Bandwidth and Price Competitions of Wireless Service Providers in Two-stage Spectrum Market

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Abstract—Significant technology progress has been witnessed in the research area of dynamic spectrum access. However, the success of dynamic spectrum access will not be possible without the evolution of an spectrum and service market which is both stable and efficient. In this paper, we investigate the dynamic access spectrum from the economic point of view. We study three-layer spectrum market with spectrum holder, wireless service provider and end users. In a duopoly situation, two wireless service providers participate in bandwidth competition to purchase spectrum and price competition to attract end users, with the aim of maximizing their own profit. We formulate the wireless service providers’ competition as a two-stage game. Under general assumptions about the pricing and demand functions, a unique equilibrium is identified as the outcome of the game, which shows the stability of the market. We further evaluate the market efficiency in a special case of symmetric wireless service providers and affine pricing and demand functions. The result shows the efficiency of the equilibrium is of reasonable level even with non-cooperative wireless service providers.

I. INTRODUCTION

With the development of dynamic spectrum access (DSA), radio spectrum resource will be reallocated by flexible spectrum management schemes rather than the current fixed and inefficient allocation policy controlled mostly by government. Besides the intensive research efforts to design and produce advanced technology (e.g., software defined radio, cognitive radio) [1][2], economic aspects of dynamic spectrum access also draw much attention. Technology and economics are both important for the success of dynamic spectrum access. In this paper, we study the dynamic spectrum access from an economic point of view, as a spectrum market. Within a spectrum market, spectrum bands are treated as commodities flowing from owners to consumers regulated by market mechanisms. The utilization of spectrum resource can be greatly improved compared with current command and control policy due to the supply-demand relationship of the market [3].

As far as economics of spectrum management is concerned, most of the discussions are limited to the two-layer (owner and consumer) market structure. There are comprehensive literatures related to the spectrum auction [4][5]. Spectrum users bidding strategy and auctioneer revenue maximization are their main focuses. Recent development of cognitive radio and dynamic spectrum access arouses some work related to spectrum market [6][7]. In [6], they consider the problem of spectrum sharing among a primary user and multiple secondary users. They formulate the problem as an oligopoly market competition and use a Cournot game to obtain the spectrum allocation for secondary users. The competition among secondary users is limited to the spectrum purchasing. In [7], they propose a spectrum auction framework to allocate spectrum to wireless users. They propose a bidding language using piecewise linear price-quantity curves, two pricing models to address revenue and fairness, and low-complexity market-clearing algorithms to derive prices and allocations in realtime. However, the importance aspect about the strategic behavior of wireless users is not considered.

In this paper, we focus on the profit maximization strategy of non-cooperative wireless service providers. We use a three-layer model to describe the market: spectrum bands are the raw resource flowing from spectrum holders in the first layer to wireless service providers in the second layer; wireless service providers deploy infrastructures and use spectrum bands to provide wireless services to end users in the third layer. Such model can better characterize the situation wireless service providers face, since in dynamic spectrum access wireless service providers not only involve in the competition of attracting end users to subscribe their provided service as usual, but also compete with each other for the spectrum acquisition from spectrum holders. Further, these two competitions are related for wireless service providers’ profit maximization. We share the similar three-layer trading system as the one proposed in [8]. They capture the interaction among spectrum broker, service providers, and end users in a multi-provider setting. However, they do not explicitly manifest the relation between the two competitions, while we focus on this aspect.

For the ease of analysis, we consider a duopoly case, where two wireless service providers purchase spectrum bands from one spectrum holder and produce wireless services for a pool of end users. Considering the sequence of spectrum acquisition and wireless service provision, we formulate it into a two-stage game. In the first stage, wireless service providers simultaneously decide the amount of spectrum to buy from
the spectrum holder. We assume the unit spectrum price charged by the spectrum holder is related to the total demand of wireless service providers. Thus the purchasing decision of one wireless service provider must take the other into consideration. In the second stage, wireless service providers produce the services and set up their prices. We assume end users are able to switch freely between these two providers and the rationing rule for end users is always to turn to the low-price provider, with the unserved market share going to the high-price provider. Therefore, the purchasing decision in the first stage must consider the second stage price competition.

