I. INTRODUCTION

The stability problem for singular descriptor systems has long been studied. Such systems are generally characterized by the following differential equations:

\[
E \frac{dx}{dt} = f(x(t), t), \quad E x(t_0) = E x^0
\]

where \( x \in \mathbb{R}^n \), \( f(x(t), t) \in \mathbb{R}^n \), and \( E \in \mathbb{R}^{n \times n} \). The matrix \( E \) may be singular, that is, \( \text{rank}(E) = r < n \). The problems of stability and robustness for linear descriptor systems have been extensively discussed and many results have been published. It has been shown that the optimal regulators for descriptor systems with feedback gain properly selected have the same robustness properties as those in conventional systems.

Xu and Mizukami extended the Hamilton–Jacobi equation from the state space to the descriptor systems [1]. It is shown that, if the optimal performance index satisfies the Hamilton–Jacobi equation, an optimal control can be easily determined. In [5] the problem of stability and robustness of uncertain nonlinear descriptor systems is considered, in which the Lyapunov stability theory for conventional systems is extended to nonlinear descriptor systems. By making use of the stability results developed in the paper, a stabilizing state feedback controller was proposed for a class of nonlinear uncertain descriptor systems. The controller is robust under bounded uncertainties. However, in practice, many systems contain unknown parameters whose upper bounds are also unknown. For such systems, the well-used approach is adaptive control.

Numerous adaptive robust control algorithms for systems containing uncertainties have been developed [7]–[11]. Variable structure control with an adaptive law is developed for an uncertain input-output linearizable nonlinear system, where linearity-in-parameter condition for uncertainties is assumed [8]. The unknown gain of the upper bounding function is estimated and updated by adaptation law so that the sliding condition can be met and the error state reaches the sliding surface and stays on it. To deal with a class of nonlinear systems with partially known uncertainties, an adaptive law using a dead zone and a hysteresis function is proposed to guarantee both the uniform boundedness of all the closed-loop signals and uniform ultimate boundedness of the system states [9]. In both control schemes, it is assumed that the system uncertainties are bounded by a bounding function which is a product of a set of known functions and unknown positive constants. The objective of adaptation is to estimate these unknown constants.

A new adaptive robust control scheme is developed for a class of nonlinear uncertain systems with both parameter uncertainties and exogenous disturbances [7]. Including the categories of system uncertainties in [8] and [9] as the subsets, it is assumed that the disturbances are bounded by a partially known upper bounding function with unknown parameters. Furthermore, the input distribution matrix is assumed to be state related and have a bounded perturbation.

By making use of the Hamilton–Jacobi equation for the descriptor systems [1], we propose adaptive robust control schemes for a class of nonlinear uncertain descriptor systems in this paper. To reduce the robust control gain and widen the application scope of adaptive techniques, the system uncertainties are supposed to be composed of two different categories: the first can be separated and expressed as the product of known function of states and a set of unknown constants, and the other category is not separable but with partially known bounding functions. The second category still includes two parts: the bounded system disturbances and the perturbation of the input distribution matrix. The first control scheme achieves asymptotic stability of the closed-loop system, while the second control scheme guarantees the uniform boundedness of the system and at the same time, the system will enter an arbitrarily designated zone in a finite time. Moreover, the adaptation will be stopped once the system has entered the desired zone, which can be arbitrarily designated, in a finite time.

Index Terms—Adaptive robust control, descriptor systems, nonlinearity, uncertainty.

II. PROBLEM STATEMENT

Let \( x(\cdot) \) be the state of the system, \( x(0) = x^0 \).

The problem of adaptive robust control is to find a control law \( u(\cdot) \) such that

\[
E \frac{dx}{dt} = f(x(t), t) + u(t), \quad E x(t_0) = E x^0
\]

is uniformly asymptotically stable, where \( x \in \mathbb{R}^n \), \( f(x(t), t) \in \mathbb{R}^n \), and \( E \in \mathbb{R}^{n \times n} \). The matrix \( E \) may be singular, that is, \( \text{rank}(E) = r < n \). The problems of stability and robustness of linear descriptor systems have been extensively discussed and many results have been published. It has been shown that the optimal regulators for descriptor systems with feedback gain properly selected have the same robustness properties as those in conventional systems.

The objective of adaptation is to estimate these unknown constants.

