Non-Unitary Codebook Based Precoding Scheme for Multi-User MIMO with Limited Feedback

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Abstract—In this paper, we propose a non-unitary codebook based precoding scheme for multi-user MIMO (MU-MIMO) downlink transmission with limited feedback. The proposed MU-MIMO scheme will perform scheduling and precoding just according to the limited feedback of the signal to interference and noise ratio (SINR) of each user to enhance the system capacity. A precoder codebook design method is related to Grassmannian line packing criterion. We group vectors from the Grassmannian codebook into precoding matrices according to the correlation coefficient of the vectors to suppress the multi-user co-channel interference (CCI). It terms as non-unitary precoding and outperforms the traditional single user MIMO system as well as the unitary codebook based MU-MIMO scheme with limited feedback and low complexity.

Keywords-multi-user MIMO, Grassmannian line packing, non-unitary precoding, SINR, limited feedback

I. INTRODUCTION

Multiple-input multiple-output (MIMO) communication technique has attracted enormous interest for the next-generation wireless communications because of its potential for high capacity. The traditional MIMO system is focused on the point-to-point single-user MIMO (SU-MIMO) such as the spatial diversity (STC: Space-Time Coding) and the spatial multiplexing (VBLAST: Vertical Bell Lab Layered Space-Time) techniques. Spatial diversity technique is employed to combat the channel fading, meanwhile robust to various channel environments [1], but has limit to the transmission rate. Spatial multiplexing technique can obtain considerable system throughput by transmitting parallel data streams. However, it has limit to the transmission environment that when the spatial channel correlation is high, the performance degrades severely [2]. When the base station (BS) and also the mobile station (MS) are equipped with multiple antennas, the space dimension can be exploited besides time dimension and frequency dimension for transmission the data streams of multiple users. As a result, the study of point-to-multipoint MIMO system termed as the multi-user MIMO (MU-MIMO) has emerged recently as a hot research topic. Such system has many advantages that the multiple antenna diversity gain can enhance the bit error rate performance and the multiple antenna spatial multiplexing gain can enlarge the channel capacity area.

Telatar generalized the Shannon capacity of the single-input single-output channel to the MIMO case [3]. He showed that in the single-user environment, the optimal power allocation that maximizes the mutual information of the MIMO channel is achieved through waterfilling over its eigenvalues. For the MU-MIMO channel, the problem is more complex. The sum capacity in a multi-user broadcast channel is defined to be the maximum aggregation of all users’ data rates. It requires maximizing each user’s data rate and minimizing the co-channel interference (CCI) to other users. It was proven that when a transmitter has advance knowledge of the interference, it could design a code to compensate for it [4]. An approach that was defined as dirty paper coding (DPC) [4][5] by non-linear preprocessing was developed. Although DPC can achieve the sum capacity of multi-user broadcast channel, deploying DPC in real-time systems is difficult because of the complicated encoding and decoding schemes.

Many linear precoding MU-MIMO techniques are emerged recently. Such as the channel inversion method [6], the diagonalization (BD) method [7][8], the unitary codebook based precoding method [9] and other precoding methods [10][11]. Channel inversion method is simple but has capacity limit. When the spatial correlation increases, the multi-user channel capacity decreased rapidly. BD can perfectly cancel CCI, but has antenna constraint at the base station and the mobile station [8] and the computation burden for system is very heavy when the number of users is large. Both channel inversion method and the BD method are based on the feedback of the MIMO channel matrix, so the feedback is very large. The unitary codebook based precoding method is firstly proposed in 3GPP LTE (Long Term Evolution) [12]. It can suppress the CCI efficiently through orthogonal precoding vectors and increase the MU-MIMO system capacity with the limited feedback. It is much more practical for its simplicity, low feedback and has become a hot research topic in LTE.

In this paper, we proposed a non-unitary codebook based precoding method for MU-MIMO downlink transmission with limited feedback and low complexity. The proposed non-unitary codebook based precoding scheme exploits efficient multi-user diversity gain and precoding gain by scheduling and precoding at the BS. The adoption of Grassmannian precoding [13] aims to provide strong precoding gain for each user. We construct the precoding matrices according to the correlation coefficient of the Grassmannian vectors to suppress the CCI. So, it terms as non-unitary precoding since the vector pair in Grassmannian codebook is not orthogonal. The proposed
scheme shows considerable system capacity improvement than the traditional SU-MIMO system and outperforms the unitary codebook based precoding scheme [9].