This problem is similar to a classic two-stage Bertrand problem [9] within the theory of industrial organization [10]. The difference lies in the first stage: while the classic problem assumes the independent production or acquisition of resource by two firms, our problem assumes there is an upstream spectrum holder as the common supplier. Nevertheless, we can still prove the existence and uniqueness of a Nash equilibrium with the backward induction method used in [9]. The equilibrium is a pure strategy in which both provider will purchase the amount the same as the equilibrium of quantity game (a variation of the Cournot game). This result shows the stability of the duopoly spectrum market. To evaluate the efficiency of the market, we analyze a special case with affine pricing function of spectrum holder and affine demand function of end users. We prove that the ratio between the market with such non-cooperative wireless service providers and the market where providers are coordinated to maximize the total profit is 8/9.

The rest of the paper is organized as follows. In Section II, we give a detailed description of the system model. The game models and equilibria analysis are presented in Section III. We analyze the efficiency of the market in Section IV. Conclusion is drawn in the last section.

II. SPECTRUM TRADING AND SERVICE PROVISION MODEL

In this section, we present the detailed market model and corresponding assumptions. We consider a duopoly case in which two wireless service providers (provider), both with infrastructures deployed in the same geographical area, buy spectrum from a common spectrum holder (holder) and compete for a pool of end users (user). Providers are non-cooperative in that they both want to optimize their own profit. They participate in the following two-stage competitions:

A. First Stage: Spectrum Holder Side, Bandwidth Competition

At the first stage, providers simultaneously buy spectrum bandwidth from the holder, \(b_1\) and \(b_2\) respectively. The holder can be spectrum management regulator of the country (e.g. FCC, Ofcom) or primary user, who experiences low utilization of its allocated spectrum whilst wishes to generate extra revenue from it. The holder charges providers for the leased spectrum according to pricing function \(C(b_1 + b_2)\). We make the following assumption for this function,

Assumption 1: The function \(C(b)\) is strictly positive, non-decreasing and convex for \(b > 0\).

Holder does not differentiate between two providers, charging them at the same price. Each provider chooses its purchasing amount according to the payment from service provision and purchasing strategy of its rival.

B. Second Stage: End User Side, Price Competition

After the first stage of spectrum purchasing, each provider learns how much spectrum its opponent buys. Then, in the second stage, they simultaneously and independently choose price \(p_i\) to provide service to end users. The users are assumed to have the ability to freely switch to any of the providers. They can either be traditional wireless devices communicating with similar systems of the two providers or be the advanced devices equipped with software defined radio / cognitive radio.

The demand of end users is determined by Bertrand-like price competition, subject to capacity constraints generated by the first stage. The system capacities \(k_i\) produced by provider \(i\) is determined by the spectrum bandwidth bought in the first stage,

\[
k_i = \theta_i b_i,
\]

where \(\theta_i\) is provider \(i\)'s spectrum usage efficiency factor. Here, capacity is simply interpreted as the maximum amount of throughput a user can support.

The services of the providers are perfectly substitutable such that the market demand can be expressed by \(P(d)\) (price as a function of demand \(d\)) or \(D(p) = P^{-1}(p)\) (demand as a function of price \(p\)). We make the following assumption for the demand function:

Assumption 2: The function \(P(d)\) is strictly positive for \(0 < d < K\), on which it is twice-continuously differentiable, strictly decreasing and concave. For \(d \geq K\), \(P(d) = 0\).

We also make an assumption about end users’ rationing rule,

Assumption 3: The rationing rule used by users is that users will buy first from the cheapest provider and any unsatisfied demand goes to the other provider.

