The Strategic Perils of Low Cost Outsourcing

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The existing outsourcing literature has generally overlooked the cost differential and contract negotiations between manufacturers and suppliers (by assuming identical cost structures and adopting Stackelberg-game based models). One fundamental question that yet to be addressed is whether upstream suppliers’ cost efficiency is always beneficial to downstream manufacturers in the presence of competition and negotiations. In other words, does low cost outsourcing always lead to a win–win outcome? To answer this question, we adopt a multiunit bilateral bargaining framework to investigate competing manufacturers’ sourcing decisions. We analyze two supply chain structures: one-to-one channels, in which each manufacturer may outsource to an exclusive supplier; and one-to-two channels, in which each manufacturer may outsource to a common supplier. We show that, under both structures, low cost outsourcing may lead to a win–lose outcome in which the suppliers gain and the manufacturers lose. This happens because suppliers’ cost advantage may backfire on competing manufacturers through two negative effects. First, a decrease of upstream cost weakens a manufacturer’s bargaining position by reducing her disagreement payoff (i.e., her insourcing profit) because the competing manufacturer can obtain a low cost position through outsourcing. Second, in one-to-two channels, the common supplier’s bargaining position is strengthened with a lower cost because his disagreement payoff increases (i.e., his profit from serving only one manufacturer increases). The endogeneity of disagreement payoffs in our model highlights the importance of modeling firm negotiations under competition.

Moreover, we identify an interesting bargaining externality between competing manufacturers when they outsource to a common supplier. Because the supplier engages in two negotiations, his share of profit from the trade with one manufacturer affects the total surplus of the trade with the other manufacturer. Due to this externality, surprisingly, as a manufacturer’s bargaining power decreases, her profit under outsourcing may increase and it may be more likely for her to outsource.

Key words: Outsourcing; Multiunit Bilateral Bargaining; Competition; Supplier Cost Advantage.
1 Introduction

Production outsourcing is common in many industries, including computer, pharmaceutical, automotive, and food and beverage (Tully 1994). The widespread of production outsourcing has fueled the growth of contract manufacturing—worldwide revenue from contract manufacturers reached over $250 billion in 2009, and is forecasted to exceed $370 billion by 2014 (IDC 2010). It is well acknowledged that seeking cost savings is a major, if not the most important, driver for outsourcing (Gareiss 2002, Engardio 2006).

Firms’ pursuit of low cost outsourcing is heavily influenced by their competitive strategies. In the computer industry, large Asian contract manufacturers provide a range of services to PC makers, including sourcing, component production, and final assembly. These suppliers generally have a lower cost structure than PC makers due to factors such as manufacturing expertise, scale economy, as well as location in low-wage countries (Dedrick and Kraemer 2006, iSuppli 2009b). Companies like Hewlett-Packard and Acer have long been relying on such contract manufacturers to assemble final products. Dell, in contrast, kept its own assembly plants in North America to support its build-to-order model. Facing an increasing pressure to achieve cost competitiveness, Dell has recently shifted away from the build-to-order model by outsourcing assembly work to contract manufacturers, the strategy already adopted by its competitors (see Ladendorf 2009a, Ladendorf 2009b, iSuppli 2009a).

Despite its important role in manufacturers’ decision making, the cost-saving benefit of outsourcing seems very intuitive. Partly because of this, the academic literature has largely focused on identifying alternative explanations for the wide adoption of outsourcing by assuming away suppliers’ cost advantage. It is well established that manufacturers are willing to outsource in the absence of suppliers’ cost advantage because outsourcing mitigates market competition (see, e.g., Cachon and Harker 2002, Gilbert, Xia and Yu 2006, Liu and Tyagi 2011). In addition, manufacturers’ tendency to outsource is linked to their bargaining power—some studies find that noncompeting manufacturers with a larger bargaining power obtain a higher outsourcing profit and are more likely to outsource (e.g., Van Mieghem 1999, Plambeck and Taylor 2005). Building on these studies, we attempt to understand how upstream cost advantage impacts competing manufacturers’ outsourcing profit and their negotiations with suppliers. Our central research question is: Do manufacturers always benefit from low cost outsourcing in the presence of competition and negotiation?

To answer this question, we identify a trade-off between two conflicting forces that drive a manufacturer’s outsourcing decision. On the one hand, a manufacturer can potentially enjoy cost savings as a result of its supplier’s cost efficiency, which in turn puts an insourcing competitor at a cost disadvantage. On the other hand, the manufacturer has to share part of its profit with the supplier for the outsourcing service. Naturally, profit sharing mechanisms depend on the bargaining power of the manufacturer and the supplier as well as their disagreement payoffs (i.e., their profits in the case of negotiation breakdown). The manufacturer’s disagreement payoff is its insourcing profit and thus depends critically on the degree of market competition. The existing outsourcing
literature apparently has not modeled firm negotiations in a competitive setting, thereby unable to provide insights into this important trade-off in competing manufacturers’ outsourcing decisions.

In this paper, we uncover that outsourcing to low cost suppliers may backfire on competing manufacturers, leading to a *win–lose* outcome, in which the suppliers gain and the manufacturers lose. In some cases, manufacturers with larger bargaining power may obtain a lower outsourcing profit and thus may be *less likely* to outsource than those with smaller bargaining power. We derive these results in a two-tier supply chain model with two competing manufacturers, who may trade with separate suppliers, i.e., in *one-to-one channels*, or with a common supplier, i.e., in *one-to-two channels*. We use a *multiunit bilateral bargaining* framework to model negotiations of outsourcing agreements. Each manufacturer negotiates bilaterally with her supplier over an output quantity and profit allocation. Under competition, the bargaining outcome of one channel naturally depends on that of the other.