II. SYSTEM MODEL OF THE CODEBOOK BASED MU-MIMO

We consider a MU-MIMO system with \( M \) transmit antennas at the BS and \( N \) receive antennas at each MS as shown in figure 1. There is a codebook known prior by both the BS and the MS. If \( X \) users are waiting to be scheduled at the BS, the scheduler will determine \( K \) (\( K \leq M \)) users to be transmitted according to the scheduling criterion and the feedback information. At the same time, the scheduler will select a precoding matrix \( \mathbf{w} \) from the codebook to precode for the scheduled \( K \) users before transmission.

The MIMO channel matrix \( \mathbf{H}_k \) for the \( k \)-th MS is as follow.

\[
\mathbf{H}_k = \begin{bmatrix}
    h_{k,1} & \cdots & h_{k,M} \\
    \vdots & \ddots & \vdots \\
    h_{N,k} & \cdots & h_{N,M}
\end{bmatrix}
\]  

(1)

Where \( h_{i,j} \) indicates the channel impulse response coupling the \( j \)-th antenna at the BS to the \( i \)-th antenna at the MS and its amplitude obeys independent and identically Rayleigh-distribution.

The orthogonal frequency division multiplexing (OFDM) technique has become one of the brilliant techniques in the next generation of wireless communications. Since OFDM technique can deal frequency selective fading as flat fading, so in this paper, we model the MIMO channel as the flat MIMO fading.

The received signal at the \( k \)-th MS can be denoted as

\[
y_k = \mathbf{H}_k \mathbf{w} \mathbf{s} + \mathbf{n}_k \tag{2}
\]

where \( \mathbf{s} \) is the transmit vector with \( K \) data streams, \( \mathbf{n}_k \) obeys distribution \( CN(0, N_0) \) and is spatially and temporally white. \( \mathbf{w} \) is the \( M \times K \) precoding matrix, which contains \( K \) precoding vectors. \( \mathbf{v}_i \) is the \( M \times 1 \) column vector with unit norm \( \mathbf{v}_i^\dagger \mathbf{v}_i = 1 \) and \( [\cdot]^\dagger \) denotes the matrix transposition. \( \mathbf{s} \) is the transmit vector with \( K \) data streams.

For the MU-MIMO system, the virtual MIMO channel matrix of user \( k \) after precoding can be assumed as

\[
\mathbf{H}_k = \mathbf{H}_k \mathbf{w} \tag{6}
\]

For traditional MIMO detection, a linear receiver is designed to detect the transmit data. Zero Forcing (ZF) or Minimum Mean Squared Error (MMSE) detection criterions are commonly employed. In order to obtain good performance, linear MMSE receiver is adopted at MS in this paper.

The linear MMSE receiver \( \mathbf{\hat{G}}_k \) for the \( k \)-th user or the \( k \)-th data stream is

\[
\mathbf{\hat{G}}_k = \mathbf{h}_k^\dagger \left( \mathbf{H}_k^\dagger \mathbf{H}_k + \frac{KN_0}{p_0} \mathbf{I}_N \right)^{-1} \tag{7}
\]

\[
\mathbf{\hat{h}}_k = \mathbf{H}_k \mathbf{v}_k = \left[ \mathbf{H}_k \mathbf{w} \right]_k \tag{8}
\]

Where \( (\cdot)^{-1} \) indicates the inverse of the matrix, \( (\cdot)^\dagger \) is the matrix conjugation transposition and \( \mathbf{I}_N \) is the \( N \times N \) identity matrix. \( \mathbf{v}_k \) is the precoding vector for the \( k \)-th user and \( [\cdot]_k \) denotes the \( k \)-th column of the matrix.

The received signal for \( k \)-th user can be indicated as

\[
y_k = \sqrt{p_k} \mathbf{H}_k \mathbf{v}_k s_k + \sum_{i=1, i \neq k}^K \sqrt{p_i} \mathbf{H}_k \mathbf{v}_i s_i + \mathbf{n}_k \tag{9}
\]

The middle part of (9) is the CCI from other users and the last part is the noise. Then the detected data stream \( \hat{s}_k \) for the \( k \)-th MS is

\[
\hat{s}_k = \mathbf{\hat{G}}_k y_k \tag{10}
\]

The detected SINR for the \( k \)-th user with the linear receiver is

\[
\text{SINR}_k = \frac{p_k \| \mathbf{\hat{G}}_k \mathbf{H}_k \mathbf{v}_k \|^2}{\sum_{i=1, i \neq k}^K p_i \| \mathbf{\hat{G}}_k \mathbf{H}_k \mathbf{v}_i \|^2 + \| \mathbf{\hat{G}}_k \|^2 N_0} \tag{11}
\]

Where \( \| \cdot \|_2 \) denotes the matrix two-norm.