Formally, if \(p_1 < p_2\), provider 1 first sells capacity

\[
d_1 = \min(\theta_1 b_1, D(p_1))
\]

at price \(p_1\), then provider 2 sells capacity

\[
d_2 = \min(\theta_2 b_2, \max(0, D(p_2) - \theta_1 b_1))
\]

at price \(p_2\). It is similar for \(p_2 < p_1\). If \(p_1 = p_2\), we assume the two provider will equally share all or part of the user market,

\[
d_i = \min(\theta_i b_i, \max(D(p_i)/2, D(p_i) - \theta_i b_i))
\]

at price \(p_i\). We assume each provider seeks to maximize the expectation of its profit, and the above competition structure is common knowledge between the providers.
III. Static Game Analysis

To study the strategic behavior of wireless service providers in the market described previously and characterize market status, we in this section formalize the competition between the duopolies as a two-stage providers’ game. In short, the game is as follows:

1) In stage 1, two providers decide their purchasing bandwidth \( b_1 \) and \( b_2 \), given the holder’s pricing function \( C(\cdot) \).
2) In stage 2, two providers decide their price \( p_1 \) and \( p_2 \), given the bandwidth constraints \( (b_1, b_2) \), according to the market demand function \( P(\cdot) \).

We solve for subgame perfect equilibrium of this game by applying the concept of backward induction. Given a fixed spectrum purchasing amount of \((b_1, b_2)\), which is a subgame of the full game, providers compete with pricing and result in a game outcome related to \((b_1, b_2)\). Then they calculate the best strategy \((b_1^*, b_2^*)\) of the full game. The detailed analysis is as follows.

A. Subgame with Price Competition

After stage 1 providers buy \( b_1, b_2 \) respectively and \((b_1, b_2)\) becomes common knowledge. From this point providers play a \((b_1, b_2)\) capacity-constrained subgame, which is the Edgeworth competition [11]. Provider \( i \) can provide service capacity \( x_i \leq \theta_i b_i \) at unit price \( c \). Here we assume \( c = 0 \) for simplicity. The rationing rule is according to Assumption 3.

According to the seminal paper [9], the analysis can be briefly summarized as follows. For details, please refer to [9].

1) First consider the existence of a pure-strategy equilibrium. It can be shown that such an equilibrium exists if and only if the capacities belong to some region just above the origin in the capacity space. The equilibrium is for providers to dump their capacities.

Lemma 1: In a pure-strategy equilibrium, \( p_1 = p_2 = P(\theta_1 b_1 + \theta_2 b_2) \). Providers sell up to capacity. Denote \( r_i(b_j) \) as provider \( i \)'s optimal reaction to \( j \)'s action \( b_j \) in the one-stage capacity game:

\[
r_i(b_j) = \arg\max_{b_i} \theta_i b_i P(\theta_i b_i + \theta_j b_j)
\]

Denote \( R_i(b_j) \) as the provider \( i \)'s collected payment associated with \( r_i(b_j) \):

\[
R_i(b_j) = \theta_i r_i(b_j) P(\theta_i r_i(b_j) + \theta_j b_j).
\]

Lemma 2: In a pure-strategy equilibrium, provider \( i \) never charge less than \( P(\theta_i r_i(b_j) + \theta_j b_j) \) in the subgame. Lemma 1 and 2 imply that a pure-strategy equilibrium exists only if, for all \( i \),

\[
\bar{b}_i \leq r_i(\bar{b}_j)
\]

2) Next is to characterize the equilibrium when capacities are “high”. It is proved that the profit of the highest-capacity provider is equal to the Stackelberg follower profit. Outside the region of pure-strategy, there exists mixed-strategy equilibrium. Lemma 3: In the mixed-strategy region \((\bar{b}_1 > r_i(\bar{b}_j))\) for at least one provider \( i \), the highest-capacity provider \( i \) makes a profit equal to:

\[
\theta_i r_i(\bar{b}_j) P(\theta_i r_i(\bar{b}_j) + \theta_j \bar{b}_j)
\]