Our analysis of manufacturer–supplier negotiations highlights the strategic perils of low cost outsourcing. In one-to-one channels, the manufacturers’ dominating strategy is to outsource due to the fact that lower production cost attained from outsourcing enhances their competitiveness in the downstream market. However, the manufacturers may earn *less* profits when they both outsource than when they both insource. This outcome stems from two opposite effects of upstream cost advantage on the manufacturers’ outsourcing profits: First, a *product profit effect* represents increased product profitability as a result of cost reduction. Second, a *downstream bargaining effect* captures the fact that the suppliers’ cost advantage weakens the manufacturers’ bargaining positions by reducing their disagreement payoffs. The reason is that if a manufacturer were to insource, she would compete unfavorably against the rival manufacturer, who enjoys a low cost position through outsourcing. This negative bargaining effect dominates the positive product profit effect when a manufacturer has low bargaining power vis-à-vis her supplier. In this case, outsourcing leads to a win–lose outcome. Therefore, the suppliers’ cost advantage, while always benefiting themselves, can be a double-edged sword for the competing manufacturers.

In one-to-two channels, the aforementioned two effects of upstream cost advantage preserve. In addition, there is a third effect—the *upstream bargaining effect*. Specifically, the common supplier enjoys an enhanced bargaining position when his production cost decreases. This effect stems from the common supplier’s advantage to earn a positive profit even when one of the two negotiations breaks down. Furthermore, we identify a *bargaining externality* between the two negotiations—each manufacturer enjoys part of the competing channel’s profit because they trade with a common supplier. The bargaining externality may cause a manufacturer’s profit to decrease in her bargaining power. This counterintuitive behavior is caused by the fact that an increase of a manufacturer’s bargaining power reduces the bargaining externality she enjoys.

In the sourcing equilibrium of the one-to-two channels, both products are outsourced to the common supplier when the manufacturers have similar in-house production costs. If, however, the
manufacturers differ significantly in their cost positions, the high-cost one always outsources and the low cost one tends to insource when her bargaining power is large. In this case, outsourcing does not significantly improve the market competitiveness of the low cost manufacturer. Moreover, with a large bargaining power, the low cost manufacturer enjoys a small bargaining externality in the outsourcing negotiation (as we explained earlier). Consequently, the powerful, low cost manufacturer may find insourcing a better option than outsourcing. This observation makes an interesting contrast to earlier studies on sourcing decisions of monopolist manufacturers (see, e.g., Van Mieghem 1999, Plambeck and Taylor 2005), which find powerful manufacturers more likely to outsource. Our result suggests that bargaining power may affect manufacturers’ sourcing decisions differently in the presence of competition and upstream cost advantage.

The remainder of the paper is organized as follows. We discuss related literature and our contributions in §2 and present the model in §3. We analyze the one-to-one channels in §4 and the one-to-two channels in §5. We conclude in §6 and relegate all proofs to the Appendix.

2 Literature Review

This paper serves as a commentary on the broad literature of outsourcing. The widespread use of contract manufacturing in various industries has generated diverse interest on production outsourcing in the operations management literature. Lee and Tang (1996) and Iyer, Schwarz and Zenios (2005) study the managerial issues that arise in managing suppliers and the outsourced production processes. Others investigate how a manufacturer’s outsourcing decision is affected by factors such as contract type (Van Mieghem 1999), scale economies (Cachon and Harker 2002), capacity pooling (Plambeck and Taylor 2005), timing of market entry (Ülkü, Toktay and Yucesan 2005), allocation of demand risks (Ülkü, Toktay and Yucesan 2007), product substitution (Liu and Tyagi 2011), and learning-by-doing (Anderson and Parker 2002, Gray, Tomlin and Roth 2009). Beyond manufacturing, service outsourcing has also received growing attention. A series of studies construct contracts in a bilateral-monopoly setting to coordinate staffing, quality, and effort decisions (e.g., Hasija, Pinker and Shumsky 2008, Ren and Zhou 2008, Akşin, de Véricourt and Karaesmen 2008). In competitive settings, the research work centers on the benefit of outsourcing (Allon and Federgruen 2008) and the value of competition to promote service quality (Benjaafar, Elahi and Donohue 2007).

Moreover, several papers identify the strategic benefit of outsourcing in alleviating competitive pressure. Cachon and Harker (2002) show that outsourcing mitigates competition between two firms who face scale economies in production. Gilbert et al. (2006) demonstrate a similar effect when two firms engage in product competition with the opportunity to invest in cost reduction. Arya, Mittendorf and Sappington (2008) suggest that a retailer may outsource production to a common supplier to raise its rival’s cost. These results share a similar spirit with the findings of the strategic decentralization literature (see, e.g., McGuire and Staelin 1983, Coughlan and Wernerfelt 1989).
In contrast to the aforementioned studies, we focus on the strategic implications of low cost outsourcing by explicitly modeling the production cost difference between upstream and downstream firms. For this purpose, we intentionally leave out various factors analyzed in those papers that may interact with the cost effect. Most of the aforementioned outsourcing studies consider industry structures consisting of either one-to-one channels (e.g., McGuire and Staelin 1983, Gilbert et al. 2006, Liu and Tyagi 2011) or one-to-many channels (e.g., Cachon and Harker 2002, Plambeck and Taylor 2005, Ülkü et al. 2005, Ülkü et al. 2007). These two structures have also been widely adopted in other supply chain contexts (e.g., Cachon and Lariviere 2005, Ha and Tong 2008, Cachon and Kök 2010). We analyze both structures in this paper and point out the fundamental differences between them in determining negotiation outcomes.