III. THE UNITARY PRECODING CODEBOOK

The unitary precoding codebook contains a set of unitary precoding matrices [9] as

\[
\mathbf{E} = \{ \mathbf{E}^0, \ldots, \mathbf{E}^{L-1} \} \tag{12}
\]
Where $E'$ is the $(l+1)$-th precoding matrix in the codebook $E$. 

$$E' = \begin{bmatrix} e_0^{(l)} & \cdots & e_{M-1}^{(l)} \end{bmatrix}$$ (13)

Where $e_m^{(l)}$ is the $(m+1)$-th precoding vector of the $(l+1)$-th precoding matrix and $M$ is the transmit antenna number.

$$e_m^{(l)} = \frac{1}{\sqrt{M}} \begin{bmatrix} u_{0m}^{(l)} & \cdots & u_{(M-1)m}^{(l)} \end{bmatrix}^T$$ (14)

$$u_{nm}^{(l)} = \exp\{j \frac{2\pi n}{M} (m+\frac{l}{L})\}$$ (15)

The distance is defined as follow

$$\delta(V) = \min_{1 \leq k \leq L} \sqrt{1 - |v_k^H v_j|^2}$$ (16)

And the minimum distance between any pair of lines [13].

The generation of the precoding matrices is based on Fourier transform. The precoding matrices are unitary matrices since $EE'^H = I$. According to the above criterion, the unitary precoding codebook with arbitrary antenna number $M$ and matrix number $L$ could be generated.

IV. GRASSMANNIAN LINE PACKING CRITERION

The Grassmannian line packing problem is the problem of optimally packing one-dimensional subspaces, which results in finding the set or packing of $N_t$ lines in $\mathbb{C}^M$ that has maximum minimum distance between any pair of lines [13]. Where $N_t$ is the packing number or vector number of the Grassmannian codebook. We represent a packing of $N_t$ lines in $\mathbb{C}^M$ by a $P \times N_t$ matrix $V = [v_1, v_2, \ldots, v_{N_t}]$ whose column space $v_j$ is the $i$-th line in the packing. $v_j$ is a $P \times 1$ unit vector with $v_j^H v_j = 1$ and $v_i^H v_j \neq 1 (i \neq j)$. It is proved that maximizing the transmitting precoding gain $G_x(\mathbf{H}) = \max_{i \in [N_t]} \| \mathbf{H} v_i \|_2^2$ with a finite codebook vectors corresponds to maximizing the minimum distance between any pair of lines spanned by the codebook vectors.

Defining a distance function $d(v_i, v_j)$ by letting the distance between the two lines generated from unit vectors $v_i$ and $v_j$ be the sine of the angle $\theta_{i,j}$ between the two lines. The distance is

$$d(v_i, v_j) = \sin(\theta_{i,j}) = \sqrt{1 - |v_i^H v_j|^2}$$ (16)

The minimum distance of a packing is the sine of the smallest angle between any pair of lines.

Grassmannian packing criterion is that designing the set of codebook vectors $\{v_j\}_{j=1}^{N_t}$ such that the corresponding codebook $V$ maximizes

$$\delta(V) = \min_{1 \leq k \leq L} \sqrt{1 - |v_k^H v_j|^2}$$ (17)

So, a Grassmannian line packing codebook is designed by solving [13]

$$V = \arg \max_{X \in \mathcal{U}_L} \delta(X)$$ (18)

The codebook $V$ consists of $N_t$ constant transmit precoding weights, which can be denoted as a $P \times N_t$ Grassmannian codebook.

According to the above criterion, the Grassmannian line packing codebook with different $P$ and $N_t$ could be obtained.