B. Full game

First we define a WSP-bandwidth game: two providers compete in a single stage bandwidth game with \( b_1, b_2 \). The production cost is the spectrum cost \( C(\cdot) \) and the market price is \( P(\cdot) \). The profit of provider \( i \) is

\[
u_i(b_1, b_2) = \theta_i b_i P(\theta_i b_1 + \theta_2 b_2) - b_i C(b_1 + b_2)
\]

Since \( P(\cdot) \) is concave and \( C(\cdot) \) is convex, a unique equilibrium exists which satisfies:

\[
\frac{\partial u_i}{\partial b_i} = 0.
\]

We denote this equilibrium using the bandwidth pair \( b_i^* \) and \( b_j^* \). The corresponding prices are \( p_i^* = p_j^* = P(\theta_i b_i^* + \theta_j b_j^*) \).

Theorem 1: The equilibrium in the full game is the bandwidth outcome of the responding WSP-bandwidth game.

Proof: We basically follow the proof in [9].

Step 1: Consider any equilibrium. As part of this equilibrium provider \( i \) chooses bandwidth according to some probability measure \( \mu_i \), with support \( S_i \). Let us denote by \( \Phi(\cdot) \) the strategy used by provider \( i \) in the \((b_1, b_2)\) subgame. Except for a \( \mu_1 \times \mu_2 \) null subset of \((S_1 \times S_2) \), \( \Phi(\cdot) \) must be an optimal response to \( \Phi(\cdot) \). Let \( \pi_i(\cdot) \) denote the expected profit of provider \( i \) in this equilibrium. Let \( \bar{b}_1 \) and \( \bar{b}_2 \) denote the supremum and infimum. Because the subgame equilibrium profit functions are continuous in \( b_1 \) and \( b_2 \), and because profits are bounded in any event, \( \bar{b}_1 \) and \( \bar{b}_2 \) must yield expected profit \( \pi_i \) if provider \( j \) uses its equilibrium bandwidth strategy \( \mu_j \) and providers subsequently use subgame equilibrium price strategies.

Step 2: We must have \( \bar{b}_1 \geq r_1(\bar{b}_2) \). Suppose that \( \bar{b}_1 < r_1(\bar{b}_2) \). For every \( b_1 < \bar{b}_1 \), the subgame equilibrium profit of provider 2 is \( \theta_2 b_2 P(\theta_1 b_1 + \theta_2 b_2) \), if it buys bandwidth \( b_2 \). Then

\[
\pi_2 = \int_{\bar{b}_1}^{\bar{b}_2} (\theta_2 b_2 P(\theta_1 b_1 + \theta_2 b_2) - b_2 C(b_1 + b_2)) \mu_2(\text{d}b_2).
\]

If provider 2 increases its bandwidth slightly to \( b_2 + \epsilon \) such that \( b_1 < r_1(b_2 + \epsilon) \) is still hold, then the worst that can happen to provider 2 (for each level of \( b_1 \) is that provider 2 will get \( \theta_2(b_2 + \epsilon) P(\theta_1 b_1 + \theta_2(b_2 + \epsilon)) - (b_2 + \epsilon) C(b_1 + b_2 + \epsilon) \). Since for all \( b_1 < \bar{b}_1, b_2 + \epsilon < r_2(b_1) \) and we have Assumption 1 and Assumption 2, it follows that

\[
\theta_2(b_2 + \epsilon) P(\theta_1 b_1 + \theta_2(b_2 + \epsilon)) - (b_2 + \epsilon) C(b_1 + b_2 + \epsilon) > \theta_2 b_2 P(\theta_1 b_1 + \theta_2 b_2) - b_2 C(b_1 + b_2),
\]

and this variation will raise provider 2’s profit. Contradiction.

