The equivalent control gain is obtained from (9) as

\[
K_E = \hat{b}^{-1}[-\hat{a}_1 \ (c_1 - \hat{a}_2) \ (c_2 - \hat{a}_3)].
\]

### Table I: Hydraulic Servo System Parameters

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piston effective area</td>
<td>A</td>
<td>0.0013</td>
<td>m²</td>
</tr>
<tr>
<td>Viscous damping coefficient</td>
<td>B</td>
<td>6400</td>
<td>N/m²</td>
</tr>
<tr>
<td>Discharge coefficient</td>
<td>C_d</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>Coulomb friction</td>
<td>F_c</td>
<td>120</td>
<td>N</td>
</tr>
<tr>
<td>Tangential stiffness</td>
<td>G</td>
<td>100</td>
<td>N/m</td>
</tr>
<tr>
<td>Total mass of cylinder rod and load</td>
<td>M</td>
<td>30</td>
<td>kg</td>
</tr>
<tr>
<td>Leakage coefficient</td>
<td>C_p</td>
<td>2.4 x 10⁻⁷</td>
<td>m²/s/MPa⁻¹</td>
</tr>
<tr>
<td>Valve gain</td>
<td>K</td>
<td>0.2 x 10⁻⁴</td>
<td>m³/V⁻¹</td>
</tr>
<tr>
<td>Supply pressure</td>
<td>P_s</td>
<td>18.6</td>
<td>MPa</td>
</tr>
<tr>
<td>Piston stroke</td>
<td>L</td>
<td>0.442</td>
<td>m</td>
</tr>
<tr>
<td>Effective bulk modulus</td>
<td>( \beta )</td>
<td>345</td>
<td>MPa</td>
</tr>
<tr>
<td>Fluid mass density</td>
<td>( \rho )</td>
<td>850</td>
<td>kg/m³</td>
</tr>
</tbody>
</table>

C. Simulation Results

Consider the axis control problem of the experimental excavator arm actuated by the hydraulic servo system described in Table I. With desired eigenvalues chosen at \( \{ -10, -10 \} \), the constants \( c_1 = 100 \) and \( c_2 = 20 \) are obtained. Assuming that the elements of the state equation (29) may fluctuate by 30% around their nominal values, the bounds of uncertainties in the matching conditions (30) can be determined as \( \delta_i = 0.3 \hat{a}_i / \hat{b}, \ i = 1, 2, 3, \) where \( \hat{a}_i \) and \( \hat{b} \) are the nominal values, \( a_i \) and \( b \), calculated from Table I. The bound \( v \) is determined by the maximal load that the arm cylinder can tolerate, with our robotic excavator \( v = 0.11 \). After some trials, the fuzzy tuning parameters are selected as \( \sigma_t = 100 \) and \( u_{F_m} = 0.3 \).

A time step of 0.001 s is used in our simulation. We consider first the free-motion (no load) tracking of a step input; for example, stroking the arm cylinder from 0.144 to 0.070 m. Fig. 3 shows the position, control voltage, and differential pressure responses with SMC (13) in which the equivalent control and the switching control are given respectively by (34) and (35). Fig. 4 depicts these responses under the same no load condition using fuzzy SMC (FSMC) in which the control action now consists of
the equivalent control (34), the switching control (35), and the fuzzy control (36). It can be seen that fast tracking and a significant reduction of chattering are obtained by introducing the fuzzy control component.

Toward autonomous excavation, our objectives focus on the execution of common excavation tasks such as digging building footings, or loading haul trucks from an open-cut mine bench. Let us consider next the practical problem of digging a certain soil, for example, “sandy loam,” with the excavator in a teleoperated form. For our experimental mini-excavator, the digging force for a cut depth of about 0.2 m can be estimated [22] to be about 3.4 kN. When rotary cutting [22] with the arm, the arm cylinder piston has to track square motion sequences between 0.07–0.17 m. The tracking responses with SMC are shown in Fig. 5. Fig. 6 presents the piston stroke and the control voltage under the same condition with the use of FSMC. Chattering in the control voltage is practically eliminated.

D. Experimental Results

Experiments have been conducted to validate the simulation results obtained. Data acquisition and control algorithms are written in C++ and executed under the Windows NT operating system. The sampling time is chosen to be 0.010 s. The CAN bus communication rate is set at 250 kb/s. The free-motion responses with SMC and FSMC are shown in Figs. 7 and 8, respectively. It should be noted that the fuzzy control parameters can be further tuned in practice to achieve a smoother control action.