A convention in the outsourcing and supply chain contracting literatures is to either assume exogenous contract parameters or derive endogenous contract parameters using the Stackelberg framework. As Lovejoy (2010) points out, a bargaining model would be more appropriate in many supply chain contexts because the solutions derived from the Stackelberg framework can be highly impractical due to various issues. As an attempt to construct a more realistic model, we adopt a multiunit bilateral bargaining framework to capture the interactions between trading parties in outsourcing relationships. Specifically, our model features two interdependent bilateral bargaining units. The negotiation outcomes are derived as a Nash equilibrium of two parallel Nash bargaining problems. This solution concept, known as the Nash–Nash approach, has been adopted by many researchers in economics (e.g., Davidson 1988, Dobson and Waterson 1997, Horn and Wolinsky 1988, Gal-Or 1999, O’Brien and Shaffer 2005, Milliou and Petrakis 2007, Björnerstedt and Stennek 2007, Symeonidis 2008) and marketing (e.g., Chipty and Snyder 1999, Shaffer and Zettelmeyer 2002, Dukes and Gal-Or 2003, Shaffer and Zettelmeyer 2004, Dukes, Gal-Or and Srinivasan 2006). We are arguably the first to apply this framework to an outsourcing problem, and we have identified novel effects of bargaining on outsourcing decisions.

3 The Model

We study two competing supply chains, indexed by $i, i = 1, 2$. Each supply chain consists of a manufacturer (she) and a supplier (he), both indexed by $i$ as well. The manufacturers sell differentiated products and have the option to produce in-house, or to outsource production to an upstream supplier. We analyze two industry structures that have been commonly employed in the literatures of outsourcing and channel management (see §2). The first structure consists of two one-to-one channels, in each of which a single supplier serves a single manufacturer. The second structure consists of one-to-two channels, in which a common supplier serves both manufacturers. Both structures, although an abstraction from reality, find their resemblance in outsourcing practices. For example, in the computer industry, one of the largest contract manufacturers, Flextronics, manufactured products for Dell, Hewlett-Packard, and others, while another major contract man-
ufacturer, Solectron, made products for IBM and NEC (Ruffolo 2007). When Flextronics acquired Solectron in 2007, competing OEMs like Hewlett-Packard and IBM, used to source from the two suppliers separately, started to work with the same firm.

**Cost Structure.** If manufacturer \( i \) produces in-house, she incurs a constant marginal cost, denoted by \( c_i \). If manufacturer \( i \) outsources production, the supplier incurs a constant marginal cost, denoted by \( c_S \). For simplicity, we assume that the suppliers have symmetric cost structure. Consistent with the reality that outsourcing is generally associated with cost savings, we assume that \( 0 < c_S < c_i \). We exclude the trivial case of \( c_S = c_i \), under which manufacturer \( i \) becomes indifferent between insourcing and outsourcing in our model.

**Bargaining.** The manufacturer and the supplier in each channel negotiate on the output quantity and on the split of the product profit. When an outsourcing agreement is reached, the manufacturer outsources production to the supplier. If either party chooses not to trade, the manufacturer produces in house. We adopt the asymmetric Nash bargaining solution (Nash 1950, Osborne and Rubinstein 1990) to determine the negotiation outcome. Let \( \theta_i \in (0, 1) \) denote manufacturer \( i \)'s bargaining power vis-à-vis her supplier. The economic meaning of bargaining power has been vigorously investigated (see, e.g., Binmore, Rubinstein and Wolinsky 1986, Nagarajan and Bassok 2008). It is worth noting that the supply chain management literature has long acknowledged the shortcomings of using a Stackelberg game to determine profit allocation—letting either the supplier or the manufacturer extract all the surplus of a trade does not reflect the reality that both firms possess some bargaining power (see Cachon and Harker 2002, Lovejoy 2010).

**Product Competition.** The demand function for product \( i \) is given by

\[
p_i = a - bq_i - \gamma bq_j,
\]

\( i = 1, 2 \) and \( j \in \{1, 2\} \setminus i \), where \( a > c_i, b > 0, \) and \( \gamma \in [0, 1] \) measures the degree of substitutability. When \( \gamma \) approaches 0, the two products become independent; when \( \gamma \) approaches 1, they become perfect substitutes.

We introduce the index of outsourcing cost advantage given by

\[
\Delta_i \equiv \frac{a - c_S}{a - c_i}, \quad i = 1, 2.
\]

This index allows us to express the upstream's and the downstream's relative cost positions in the perspective of their potential margins, captured by \( a - c_S \) and \( a - c_i \), respectively. For our subsequent analysis, it is convenient to work with \( (\Delta_i, \Delta_j) \) instead of \( (c_i, c_j) \). The index of outsourcing cost

1. This assumption allows us to conduct equilibrium analysis without assuming any specific contract form, which precludes the associated inefficiency issues.
2. We omit the case of \( \theta_i = 0 \) or \( \theta_i = 1 \), under which either manufacturer \( i \) or her supplier is indifferent between trade and no trade.
advantage for manufacturer $i$ would increase when the upstream’s production cost decreases and/or when the manufacturer’s in-house production cost increases.

4 Independent Suppliers

In this section, we analyze the model involving two one-to-one channels, in which each manufacturer has an exclusive supplier. We first introduce the bargaining framework in §4.1 and then analyze the equilibrium of the outsourcing game in §4.2.