Step 3: We must have \( \bar{b}_1 \leq r_1(\bar{b}_2) \). Suppose that \( \bar{b}_1 > r_1(\bar{b}_2) \). Provider 1 buys \( \bar{b}_1 \), and gets \( R_1(b_2) \) if \( \bar{b}_1 > r_1(b_2) \), and \( \bar{b}_1 P(\bar{b}_1 + b_2) \) if \( \bar{b}_1 \leq r_2(b_2) \), assuming that a subgame equilibrium ensues. Then:
\[ \pi_1 = \int_{r_1^{-1}([\bar{b}_1], \bar{b}_2]} (R_1(b_2) - \bar{b}_1 C(\bar{b}_1 + b_2)) \mu_2(db_2) + \int_{[\bar{b}_2, r_1^{-1}([\bar{b}_1], \bar{b}_2])] (\theta_1 \bar{b}_1 P(\theta_1 \bar{b}_1 + \theta_2 b_2) - \bar{b}_1 C(\bar{b}_1 + b_2)) \mu_2(db_2) \] (1)

Consider what happens to provider 1’s expected profit if it lowers its capacity from \( \bar{b}_1 \) to \( \bar{b}_1 - \epsilon \), where \( \bar{b}_1 - \epsilon > r_1(\bar{b}_2) \). Then the worst that can happen to provider 1 is that provider 2 (after buying bandwidth according to \( \mu_2 \)) names price zero. This would leave provider 1 with residual demand \( D(p) - \theta_2 b_2 \) (where \( b_2 \leq \bar{b}_2 \)). Provider 1 can still accrue profit \( R_1(b_2) \) if \( \bar{b}_1 - \epsilon > r_1(\bar{b}_2) \) and \( \bar{b}_2 P(\bar{b}_1 - \epsilon + \theta_2 b_2) \) otherwise. Thus, the expected profit of provider 1 in this variation is at least

\[ \int_{[r_1^{-1}(\bar{b}_1 - \epsilon), \bar{b}_2]} (R_1(b_2) - (\bar{b}_1 - \epsilon) C(\bar{b}_1 - \epsilon + b_2)) \mu_2(db_2) + \int_{[\bar{b}_2, r_1^{-1}([\bar{b}_1], \bar{b}_2])] (\theta_1 (\bar{b}_1 - \epsilon) P(\theta_1 (\bar{b}_1 - \epsilon) + \theta_2 b_2) - (\bar{b}_1 - \epsilon) C(\bar{b}_1 - \epsilon + b_2)) \mu_2(db_2) \] (2)

The difference (2) minus (1) can be analyzed by breaking the integrals into three intervals: \( [r_1^{-1}(\bar{b}_1 - \epsilon), \bar{b}_2] \), \( [\bar{b}_2, r_1^{-1}(\bar{b}_1), \bar{b}_2] \), and \( (r_1^{-1}(\bar{b}_1), r_1^{-1}(\bar{b}_1 - \epsilon)) \).

1) \([r_1^{-1}(\bar{b}_1 - \epsilon), \bar{b}_2], \) the difference in integrands is

\[ (R_1(b_2) - (\bar{b}_1 - \epsilon) C(\bar{b}_1 - \epsilon + b_2)) - (R_1(b_2) - \bar{b}_1 C(\bar{b}_1 + b_2)) = \epsilon (\bar{b}_1 C'(\bar{b}_1 - \epsilon + b_2) + C(\bar{b}_1 - \epsilon + b_2)) + o(\epsilon) \]

which is positive when \( \epsilon \) goes to zero since \( C'(\cdot) \) and \( C'(\cdot) \) are positive.