V. NON-UNITARY CODEBOOK BASED PRECODING FOR MU-MIMO WITH LIMITED FEEDBACK

We construct a codebook $W = \{w_1, \ldots, w_L\}$ of $L$ precoding matrices provided for users. It is known prior at both the BS and each MS. The generation of a precoding matrix $w_l$ $(l=1, \ldots, L)$ is processed by selecting $M$ $(M \leq N_t)$ different vectors from a $M \times N_t$ Grassmannian codebook. So $w_l$ is a $M \times M$ dimensional matrix. There will be $Q = C_t^{N_t}$ different precoding matrices in total. Where $C_t^{N_t}$ denotes the total methods of selecting $M$ vectors from $N_t$ vectors. The precoding terms as non-unitary precoding since $w_l w_l^H \neq I$ and there is $v_i^H v_l = 1$. We order these precoding matrices according to $\rho$ as follow

$$\rho = \sum_{j=1}^{K} \sum_{l=1}^{K} v_i^H v_j$$ (19)

The $L$ precoding matrices are selected prior with small $\rho$ from the totally $Q$ matrices, since small $\rho$ will result in small CCI.

For each user, the received SINR not only depends on its own precoding vector, but also on other users’ precoding vectors of the same precoding matrix according to (11). If user $k$ obtains information about the precoding vectors and its channel matrix $H_k$, the SINR of user $k$ can be computed. Thus, for a precoding matrix $w_l$, it results in $M$ different SINR for each user. Each SINR corresponds to precoding by one of the vectors in $w_l$. The codebook $W = \{w_1, \ldots, w_L\}$ with $L$ non-unitary matrices results in $M \times L$ different SINR for each user. Then, each user selects one best SINR of a precoding matrix $w_l$, and $L$ best SINR are selected for each user with the codebook $W = \{w_1, \ldots, w_L\}$. And they will feed back $L$ best SINR and the corresponding vector index of the codebook to the BS.

We propose a criterion to maximize the system capacity of MU-MIMO according to the feedback SINR and vector index of each user. The aim of the proposed scheme is to select proper users and a precoding matrix $w_l$ from the codebook $W$. The selected users will be precoded by $w_l$ to share the same time and frequency resources to maximize the system capacity.
The system capacity of the MU-MIMO depends on the received SINR of the scheduled users as
\[
C_{MU} (\mathbf{H}_k, \mathbf{w}_i) = \sum_{k=1}^{K} \log_2 \left( 1 + SINR_k (\mathbf{H}_k, \mathbf{w}_i) \right)
\] (20)

We proposed maximum capacity criterion to select precoding matrix and the scheduled users as
\[
\{ \mathbf{\bar{w}}, k = 1, ..., K \} = \arg \max_{\mathbf{w} \in \mathbb{C}^{N \times L}} C_{MU} (\mathbf{H}_k, \mathbf{w}_i)
\] (21)

The detail procedure is as follow
1) Constructing the non-unitary precoding codebook \( \mathbf{W} = \{ \mathbf{w}_1, ..., \mathbf{w}_L \} \) according to the above illustration.
2) Each MS computes \( M \times L \) SINR according to the non-unitary precoding codebook and its channel matrix, then feeds back \( L \) best SINR correspond to \( L \) non-unitary matrices and the \( L \) vector indexes to the BS.
3) The BS gathers the feedback information from each MS.
4) According to all feedback SINR values and index information, the scheduler at the BS will select \( k = 1, ..., K \) users who declare the same precoding matrix \( \mathbf{\bar{w}} \) with different precoding vector index in the precoding matrix to maximize the system capacity. That is the selected users declare their precoding vectors in a same precoding matrix \( \mathbf{\bar{w}} \) and its precoding vector with \( \mathbf{v}_k = [\mathbf{\bar{w}}]_k \) to obtain the maximal system capacity \( C_{MU} \).
5) The precoder at the BS will perform precoding to the selected users with the selected precoding matrix \( \mathbf{\bar{w}} \) before transmission.

VI. SIMULATION RESULTS

The proposed non-unitary codebook based MU-MIMO scheme, the traditional SU-MIMO scheme and the unitary codebook based MU-MIMO scheme were simulated.

For SU-MIMO system, the scheduler selects the user with the maximum Shannon capacity among the users waiting to be scheduled, so the maximum SU-MIMO system capacity with equal power allocation is [14]
\[
C_{SU} = \max_k \left[ \sum_{i=1}^{n} \log_2 \left( 1 + \frac{P_0}{N_0 M} \lambda_i \right) \right]
\] (22)

Where \( \lambda_i \) is the \( i \)-th eigenvalues of \( \mathbf{H}_k^H \mathbf{H}_k \), \( n = \min \{ M, N \} \).