4.1 Bargaining

Consider the negotiation between manufacturer $i$ and supplier $i$ over the profit of product $i$, which we will refer to as bargaining unit $i$. Two possible outcomes arise from the negotiation: (1) one or both parties withdraw from the trade and the manufacturer produces in-house, or (2) an agreement is reached and the supplier makes the product for the manufacturer. An outsourcing agreement should specify the trading quantity $q_i$, the manufacturer’s profit $\Pi_i$, and the supplier’s profit $\pi_i$.

Under this agreement, the profit of product $i$ is given by

$$T_i \equiv q_i \cdot (a - bq_i - \gamma bq_j - c).$$

To define the bargaining problem, we need to specify the trading parties’ disagreement points, i.e., their profits when the negotiation breaks down (Binmore et al. 1986). Let $D_i$ and $d_i$ denote the disagreement point of manufacturer $i$ and supplier $i$, respectively. By definition, $D_i$ is manufacturer $i$’s insourcing profit, and $d_i = 0$ because supplier $i$ makes zero profit under no trade. In the presence of the competing product, i.e., product $j$, manufacturer $i$’s disagreement point $D_i$ depends on whether or not an outsourcing agreement is reached between manufacturer $j$ and supplier $j$. Specifically, manufacturer $i$ would compete against a rival with production cost $c_j$ if manufacturer $j$ insources and a rival with production cost $c_S$ if manufacturer $j$ outsources.

For given $(q_i, q_j)$, the set of feasible profit allocations for bargaining unit $i$ can be defined as

$$\mathcal{B}_i(q_i; q_j) = \{(\Pi_i, \pi_i) : \Pi_i \geq D_i, \pi_i \geq d_i, \Pi_i + \pi_i \leq T_i\}. \quad (1)$$

To have a well-defined bargaining problem, one needs to verify the existence of a bargaining solution in the feasible set $\mathcal{B}_i(q_i; q_j)$. This is trivially true in our problem because $\mathcal{B}_i(q_i; q_j)$ is convex. Thus, if a deal is settled, the order quantity $q_i \geq 0$ and the profit pair $(\Pi_i, \pi_i) \in \mathcal{B}_i(q_i; q_j)$ maximize the following Nash product

$$\Omega_i \equiv (\Pi_i - D_i)^{\theta_i}(\pi_i - d_i)^{1-\theta_i}. \quad (2)$$

It is straightforward to show that bargaining leads to the following profit allocation

$$\begin{align*}
\Pi_i &= \theta_i(T_i - D_i - d_i) + D_i = \theta_i T_i + (1 - \theta_i)D_i, \\
\pi_i &= (1 - \theta_i)(T_i - D_i - d_i) + d_i = (1 - \theta_i)(T_i - D_i). \quad (3)
\end{align*}$$
These relationships indicate that the trade surplus $T_i - D_i - d_i$ is allocated proportionately based on the parties’ bargaining power. Consequently, bilateral coordination of the supplier–manufacturer pair is achieved by choosing a $q_i$ to maximize the channel profit $\Pi_i + \pi_i = T_i$. It is straightforward to show that when $\theta_i$ increases, manufacturer $i$’s profit increases and supplier $i$’s profit decreases.

Clearly, the negotiation outcome $(\Pi_i, \pi_i, q_i)$ depends on the order quantity $q_j$ (recall the expression of $T_i$), which is part of the negotiation outcome between manufacturer $j$ and supplier $j$. The negotiation outcomes of the two competing supply chains form a Nash equilibrium. This solution concept for our multiunit bilateral bargaining problem is known as the Nash–Nash solution (see §2). This approach is viewed as a direct extension of the single-unit Nash bargaining solution to multiple bargaining units. The validity of this approach has been justified by two theoretical interpretations. First, the Nash–Nash solution corresponds to the subgame perfect equilibrium derived from a dynamic bargaining game with a sufficiently short period length. In the dynamic game, the two parties of each bargaining unit make alternating offers until an agreement is reached or one party withdraws from the negotiation (Davidson 1988, Björnerstedt and Stennek 2007). The second interpretation takes into account the possibility that one bargaining unit may not observe but may form a belief about the bargaining outcome of others. Given this belief, the two parties of a bargaining unit make alternating offers until they settle on an agreement or break up. The rational expectation equilibrium thus derived coincides with the Nash–Nash solution (Horn and Wolinsky 1988, Inderst and Wey 2007).

4.2 Sourcing Equilibrium

Depending on whether the negotiations are successful, four possible equilibrium sourcing structures may arise: (Outsource, Outsource), (Insource, Insource), (Outsource, Insource), and (Insource, Outsource). We denote them with $OO$, $II$, $OI$, $IO$, respectively. We use $\Pi_i^X$ and $\pi_i^X$, $i = 1, 2$, to denote, respectively, manufacturer $i$ and supplier $i$’s profits under structure $X \in \{OO, II, OI, IO\}$.

**Theorem 1 (Equilibrium Sourcing Decisions)** $OO$ arises as a unique equilibrium.

For a given sourcing decision of the rival supply chain, outsourcing always leads to an increased channel profit because the supplier possesses a production cost advantage. Furthermore, the increased channel profit is shared between the manufacturer and the supplier proportionally according to their bargaining power (recall Eq. (3)). Therefore, the manufacturers’ dominating strategy is to outsource.