2) \([\bar{b}_2, r_1^{-1}(\bar{b}_1)], \) the difference in integrands is

\[ (\theta_1 (\bar{b}_1 - \epsilon) P(\theta_1 (\bar{b}_1 - \epsilon) + \theta_2 b_2) - (\bar{b}_1 - \epsilon) C(\bar{b}_1 - \epsilon + b_2)) - (\theta_1 \bar{b}_1 P(\theta_1 \bar{b}_1 + \theta_2 b_2) - \bar{b}_1 C(\bar{b}_1 + b_2)) = \epsilon (\bar{b}_1 C'(\bar{b}_1 - \epsilon + b_2) + C(\bar{b}_1 - \epsilon + b_2)) + o(\epsilon) \]

Here the term premultiplied by \( \epsilon \) is strictly positive except possibly at the lower boundary (where it is nonnegative), since by step 2, \( \bar{b}_1 \geq r_1(\bar{b}_2) \geq r_1(b_2) \).

3) \((r_1^{-1}(\bar{b}_1), r_1^{-1}(\bar{b}_1 - \epsilon)), \) the difference in integrands is no more than \( O(\epsilon), \) because of

\[ \theta_1 b_1 P(\theta_1 b_1 + \theta_2 b_2) - b_1 C(b_1 + b_2) \]

is continuous.

Hence for small enough \( \epsilon \), the difference between (2) and (1) will be strictly positive. Contradiction.

**Step 4:** The rest is easy. Steps 2 and 3 imply that \( \bar{b}_1 = r_1(\bar{b}_2) = r_1(b_2) \), and hence that provider 2 uses a pure strategy in the first round. But then provider 1’s best response in the first round is the pure strategy \( r_1(b_2) \). And provider 2’s strategy, which must be a best response to this, must satisfy \( b_2 = r_2(\bar{b}_1) = r_2(r_1(b_2)) \). This implies that \( b_2 = b_2' \), and, therefore, \( b_1 = r_1(b_2') = b_1' \). Finally, the two providers will announce prices \( p_1' \) and \( p_2' \) in the second round.

As a summary of this section, we prove the existence and uniqueness of equilibrium for the two-stage game, which stands for the stable of the duopoly spectrum market.

**IV. Efficiency Analysis**

In this section, we further evaluate the efficiency of the market stable point. The fact that Nash equilibria of a game may not achieve full efficiency has been well known in the economics literature [12]. Recent research efforts have focused on quantifying this loss for specific game environments; the resulting degree of efficiency loss is known as the Price of Anarchy [13].

We now calculate the global optimal, that is we assume the providers are coordinated so that maximization of total profit generated by the two providers is the target:

\[ \max R_{opt} = \max \sum_{i=1,2} p_i d_i(b_1, b_2) - b_i C(b_1 + b_2) \]

**Lemma 4:** For the global optimal solution \((b_1^*, b_2^*; p_1^*, p_2^*)\), it holds that,

\[ p_1^* = p_2^* = P(\theta_1 b_1^* + \theta_2 b_2^*) \]

**Proof:** It must be the case that the providers will set bandwidth and price so that the capacities they produce are just sold out. Otherwise, if the provider \( i \) ends up with abundant capacity, it can just reduce its purchasing amount \( b_1^* \) by \( \epsilon \), which keeps the total payments unchanged while cuts the cost of buying the spectrum; if the capacity supply cannot catch up with user demand, providers can just raise the price with the payment increased and cost unchanged. In both cases, the total profit increases, which is a contradiction.

Given a fixed bandwidth pair \( b_1^p \) and \( b_2^p \), the total profit is

\[ R_{opt} = p_1^p \theta_1 b_1^p + p_2^p \theta_2 b_2^p - (b_1^p + b_2^p) C(b_1^p + b_2^p) \]

Suppose \( p_1^p \leq p_2^p \), according to the rationing rule we must have the following constraints

\[ p_1^p \leq P(\theta_1 b_1^p), \quad p_2^p \leq P(\theta_1 b_1^p + \theta_2 b_2^p) \]

We can get

\[ p_1^p \leq p_2^p \leq P(\theta_1 b_1^p + \theta_2 b_2^p) \]

so that the capacities are just sold out. Since \( R_{opt} \) is strictly increasing in terms of \( p_1^p \) and \( p_2^p \). Thus to maximize \( R_{opt} \), we have (4).