A new adaptive robust control scheme is developed for a class of nonlinear uncertain descriptor systems to be discussed. Section II describes the class of nonlinear uncertain descriptor systems to be discussed. Section III gives the design procedure of the first adaptive robust control scheme and the stability analysis. Section IV gives the design and analysis of

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Adaptive Robust Control Schemes for a Class of Nonlinear Uncertain Descriptor Systems

Jian-Xin Xu, Qing-Wei Jia, and Tong-Heng Lee

Abstract—This paper presents new adaptive robust control schemes for a class of nonlinear uncertain descriptor systems. Taking both the structured and unstructured uncertainties into consideration, the first control scheme achieves asymptotic stability of the closed-loop descriptor system. To avoid any potential problems such as parameter drift by unmodeled dynamics, in the second control scheme the adaptation is ceased when the control performance gets into the desirable region. The new control scheme guarantees the uniform boundedness of the system and, at the same time, the system will enter the desired zone, which can be arbitrarily designated, in a finite time.

Index Terms—Adaptive robust control, descriptor systems, nonlinearity, uncertainty.

The stability problem for singular descriptor systems has long been studied. Such systems are generally characterized by the following differential equations:

\[
E \frac{dx}{dt} = f(x(t), t), \quad E x(t_0) = E x^0
\]

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Manuscript received July 20, 1998; revised July 15, 1999 and October 14, 1999. This paper was recommended by Associate Editor L. Fortuna.


1057–7122/00$10.00 © 2000 IEEE
the second adaptive robust control scheme. Section V gives a simulation study which shows the effectiveness of the proposed method.

II. PROBLEM FORMULATION

Consider a class of uncertain dynamical system described by

\[ E \frac{dx}{dt} = f(x, t) + B(x, t) \{ [I + \Delta B(x, p, t)] u(t) \}
+ g(x, p, t) + \Delta g(x, p, \omega, t) \] (2)

\[ Ex(t_0) = Ex^0 \] (3)

where \( x \in R^n \) is the measurable state vector of the system; \( u(t) \in R^m \) is the control input of the system; \( p \in P \) is an unknown system parameter vector; \( P \) is the set of admissible system parameters; \( B(x, t) \) is the input distribution matrix which is continuous; \( f(x, t) \in R^n \) is a known nonlinear function vector; \( g(x, p, t) \in R^m \); and \( \Delta g(x, p, \omega, t) \in R^m \) are nonlinear uncertain function vectors of the state \( x \), unknown parameter \( p \), time \( t \), as well as a set of random parameters \( \omega \). Here we make the following assumptions.

Assumption A1: For any initial condition \( Ex_0 \) and any control \( u(t) \), there exists a unique solution \( x(t) \) \[1\], \[3\].

Assumption A2: A control law \( \nu(x, t) \in \mathcal{Y} \) is called an admissible control law if, for any initial condition \( Ex^0 \), the closed-loop descriptor system obtained by using it has no impulsive solution. Correspondingly, \( \mathcal{Y} \) is called the admissible control law set \[2\].

Assumption A3: There exist admissible control laws for the descriptor system (2)–(3). That is, the admissible control law set is nonempty. In a linear descriptor system, this is equivalent to the impulsive controllability of the system \[1\].

Assumption A4: For \( \Delta B(x, p, t) \in R^{n \times m} \)

\[ \forall t \in [0, \infty) \quad \forall x \in D \quad \forall p \in P \]
\[ r_{\text{max}} \geq \lambda \left( \frac{1}{2} \Delta B + \frac{1}{2} \Delta B^T \right) \] (4)

where \( \lambda(\cdot) \) indicates the eigenvalues of \( \cdot \).

Assumption A5: The structured uncertainty \( g(x, p, t) \in R^m \) is a nonlinear function vector which can be expressed as

\[ g(x, p, t) = \xi(x, t) \theta(p) \]
\[ \theta = [\theta_1, \ldots, \theta_m]^T \]
\[ \xi = \text{diag}(\xi_1, \xi_2, \ldots, \xi_m) \] (5)

where \( \theta \) is an unknown parameter vector and \( \xi \) is a known function matrix. The nonstructured uncertainty \( \Delta g(x, p, \omega, t) \) is bounded such that \( ||\Delta g(x, p, \omega, t)|| \leq \rho\omega(x, t) \), where \( ||\cdot|| \) represents the Euclidean norm for vectors and the spectral norm for matrices and \( \rho\omega(x, t) \) is a known upper bounded function.