In view of the strong incentive to outsource, an immediate question is: Does outsourcing lead to a win–win outcome for every firm? For that we compare the firms’ profits under $OO$ to those under $II$. It is clear that the suppliers always gain from outsourcing because they obtain zero profit otherwise. For the manufacturers, however, the answer depends, as specified in the next theorem.
Theorem 2 (Win–Win or Prisoner’s Dilemma) There exists a threshold $\hat{\theta}_i$ such that $\Pi_i^{OO} < (>)\Pi_i^{II}$ when $\theta_i < (>)\hat{\theta}_i$. Moreover, $\hat{\theta}_i$ is independent of $\theta_j$, decreasing in $\Delta_i$, and increasing in $\Delta_j$. If $\Delta_i = \Delta_j = \Delta$, then $\hat{\theta}_i = \hat{\theta}_j$ is decreasing in $\Delta$ and increasing in $\gamma$.

Figure 1: The impact of outsourcing on profits depends on the manufacturers’ bargaining power and the indices of outsourcing cost advantage, as characterized in Theorem 2 ($\gamma = 0.85$, Mi: manufacturer $i$).

We illustrate the equilibrium outcomes in the space of the manufacturers’ bargaining power and the indices of outsourcing cost advantage in Figure 1. The left panel depicts the case when the two supply chains are symmetric, i.e., $\theta_1 = \theta_2 = \theta$ and $\Delta_1 = \Delta_2 = \Delta$. We observe that outsourcing, compared to insourcing, may lead to a win–lose outcome with the suppliers being better off and the manufacturers worse off. In this case, because their profits would be higher if both insource, the manufacturers are trapped in a prisoner’s dilemma to outsource in equilibrium. Note that even a significant cost advantage under outsourcing, i.e., a large $\Delta$, would not save the manufacturers from a losing position when their bargaining power is low. As the products become more substitutable (i.e., as $\gamma$ increases), the manufacturers are more likely to end up with a losing outcome.

The middle and right panels of Figure 1 further demonstrate the game outcome when the manufacturers may differ in their bargaining power or in-house production cost. We observe that whether or not a manufacturer loses from outsourcing depends only on her own bargaining power, but not on the rival manufacturer’s bargaining power. Furthermore, a manufacturer is more likely to lose from outsourcing if she has a relatively low cost position (i.e., with a low index of outsourcing cost advantage) or if the rival manufacturer has a relatively high cost position (i.e., with a high index of outsourcing cost advantage).

What causes the manufacturers to lose when outsourcing to a low cost supplier? To answer this question, we express the firms’ profits under OO as (recall Eq. 3)

$$\Pi_i^{OO} = \theta_i T^{OO}(c_S) + (1 - \theta_i) D_i^{OO}(c_S), \quad (4)$$

$$\pi_i^{OO} = (1 - \theta_i) T^{OO}(c_S) + (1 - \theta_i) D_i^{OO}(c_S), \quad i = 1, 2. \quad (5)$$
where $T^{OO}(c_S)$ is the product profit and $D_i(c_S)$ is manufacturer $i$’s disagreement point. The above profit decompositions allow us to characterize the effects of upstream cost advantage by taking the first derivatives with respect to $c_S$:

$$\frac{d\Pi^{OO}_{i}}{dc_S} = \theta_i \frac{dT^{OO}}{dc_S} + (1 - \theta_i) \frac{dD^{OO}_{i}}{dc_S},$$

(6)

$$\frac{d\pi^{OO}_{i}}{dc_S} = (1 - \theta_i) \frac{dT^{OO}}{dc_S} - (1 - \theta_i) \frac{dD^{OO}_{i}}{dc_S}.$$

(7)

We refer to $\frac{dT^{OO}}{dc_S}$ as the product profit effect, which captures the change in the product profit due to an increased upstream cost. We refer to $\frac{dD^{OO}_{i}}{dc_S}$ as the downstream bargaining effect, which measures the change in the manufacturer’s disagreement point as a result of an upstream cost increase. We characterize the signs of these effects in the next theorem.

**Theorem 3 (Effects of Upstream Cost Advantage)** The product profit effect $\frac{dT^{OO}}{dc_S} < 0$ and the downstream bargaining effect $\frac{dD^{OO}_{i}}{dc_S} > 0$. When $c_S$ decreases,

i) supplier $i$’s profit increases, and

ii) there exists a $\tilde{\theta}_i \in [0, 1]$ such that manufacturer $i$’s profit increases (decreases) for $\theta_i > (<) \tilde{\theta}_i$.

Moreover, $\tilde{\theta}_i$ is weakly decreasing in $\Delta_i$ and is independent of $\theta_j$ and $\Delta_j$.

It is intuitive that the product profit is always increasing as the upstream suppliers become more cost efficient. Meanwhile, a decreased upstream cost always weakens the manufacturers’ bargaining positions. Namely, in the presence of product competition, a manufacturer’s disagreement point in the bilateral negotiation, i.e., her insourcing profit, is endogenous and decreases when the upstream cost reduces. This effect stems from the improved cost position of the competing manufacturer who outsources. For a manufacturer with low bargaining power, the effect of weakened bargaining position dominates (note that the last term in Eq.(6) is decreasing in $\theta_i$), leading to a losing outcome for her. Overall, the suppliers always benefit from their own cost reduction, whereas the manufacturers may gain or lose depending on their bargaining power. The behavior of the competing manufacturers’ disagreement points contrasts sharply with that in a bilateral monopoly setting where the monopolist manufacturer’s reservation profit is fixed. This bargaining effect highlights the importance of modeling firm negotiations in a competitive supply chain setting.