It is similar for \( p_1^p \leq p_2^p \)

**Lemma 5:** For the global optimal solution \((b_1^*, b_2^*; p_1^*, p_2^*)\), if \( \theta_1 < \theta_2 \), then \( b_1^p = 0 \).

**Proof:** Suppose \( b_1^p \neq 0 \), we can let provider \( j \) to purchase \( b_1^p \) such that

\[ \theta_j b_j^p = \theta_i b_i \]


Since $\theta_i < \theta_j$, we get $b_i^c > b_j^c$. From the previous lemma, we further have $p_i^c = p_j^c = \frac{1}{\beta} \alpha + (\beta p_i + \beta b_i^c) = \frac{1}{\beta} \alpha + \beta (\theta_i b_i^c + \theta_j b_j^c)$. The revenue $R_{\text{opt}}$ is then increased.

**Theorem 2:** The optimal solution is: if $\theta_i < \theta_j$, then

$$b_i^c = 0$$

$$b_j^c = \arg\max_{b_j} \theta_j b_j P(\theta_j b_j) - b_j C(0 + b_j)$$

It is similar for $\theta_i > \theta_j$ and equal share is reasonable for $\theta_i = \theta_j$.

**Proof:** It is obvious from Lemma 4 and Lemma 5. □

We next consider a symmetric case and present its analytical result. There are some assumptions:

1) We assume the similar format of pricing function as in [6]:

$$C(b) = \alpha + \beta b^\gamma$$

where $\alpha$, $\beta$, and $\gamma$ are non-negative constants. We assume $\gamma \geq 1$, therefore $C(\cdot)$ is convex. We check for the case where $\gamma = 1$.

$$C(b) = \alpha + \beta b$$

2) The market reverse demand function of end users is also affine,

$$P(k) = A - Bk$$

where $A, B$ are positive constants.

3) The two providers are symmetric, $\theta_1 = \theta_2 = 1$.

The profit of provider $i$ is

$$u_i = b_i P(b_1 + b_2) - b_i C(b_1 + b_2) = b_i [(A - \alpha) - (B + \beta)(b_1 + b_2)]$$

For the Nash equilibrium, from Theorem 1 we obtain the following equations:

$$\frac{\partial u_i}{\partial b_i} = 0, \quad i = 1, 2$$

that is

$$2b_1 + b_2 = \frac{A - \alpha}{B + \beta}, \quad b_1 + 2b_2 = \frac{A - \alpha}{B + \beta}$$

The equilibrium point is

$$b_1^* = b_2^* = \frac{1}{3} \frac{A - \alpha}{B + \beta}$$

For the global optimal, since we assume the symmetric case $b_1 = b_2 = b$, from Theorem 1 we obtain the following equations:

$$R_{\text{opt}} = 2bp_2(2b) - 2bC(2b) = 2b[(A - \alpha) - 2b(B + \beta)]$$

We get the optimal point from $\frac{\partial R_{\text{opt}}}{\partial b} = 0$:

$$b_1^* = b_2^* = \frac{1}{4} \frac{A - \alpha}{B + \beta}$$

The efficiency of such market is

$$\text{Efficiency} = \frac{u_1^* + u_2^*}{R_{\text{opt}}} = \frac{8}{9}$$

V. Conclusion

In this paper, we investigate the revenue maximization problem of wireless service providers in dynamic access spectrum. We focus on a duopoly competition between two wireless service providers where they are fed by an upstream spectrum holder and provide wireless access service for a common pool of end users. We explicitly study the relation of such two layer competition and formulate this problem as a two-stage game, which is not paid enough attention in the existing work. Under mild assumptions, we prove that there is a unique equilibrium of the two-stage game, which shows the stability of the market. We also evaluate the market efficiency with symmetric wireless service provider and affine pricing and demand function. The analytical results can provide some insights about investment and service provision for the wireless service providers in the next generation wireless communication.

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