Figure 2 is the cumulative distribution function (CDF) of the system capacity for SU-MIMO and the proposed non-unitary codebook based MU-MIMO with different codebook size \( L \). \( M = 2 \), \( N = 2 \), equal power allocation is applied at BS, linear MMSE detection is applied at each MS, \( X = 10 \) users at the base station waiting to be scheduled and \( p_0 / N_0 = 5dB \) for both schemes. In order to obtain small \( \rho \) for the non-unitary precoding matrices, \( 2 \times 64 \) Grassmannian codebook is adopted for the proposed scheme. For SU-MIMO, the BS selects just one user to transmit \( K = 2 \) data streams for a single user. However, for MU-MIMO, the BS selects \( K = 2 \) uses to transmit one data stream for each user simultaneously.

From the simulation results, we can see that the proposed scheme can effectively improve the MU-MIMO system capacity. It provides more than 1 \( \text{bps/Hz} \) capacity gain than the SU-MIMO system when \( L = 4 \) with the feedback of \( L \) SINR values and \( L \) vector indexes. For a MU-MIMO system with \( M \) transmit antennas, the vector index feedback just requires \( \log_2 M \) bits for each index, since each precoding matrix consists \( M \) precoding vectors. With the increasing of the codebook size \( L \), the system capacity of the proposed MU-MIMO scheme will increase as well as the feedback.

Figure 3 shows the CDF of the system capacity for the SU-MIMO, the unitary codebook based MU-MIMO and the proposed non-unitary codebook based MU-MIMO with \( M = 2 \), \( N = 2 \), \( K = 2 \), \( X = 10 \), \( p_0 / N_0 = 5dB \), linear MMSE detection and equal power allocation. The codebook size with \( L = 8 \) is employed for both the two MU-MIMO schemes. \( 2 \times 64 \) Grassmannian codebook is employed for the proposed scheme.

The proposed non-unitary codebook based MU-MIMO scheme also outperforms the unitary codebook based MU-MIMO scheme, especially when \( L \) is large. Because the non-unitary codebook will be close to the Grassmannian space.
when $L$ is large, thus the Grassmannian precoding provides larger precoding gain than the unitary precoding method, while both schemes could suppress the CCI efficiently.

![Figure 4. Average system capacity CDF of the three schemes](image)

Figure 4 describes the average system capacity of the SU-MIMO, the unitary codebook based MU-MIMO and the proposed non-unitary codebook based MU-MIMO scheme with different number of users to be scheduled at the BS. $M = 2$, $N = 2$, $K = 2$, $p_b / N_0 = 5$dB, linear MMSE detection and equal power allocation are employed for all the three schemes. The codebook size with $L=8$ is employed for both the two MU-MIMO schemes. $2 \times 64$ Grassmannian codebook is employed for the proposed MU-MIMO scheme. The user number $X$ scales from 2 to 16 for the three schemes.

When more users $X$ are waiting to be scheduled, the multi-user diversity gain for MU-MIMO system will increase and the maximum Shannon capacity of SU-MIMO system will increase too. With the increasing number of users to be scheduled at the BS, the performance of the proposed scheme will increase simultaneously. It is clear that the proposed scheme exploits multi-user diversity gain and the precoding gain to improve the MU-MIMO system capacity efficiently.

![Figure 5. BER performance of three schemes](image)

Figure 5 shows the BER performance of the SU-MIMO scheme, the unitary codebook based MU-MIMO scheme and the proposed non-unitary codebook based MU-MIMO scheme. $M = 2$, $N = 2$, $K = 2$, $X = 10$, QPSK, linear MMSE detection, equal power allocation are employed for all the three schemes and no channel coding for them. The codebook size with $L=8$ is employed for both the two MU-MIMO schemes. $2 \times 64$ Grassmannian codebook is employed for the proposed scheme. The result shows that the proposed scheme provides a significant improvement than SU-MIMO and outperforms the unitary codebook based MU-MIMO.

**VII. CONCLUSIONS**

In this paper, a non-unitary codebook based precoding scheme for multi-user MIMO downlink transmission with limited feedback is proposed. The proposed scheme constructs a codebook of non-unitary precoding matrices by grouping Grassmannian precoding vectors according to their correlation coefficient. This method just requires the limited feedback information to exploit multi-user diversity gain and the precoding gain as well as suppressing the CCI effectively. It provides significant improvement to the system capacity than the traditional SU-MIMO scheme and also outperforms the unitary codebook based MU-MIMO scheme.

**REFERENCES**