Before ending this section, we shall remark that in one-to-one channels, the interdependence between the bargaining pairs is only through product competition. The profit allocation of one supply chain, however, does not depend on that of the competing supply chain. Consequently, the thresholds of $\theta_i$ derived in Theorems 2 and 3 do not depend on $\theta_j$. This is generally not true in one-to-two channels, as we shall show in the next section.
5 A Common Supplier

Now we consider an alternative supply chain structure, in which a common supplier may provide production service to both manufacturers. The bargaining solution is described in §5.1 and the equilibrium analysis is presented in §5.2.

5.1 Bargaining

The bargaining problem can be formulated similarly to the one described in §4.1. There still exist two bargaining units, in each of which the common supplier negotiates separately with a manufacturer. This bilateral formulation of negotiations reflects the reality that competing manufacturers typically negotiate independently. The key difference lies in the specification of the supplier’s profit and disagreement point. Specifically, the supplier’s profit becomes

\[ \pi = \pi_1 + \pi_2, \]

where \( \pi_i \) is his profit from product \( i, i = 1, 2 \). If the negotiation between the supplier and manufacturer \( i \) breaks down, the supplier may still earn a positive profit by serving manufacturer \( j \). Therefore, his disagreement point \( d_i \) may be positive.

The set of feasible profit allocations between the supplier and manufacturer \( i \) can be defined as (recall that \( T_i \) is the profit of product \( i \))

\[ B_i(q_i; q_j, \Pi_j) = \{ (\Pi_i, \pi) : \Pi_i \geq D_i, \pi \geq d_i, \Pi_i + \pi \leq T_i + T_j - \Pi_j \}. \tag{8} \]

Comparing Eq.(1) and Eq.(8), we note that the constraint \( \Pi_i + \pi \leq T_i \) in the former is replaced by \( \Pi_i + \pi \leq T_i + T_j - \Pi_j \). The change reflects the fact that the common supplier earns profit from both products. It is straightforward to verify that \( B_i(q_i; q_j, \Pi_j) \) in Eq.(8) is a convex set. Thus, if a deal is settled, the order quantity \( q_i \geq 0 \) and the profit pair \( (\Pi_i, \pi) \in B_i(q_i; q_j, \Pi_j) \) maximize the following Nash product

\[ \Omega_i \equiv (\Pi_i - D_i)^{\theta_i}(\pi - d_i)^{1-\theta_i} \]

If manufacturer \( j \) insources, then \( T_j = \Pi_j \) and the problem reduces to the one in §4.1. Therefore, the change in industry structure only affects the firms’ profits when both manufacturers outsource. We focus on analyzing this case and use \( \hat{OO} \) to denote the sourcing structure of (Outsource, Outsource) with a common supplier. Under \( \hat{OO} \), we can specify the bargaining game with

\[ D_1 = \Pi_1^{IO}, \quad D_2 = \Pi_2^{IO}, \]
\[ d_1 = \pi_1^{IO}, \quad d_2 = \pi_2^{IO}. \]

It is also straightforward to show that when an agreement is reached, the trade surplus \( \Pi_i + \pi - D_i - d_i \) is allocated proportionately according to the parties’ bargaining power. That is, for \( i = 1, 2 \),

\[
\begin{align*}
\Pi_i &= \theta_i(\Pi_i + \pi - D_i - d_i) + D_i, \\
\pi &= (1 - \theta_i)(\Pi_i + \pi - D_i - d_i) + d_i.
\end{align*}
\tag{9}
\]
Therefore, manufacturer \(i\) and the supplier should agree on an order quantity \(q_i\) that maximizes their joint profit \(\Pi_i + \pi = T_i + T_j - \Pi_j\) when a deal is settled. Because \(\Pi_j\) is viewed by bargaining unit \(i\) as given, the optimal \(q_i\) maximizes the system profit \(T_i + T_j\), and thus the centralized performance is achieved in equilibrium.

Note that the relations in Eq. (10) do not reduce to their counterparts in Eq. (3) because \(d_i\) is nonzero in general. By definition, \(d_i\) depends on manufacturer \(j\)’s bargaining power \(\theta_j\). Hence, the surplus \(\Pi_i + \pi - D_i - d_i\) from the trade between the supplier and manufacturer \(i\) depends also on \(\theta_j\). This makes the bargaining problem with a common supplier fundamentally different from the one with two independent suppliers. To see that, we express the firms’ profits in terms of the product profits and the disagreement points:

\[
\begin{align*}
\Pi_i^{\hat{O}O} &= \frac{1}{1 - \theta_i \theta_j} \left[ \theta_i \left( (1 - \theta_j)T_i - d_i \right) + (1 - \theta_i)D_i + \theta_i E_j \right], \\
\pi_i^{\hat{O}O} &= \frac{1}{1 - \theta_i \theta_j} \left[ (1 - \theta_i)(T_i - D_i) + \theta_i d_i - \theta_i E_j \right],
\end{align*}
\]

where \(E_j = (1 - \theta_j)(T_j - D_j) + \theta_j d_j\). Comparing Eq. (11) and Eq. (3), we notice two major differences in the firms’ profits. First, in the one-to-two channels, the common supplier enjoys an enhanced bargaining position via a nonzero \(d_i\), which hurts the manufacturers. Second, there exists a bargaining externality between the two trades, denoted by \(E_j\). Specifically, because the supplier’s profit from both products are included in each of the two negotiations, part of product \(j\)’s profit is allocated to manufacturer \(i\), as represented by \(\theta_i E_j\) in Eq. (11). This bargaining externality hurts the supplier but benefits the manufacturers.