The control objective is to find a suitable control input \( u \) to stabilize the singular system (2).

III. ADAPTIVE ROBUST CONTROL I

The adaptive robust technique is used in this section to develop a controller which guarantees the global boundedness of the system. The design procedures are presented in detail as follows.

A. The Adaptive Robust Controller

The nominal descriptor system of (2)–(3) is described as

\[ E \frac{dx}{dt} = f(x, t) + B(x, t) u(t) \] (6)

\[ Ex(t_0) = Ex^0. \] (7)

The state feedback controller for the above nominal system can be represented by a nonlinear function

\[ u(t) = \nu(x, t). \] (8)

We now introduce a nonlinear performance index for the nominal system as follows:

\[ J(Ex(t), u, t) = \int_{t_0}^{\infty} \sigma^2(x, t) + \frac{1}{2} u^T u \, dt \] (9)

where \( \sigma(x, t) \) is a continuous nonnegative definite function which will be defined later. Denote

\[ J^*(Ex(t), u, t) = \min \{ J(Ex(t), u, t) \} \] (10)

then from the result of \[1\] we know that for the optimal control problem of (6), (7), and (9), if there exists a continuous function \( W(x, t) \in R^{n \times n} \) such that

\[ \left[ \frac{\partial J^*(Ex(t), u, t)}{\partial x} \right]^T = W(x, t) E \] (11)

the optimal strategy can be found by the following Hamilton–Jacobi equation

\[ 0 = -\frac{\partial J^*(Ex(t), u, t)}{\partial t} \]
\[ = W(x, t)f(x, t) + W(x, t)B(x, t)u^*(t) + \sigma^2(x, t) \]
\[ + \frac{1}{2} u^*(t) u^*(t) \]
\[ \leq W(x, t)f(x, t) + W(x, t)B(x, t)u(t) + \sigma^2(x, t) \]
\[ + \frac{1}{2} u(t) u(t) \] (12)

where \( u^*(t) \) represents the optimal control which minimizes the performance index \( J(Ex(t), u, t) \). The optimal controller for (6), (7), and (9) can be obtained from the standard Hamilton–Jacobi theory:

\[ u^*(t) = \nu^*(x, t) = -B^T(x, t) W^T(x) \] (13)

In our case, we assume that such a function \( W(x, t) \) that satisfies (11) exists for the control problem. Based on the works of \[1\], we propose an adaptive robust control scheme to guarantee asymptotic stability of the uncertain descriptor system (2) and (3).

The proposed control law together with the corresponding law are given below

\[ u(t) = -k B^T(x, t) W^T(x) - \xi(x, t) \hat{\theta}(x, t) \] (14)

\[ \hat{\theta}^T = \Gamma W(x, t) B(x, t) \xi(x, t) \] (15)
where the control gain $k$ satisfies that $k > 3/4$ and $\Gamma = \text{diag}(\gamma_1, \gamma_2, \ldots, \gamma_m)$ with $\gamma_i, i = 1, 2, \ldots, m$ are positive values.

B. Convergence Analysis

With the above adaptive robust control law, we have the following theorem.

Theorem 1: The adaptive robust control law described in (14) and (15) is a stabilizing controller for uncertain descriptor system (6) and (7). That is, the closed-loop system is asymptotically stable in the presence of both structured uncertainty and system disturbance.

Proof: The following positive definite function $V$ is selected:

$$V(x, \dot{\theta}, t) = J^T(E\dot{x}(t)) + \frac{1}{2} \hat{\theta}^T \Gamma^{-1} \hat{\theta}$$

where

$$\hat{\theta} = \theta - \hat{\theta}$$

and $\sigma(x, t)$ is selected as

$$\sigma(x, t) = \left\{ \begin{array}{ll}
\rho_\alpha(x, t) + \rho_\beta(x, t) - \gamma(||x||) + \gamma(||x||) \\
\rho_\alpha(x, t) + \rho_\beta(x, t)
\end{array} \right. \geq \rho_\alpha(x, t) + \rho_\beta(x, t)$$