Field trials have been performed with the fuzzy sliding-mode controller. The responses when digging “sandy loam” with a tracking pattern similar to that used for the simulation are depicted in Fig. 9. Robustness to the load force influence due to the soil resistance can be observed. Promising results obtained in a number of field tests with different types of soil enable the application of the proposed control technique to deal with tool–soil contact uncertainties of the robotic excavator in its autonomous operations.

IV. CONCLUSION

A new technique combining the features of SMC and fuzzy control has been presented in this paper. First, the equivalent
control is determined from the desired sliding eigenstructure. The reaching condition is guaranteed by the use of the robust feedback control with switching gains. Fuzzy tuning schemes are then employed to accelerate the reaching phase and reduce chattering. The practical application of fuzzy logic is proposed here as a computational-intelligence approach to the engineering problems associated with sliding-mode controllers. Fuzzy schemes with explicit expressions for tuning enable an easy adjustment of the control action and minimal computational cost. The Lyapunov stability method is used to prove the asymptotic convergence of the control strategies. The proposed technique can be applied to a number of control systems including servo hydraulic control. The axis control of an electrohydraulic excavator is used to demonstrate its validity and feasibility. The simulation and experimental results obtained for axis control of a mini-excavator demonstrate the possibility of employing the proposed FSMC method to achieve strong robustness against uncertainties such as parameter variations, external disturbances, and nonlinearities. These uncertainties are expected during the bucket–soil contact when the machine is executing autonomous excavation tasks.

**ACKNOWLEDGMENT**

The authors would like to thank the reviewers for their comments.

**REFERENCES**


Q. P. Ha (S’94–M’97) received the B.E. degree in electrical engineering from Ho Chi Minh City Polytechnic University, Ho Chi Minh City, Vietnam, the Ph.D. degree in engineering science from Moscow Power Institute, Moscow, Russia, and the Ph.D. degree in electrical engineering from the University of Tasmania, Hobart, Australia, in 1983, 1992, and 1997, respectively. From 1997 to 2000, he was a Senior Research Associate at the Centre for Field Robotics, The University of Sydney, Sydney, Australia. He is currently a Lecturer at The University of Technology, Sydney, Australia. His research interests include nonlinear control, variable-structure systems, robotics, and applications of artificial intelligence in engineering.

Q. H. Nguyen received the Bachelor in Electrical Engineering degree from Hanoi University of Technology, Hanoi, Vietnam, in 1993. He is currently working toward the Ph.D. degree at the Australian Centre for Field Robotics, The University of Sydney, Sydney, Australia.

His research interests include nonlinear control, robotics, and software engineering.

D. C. Rye (M’91) received the B.E. degree (first class honors) from the University of Adelaide, Adelaide, Australia, and the Ph.D. degree from The University of Sydney, Sydney, Australia, in 1980 and 1986, respectively, both in mechanical engineering.

From 1986 to December 1987, he was a Lecturer in mechanical engineering in the Department of Mechanical Engineering, University of Newcastle, Newcastle, Australia. Since 1988, he has been with the Department of Mechanical and Mechatronic Engineering, The University of Sydney, as a Lecturer and then a Senior Lecturer in mechanical engineering. He is also a Deputy Director of the Australian Centre for Field Robotics. His research interests include mechanics of tracked vehicles, autonomous excavation, crane dynamics and control, mechatronics and automation, nonlinear control, variable-structure control, and high-precision positioning control.

H. F. Durrant-Whyte received the B.Sc. (Eng.) degree (first class honors) in mechanical and nuclear engineering from the University of London, London, U.K., and the M.S.E. and Ph.D. degrees in systems engineering from the University of Pennsylvania, Philadelphia, in 1983, 1985, and 1986, respectively.

From 1987 to 1995, he was a University Lecturer in Engineering Science at Oxford University, Oxford, U.K. Since July 1995, he has been a Professor of Mechatronic Engineering in the Department of Mechanical and Mechatronic Engineering, The University of Sydney, Sydney, Australia, where he is also the Director of the Australian Centre for Field Robotics. His research interests include sensor data fusion, sensor systems, and mobile robotics.