Due to the bargaining externality in the one-to-two channels, a trading party’s profit does not necessarily increase with his/her bargaining power, as it does in the one-to-one channels analyzed in [4.1]. This property is formally stated in the next theorem.

**Theorem 4 (Impact of Bargaining Power on Outsourcing Profits)**

1. If \(\Delta_i = \Delta_j = \Delta\) and \(\theta_i = \theta_j = \theta\), then \(\Pi_i^{\hat{O}O} = \Pi_j^{\hat{O}O}\) and is increasing in \(\theta\), and \(\pi_i^{\hat{O}O}\) is decreasing in \(\theta\).

2. If \(\Delta_i \neq \Delta_j\) and \(\Delta_i < (>) \frac{2 + \gamma}{2 + \gamma \Delta_j}\), then \(\Pi_i^{\hat{O}O}\) is quasiconvex (quasiconcave) in \(\theta_i\) and decreasing (increasing) in \(\theta_j\), and \(\pi_i^{\hat{O}O}\) is quasiconcave (quasiconvex) in \(\theta_i\) and decreasing in \(\theta_j\).

Theorem 4 suggests that when the two supply chains are symmetric, each player gains from possessing a large bargaining power, as intuition would suggest. This, however, may not hold when the manufacturers differ in their bargaining power or cost position. To demonstrate the result, we plot each firm’s profit as a function of manufacturer \(i\)’s bargaining power in Figure 2. The

---

3 We solve for \((\pi_i, \pi_j, \Pi_i, \Pi_j)\) from (9) together with the relations \(\pi = \pi_i + \pi_j\) and \(T_i = \Pi_i + \pi_i\).

4 Note that in Eq. (11), \((1 - \theta_j)T_i\), instead of \(T_i\), appears in manufacturer \(i\)’s profit expression because part of product \(i\)’s profit is allocated to manufacturer \(j\).
upper panel provides an example for $\Delta_i < \frac{2+\gamma_{ij}}{2+2\gamma_{ij}}$ and the lower panel for $\Delta_i > \frac{2+\gamma_{ij}}{2+2\gamma_{ij}}$. From (10), we see that an increased $\theta_i$ has three opposing effects on manufacturer $i$‘s profit. First, the manufacturer obtains a larger portion of $(1-\theta_j)T_i - d_i$, as well as a larger portion of $E_{ji}$. Second, she obtains a smaller portion of $D_i$. Third, the bargaining externality $E_{ji}$ is reduced because $d_{ji}$ decreases (see the expression of $E_{ji}$). This change of $d_{ji}$ results from the fact that as $\theta_i$ increases, the supplier’s profit from trading with manufacturer $i$ alone decreases. Depending on which effect dominates, the manufacturer’s profit may increase or decrease in her bargaining power (see the left and the middle panels of Figure 2). This, in turn, may lead to an increase or a decrease of the supplier’s profit (see the right panels of Figure 2).

Further, when trading with a common supplier, a manufacturer’s outsourcing profit also depends on her competitor’s bargaining power, in contrast to the case of one-to-one channels. This effect can be readily seen from the left panels of Figure 2. When $\theta_j$ increases, manufacturer $i$ obtains a smaller share of product $i$’s profit $T_i$ and enjoys a reduced bargaining externality $E_{ji}$. However, the supplier earns a reduced disagreement payoff $d_{ji}$, which benefits manufacturer $i$. With all the
effects combined, manufacturer $i$’s profit $\Pi_i^{OO}$ increases in $\theta_j$ when she enjoys a sufficiently large cost saving via outsourcing (i.e., $\Delta_i > \frac{2 + \gamma}{2 + \gamma \Delta_j}$), and decreases when outsourcing does not improve her potential margin dramatically (i.e., $\Delta_i < \frac{2 + \gamma}{2 + \gamma \Delta_j}$). The same pattern can be seen in the middle panels of Figure 2 in terms of how manufacturer $j$’s profit depends on $\theta_i$.

5.2 Sourcing Equilibrium

With a common supplier, it is possible to obtain an equilibrium in which only one negotiation pair trades. This is established in the next theorem and demonstrated in Figure 3.

**Theorem 5 (Equilibrium Sourcing Decisions)**

i) If $\Delta_1 = \Delta_2$, then $\hat{OO}$ always arises as a unique equilibrium.

ii) If $\Delta_1 \neq \Delta_2$, then there exists $(\bar{\Delta}_1, \bar{\Delta}_2)$ such that $\hat{OO}$ is the unique equilibrium when $\Delta_1 > \bar{\Delta}_1$ and $\Delta_2 > \bar{\Delta}_2$, and IO (OI) is the unique equilibrium when $\Delta_1 < \bar{\Delta}_1$ ($\Delta_2 < \bar{\Delta}_2$). Moreover, $\Delta_1$ ($\Delta_2$) is weakly increasing in $\Delta_2$ ($\Delta_1$).

![Figure 3](image)

Figure 3: The equilibrium sourcing decisions, as characterized in Theorem 5 ($\gamma = 0.94$).

When they have the same in-house production cost, both manufacturers outsource to the supplier. When their cost positions differ significantly, however, only the one with a higher cost outsources (see the left panel of Figure 3). This is consistent with our intuition that a product is more likely to be outsourced if it leads to more cost reduction. We further observe that outsourcing is more desirable if the competing product has a lower cost advantage index of outsourcing. To see that, consider the trade between the supplier and manufacturer $j$. When $\Delta_j$ is low, an outsourcing deal on product $j$ offers a small trade surplus because the potential for cost reduction is small. As a result, the supplier’s disagreement point $d_i$ is low, leaving a large trade surplus to be shared between the supplier and manufacturer $i$. Consequently, outsourcing becomes attractive to manufacturer $i$.