(18)

$$\rho_\alpha(x, t) \triangleq r_{\max}||u(t)||$$

where $\gamma(||x||)$ is a Class $\mathcal{K}$ function. Then from (2), (11), (14), and (15) we have

$$\frac{dV(x, t)}{dt} = \frac{\partial J^T(E\dot{x}(t))}{\partial x} + \frac{\partial J^T(E\dot{x}(t))}{\partial \dot{t}} - \hat{\theta}^T \Gamma^{-1} \hat{\theta}$$

$$= W(x, t) \{f(x, t) + B(x, t) \left[I + \Delta B(x, p, t)\right] u
+ \xi(x, t) \hat{\theta}(p) + \Delta g(x, p, \omega, t)\} - \hat{\theta}^T \Gamma^{-1} \hat{\theta}$$

$$= W(x, t) f(x, t) + W(x, t) B(x, t) [-kB^T(x, t)W^T(x)]$$

$$+ \xi(x, t) \hat{\theta}(p, t) + \Delta B(x, p, t) u(t) + \Delta g(x, p, \omega, t)$$

$$= W(x, t) f(x, t) + W(x, t) B(x, t) [-kB^T(x, t)W^T(x)]$$

$$+ \Delta B(x, p, t) u(t) + \Delta g(x, p, \omega, t)$$

(20)

From (12) and (13) we have

$$W(x, t) f(x, t) = -W(x, t) B(x, t) u(t) - \sigma^2(x, t) - \frac{1}{2} u^T(t) \Gamma^{-1} \hat{\theta}^T \hat{\theta}$$

$$= \frac{1}{2} W(x, t) B(x, t) B^T(x, t) W^T(x) - \sigma^2(x, t).$$

(21)

It follows from (20) that

$$\frac{dV(x, t)}{dt} = -\delta W(x, t) B(x, t) B^T(x, t) W^T(x) - \sigma^2(x, t)$$

$$+ W(x, t) B(x, t) \Delta B(x, p, t) u(t) + \Delta g(x, p, \omega, t)$$

$$\leq -\sigma^2(x, t) - \delta ||W(x, t) B(x, t)||^2$$

$$+ \left[r_{\max}||u(t)|| + \rho_\alpha(x, t)||W(x, t) B(x, t)||\right]$$

(22)

where $\delta = k - (1/2)$ and

$$\chi(x, t) = \left[\sigma(x, t)||W(x, t) B(x, t)||\right]^T Q \chi(x, t)$$

$$Q = \left[1 - \frac{1}{\delta} \frac{1}{\delta} \right].$$

(23)

Since $k$ is chosen such that $k > 3/4$, namely, $\delta > 1/4$, it can be easily verified that the matrix $Q$ is positive definite, which implies the boundedness of $V, \chi$ and $\dot{\chi}$. From (22) we have

$$\lim_{t \to \infty} \chi(x, t) \chi(x, t) \leq \frac{1}{\lambda_{\min}(Q)} \left[ V(t = 0) - V(t = \infty) \right]$$

(24)

where $\lambda_{\min}(\cdot)$ indicates the minimum eigenvalue of $(\cdot)$. Since $\chi(x, t)$ is a continuous function because $\sigma(x, t), W(x, t)$, and $B(x, t)$ are all continuous, by Barbital’s lemma, $\lim_{t \to \infty} \chi(x, t) = 0$. From the definition of $\chi(x, t)$ in (23) and the definition of $\sigma(x, t)$ in (18), we know that $||\chi(x, t)|| = \sigma(x, t) \geq \gamma(||x||)$, which further means that

$$\lim_{t \to \infty} \gamma(||x||) = 0$$

$$\lim_{t \to \infty} ||x|| = 0.$$
where the control gain $k$ satisfies that $k > 3/4$ and $\mu$ is defined
\[
\mu = \begin{cases} 
k_1 (\varepsilon_0 - ||x||), & x \in E_0 \\
0, & \text{elsewhere}
\end{cases} 
\tag{28}
\]
with $k_1$ a positive value. $E_0$ is a set defined by
\[
E_0 \triangleq \{x : ||x|| < \varepsilon_0 \} 
\tag{29}
\]
where $\varepsilon_0$ is a positive constant specifying the desired regulating bound.

### B. Convergence Analysis

For the above adaptive robust control law, we have the following theorem.

**Theorem 2:** The proposed adaptive robust control law ensures the uniform boundedness of the closed-loop system. The system trajectory enters the set $E_0$ in a finite time, while the parameter estimation error is bounded by the set
\[
D = \left\{ \theta : \theta^T \hat{\theta} < \frac{1}{2\lambda_{\min}(\Gamma^{-1})} k_1 \varepsilon_0 \theta^T \theta \right\}. 
\tag{30}
\]

**Proof:** The same positive definite function $V$ as in (16) is selected. Through the same procedure as in Section III, we can easily reach
\[
\frac{dV(x, t)}{dt} \leq -\chi_1 (x, t) Q \chi (x, t) + \mu \hat{\theta}^T \hat{\theta}. 
\tag{31}
\]
From the definition of $\mu$ in (28) we have
\[
\frac{dV(x, t)}{dt} \leq -\chi_1 (x, t) Q \chi (x, t) \\
\leq -\lambda_{\min}(Q) \gamma^2 (||x||), \quad \forall x \in \mathbb{R}^n - E_0 
\tag{32}
\]
where $\gamma (||x||)$ is a class $\mathcal{K}$ function defined in (18). In terms of the construction of $\mu$, $V$ is a continuous function. Hence, there exists a constant $0 < \varepsilon_0 < \varepsilon_0$ such that
\[
\forall x \in \mathbb{R}^n - E_0', \quad V < 0 
\tag{33}
\]
where $E_0' \triangleq \{x : ||x|| < \varepsilon_0 \}$ is a subset of $E_0$. This means that when $x \in \mathbb{R}^n - E_0'$, the system will enter the set $E_0$ in a finite time [7]. When $x \in E_0$, it is obvious that
\[
k_1 (\varepsilon_0 - \varepsilon_0) \leq \mu \leq k_1 \varepsilon_0. 
\tag{34}
\]
From (28), (31), and (34) we obtain
\[
\frac{dV(x, t)}{dt} \leq -\gamma^2 (||x||), \quad \forall x \in \mathbb{R}^n - E_0 
\tag{35}
\]
which means that $V$ monotonically decreases in $\mathbb{R}^n - E_0$. As $V < 0$, $E_0$ is a compact set, and $V(x, t)$ is positive definite, so $\lim_{t \to \infty} V(x, t) = 0$. Hence, $x(t)$ and $\theta(t)$ are uniformly bounded. The proof is complete.
If we properly choose $\Gamma$ and $k_1$ such that $k' \geq 1$, then the residual set becomes
\[
D = \left\{ \hat{\theta} : \hat{\theta}^\top \hat{\theta} < \frac{1}{2\lambda_{\text{max}}(\Gamma^{-1})} k_1 \hat{\theta}^\top \hat{\theta} \right\}.
\] (36)

V. SIMULATION STUDY

Application of singular systems to circuit network theory can be found in many publications (see [12]–[14], for example). In this section, we use a similar circuit network model as in [13] to demonstrate the effectiveness of our control design. Using a simplified model for a transistor in the circuit of Fig. 1 yields the circuit equation
\[
c_1 u_{x_1} = i_v
\] (37)

\[
y(t) = R_2 i_v
\] (38)

\[
i_v = i_v = \alpha i_b
\] (39)

\[
u(t) + n(t) + R_1 i_b + u_{v_1} = 0
\] (40)
where \( n(t) = 0.23x_1 - 0.36x_2 + 0.015 \sin(x_1 x_2) \) represents input perturbations. Our control problem is to find a control voltage \( u(t) \) for output voltage \( y(t) \) to track the desired trajectory \( y_d(x, t) = 0.5x_1 \).

Assuming \( R_1, R_2, \) and \( c_1 \) all have values of one and the current gain \( \alpha \) be an unknown constant, we describe the system (37)-(40) in the form of (2):

\[
E \frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, t) + B(\mathbf{x}, t) [(I + \Delta B(\mathbf{x}, t, u(t)) ) \mathbf{u}(t) + \mathbf{g}(\mathbf{x}, t) u(t)]
\]

where \( \mathbf{x} \triangleq [u_1, y - y_d] \) and

\[
\mathbf{E} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \Delta \mathbf{B} = 0.015 \sin(x_1 x_2) \\
\mathbf{f} = \begin{bmatrix} x_2 + y_d \\ x_1 \end{bmatrix}, \quad \mathbf{g} = \xi \theta \\
\xi = x_2 + y_d, \quad \theta = 1/\alpha \\
\Delta \mathbf{g} = 0.13x_1 - 0.36x_2
\]

(41)

where the real value of \( \alpha \) is \( \alpha = 10.0 \) and the upper bounds of \( \Delta \mathbf{g} \) and \( \Delta \mathbf{B} \) can be easily got as \( \rho_d = 0.5\sqrt{x_1^2 + x_2^2} \) and \( \rho_u = 0.015\sqrt{u^2} \), respectively. From (18), we choose \( \sigma(\mathbf{x}, t) \) as

\[
\rho_d + \rho_u = 0.5\sqrt{x_1^2 + x_2^2 + 2\sigma^2} \\
= \sqrt{(0.5\sqrt{x_1^2 + x_2^2 + 0.015\sqrt{u^2}})^2} \\
\leq \sqrt{0.6(x_1^2 + x_2^2) + 0.1u^2} \\
\leq \sigma(\mathbf{x}, t).
\]

Then the following performance index is given by (9)

\[
J(Ex) = \int_{t_0}^{\infty} \left[ \sigma^2(\mathbf{x}, t) + \frac{1}{2}\sigma^2(t) \right] dt \\
= \frac{1}{2} \int_{t_0}^{\infty} \left[ x_1^2(t) + x_2^2(t) + u^2(t) \right] dt.
\]

(42)

Assume that \( J^*(Ex(t)) \) has the following form [1]:

\[
J^*(Ex(t)) = \frac{1}{2} [x_1 x_2] \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 & a_0 \\ a_3 & a_4 \end{bmatrix} [x_1 x_2].
\]

(43)

It is obvious that

\[
W(\mathbf{x}, t) = 1.2 [x_1 x_2] \begin{bmatrix} a_1 & a_3 \\ a_4 & a_1 \end{bmatrix}.
\]

(44)

and the optimal control law for the nominal system of (41) is

\[
u^*(t) = [-0.1 \begin{bmatrix} a_1 & 0 \\ a_3 & a_4 \end{bmatrix} [x_1 x_2].
\]

(45)

Following from (12), we have

\[
\frac{1}{2} x_1^2 + \frac{1}{2} x_2^2 + \frac{1}{2} u^2 + W(\mathbf{x}, t) + W(\mathbf{x}, t) \mathbf{b} u^* = 0.
\]

(46)

Substituting (44) and (45) into (46) yields

\[
0 = [x_1 x_2] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} [x_1 x_2] \\
+ [x_1 x_2] \begin{bmatrix} a_1 & a_3 \\ 0 & a_4 \end{bmatrix} \begin{bmatrix} 0.5 & 1 \\ 0 & 1 \end{bmatrix} [x_1 x_2] \\
+ [x_1 x_2] \begin{bmatrix} 0.5 & 1 \\ 1 & 0 \end{bmatrix} [x_1 x_2] \\
- [x_1 x_2] \begin{bmatrix} a_3 & a_1 \\ a_4 & a_3 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} [x_1 x_2].
\]

(47)

This equation holds for all \( \mathbf{x}(t) \), therefore we get

\[
\begin{bmatrix} a_1 + 2a_3 \ a_1 + a_4 \\ a_1 + a_4 \ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} a_3 & a_3a_4 \\ a_3a_4 & a_2 \end{bmatrix}
\]

(48)

Hence

\[
a_1 = 2, \quad a_3 = 3, \quad a_4 = 1.
\]

For simulation, the parameters are chosen as

\[
k = k_1 = 1, \quad \varepsilon_0 = 0.05, \quad \Gamma = \text{diag}(1, 1)
\]

and the initial states of the system are \( \mathbf{x}_0 = [-1 \quad 1.2]^T \).

Figs. 2 and 3 show the simulation results of the closed-loop descriptor system using adaptive robust Control Method I and II, respectively. We can see that the system has achieved asymptotic stability by using adaptive robust Control Method I, while uniform boundedness is guaranteed by adaptive robust Control Method II and the system enters the desired region \( ||\mathbf{x}|| < \varepsilon_0 = 0.01 \) in a very short time.

VI. CONCLUSION

This paper has presented new adaptive robust control schemes for a class of nonlinear uncertain descriptor systems. Two different categories of system uncertainties are considered here: the structured and nonstructured uncertainties. The structured uncertainty, which is dealt with by the well-used adaptive control method, can be separated and expressed as the product of known functions of states and a set of unknown constants. The nonstructured uncertainty is bounded by a known function. By making use of the Hamilton–Jacobi equation for the descriptor systems [1], adaptive robust control schemes are proposed to achieve the asymptotic stability or the uniform boundedness of the closed-loop descriptor system.

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