Furthermore, we observe an interesting dependence of the manufacturers’ sourcing decisions on their bargaining power. With the rival’s bargaining power $\theta_j$ fixed, product $i$ is more likely to
be outsourced when manufacturer $i$’s bargaining power is lower (see the right panel of Figure 3). The same pattern can also be seen from the left panel of Figure 3 by comparing the case of $\theta_1 = 0.9$ and $\theta_2 = 0.1$ with $\theta_1 = \theta_2 = 0.1$ or with $\theta_1 = \theta_2 = 0.9$. This outcome stems from the bargaining externality we explained earlier. As manufacturer $i$’s bargaining power decreases, the supplier’s disagreement point $d_j$ (in the other negotiation) is increased, thereby increasing the bargaining externality $E_j$. As a result, manufacturer $i$ enjoys a higher profit from outsourcing.

This observation makes an interesting contrast to a conclusion reached by Van Mieghem (1999) and Plambeck and Taylor (2005). Both studies show that for monopolist manufacturers, outsourcing production to a supplier is more attractive when the manufacturers’ bargaining power is large. Our analysis suggests that an opposite outcome may arise when the manufacturers engage in product competition.

Low cost outsourcing to a common supplier can also lead to a losing outcome for the manufacturers. This is formally established in Theorem 6 and demonstrated in Figure 4.

**Theorem 6 (Win–Win or Prisoner’s Dilemma)**

i) If $\Delta_i = \Delta_j = \Delta$ and $\theta_i = \theta_j = \theta$, then there exists a $\hat{\theta}$ such that $\Pi_i^{OO} = \Pi_j^{OO} < (>)\Pi_i^{II} = \Pi_j^{II}$ when $\theta < (>)\hat{\theta}$. Moreover, $\hat{\theta}_j \to 0$ as $\Delta \to 1$.

ii) If $\Delta_i = \Delta_j = \Delta$, then there exists a $\hat{\theta}_i$ such that $\Pi_i^{OO} < (>)\Pi_i^{II}$ when $\theta_i < (>)\hat{\theta}_i$.

iii) If $\Delta_i \neq \Delta_j$, then there exists a $\hat{\Delta}_i$ such that $\Pi_i^{OO} < (>)\Pi_i^{II}$ when $\Delta_i < (>)\hat{\Delta}_i$. Moreover, $\hat{\Delta}_i$ is increasing in $\Delta_j$.

At first glance, Figure 4 and Figure 1 reveal very similar patterns. There are, however, several noteworthy differences in firm behaviors under the two industry structures. We will elaborate them next.

The left panel of Figure 4 indicates that the manufacturers with low bargaining power lose from low cost outsourcing when the two supply chains are symmetric. Interestingly, the bargaining power threshold is not monotone decreasing in the outsourcing cost advantage index, as it does in the left panel of Figure 1. In the extreme case when the common supplier possesses no cost advantage (i.e., $\Delta = 1$), the manufacturers always gain from outsourcing. This is because outsourcing leads to the centralized system performance and thus a positive trade surplus.

The middle panel of Figure 4 shows the case when the manufacturers have the same in-house production cost but differ in their bargaining power. In this case, a manufacturer loses from outsourcing when her bargaining power is small. The bargaining power threshold, however, may

---

6There may be exceptions: with small $\Delta_1$ and $\Delta_2$, manufacturer $i$ may be more likely to outsource when $\theta_i$ increases. When this happens, however, the outsourcing decisions are highly insensitive to the bargaining power; see the lower left corner of the left panel of Figure 3.
Figure 4: The impact of outsourcing on profits depends on the manufacturers’ bargaining power and the indices of cost advantage, as characterized in Theorem 6 ($\gamma = 0.85$).

increase or decrease in the rival manufacturer’s bargaining power, as opposed to being constant in the middle panel of Figure 1.

The right panel of Figure 4 suggests that when the manufacturers have different in-house production costs, the one with lower cost is more likely to lose from outsourcing. We note that the threshold of outsourcing cost advantage index is not monotone in the manufacturers’ bargaining power, as it does in the right panel of Figure 1. Thus, it is possible that a manufacturer with a larger bargaining power is more likely to lose from outsourcing. This surprising outcome stems from the nonmonotonicity of the manufacturers’ outsourcing profits with respect to their bargaining power, as is demonstrated in Theorem 4.

We can further characterize the strategic effects of upstream cost advantage by taking the first-order derivatives with respect to $c_S$ in Eq. (11) and obtain

$$
\frac{d\Pi_i^{OO}}{dc_S} = \frac{1}{1 - \theta_i \theta_j} \left[ \theta_i (1 - \theta_j) \frac{dT_i}{dc_S} + (1 - \theta_i) \frac{dD_i}{dc_S} - \theta_i \frac{dd_i}{dc_S} + \theta_i \frac{dE_j}{dc_S} \right],
$$

$$
\frac{d\Pi_O^{OO}}{dc_S} = \frac{1}{1 - \theta_i \theta_j} \left[ (1 - \theta_i) \left( \frac{dT_i}{dc_S} - \frac{dD_i}{dc_S} \right) + \theta_i \frac{dd_i}{dc_S} - \theta_i \frac{dE_j}{dc_S} \right].
$$

Comparing the above equations to Eq. (11)–(7), there are two additional effects, namely, the upstream bargaining effect and the bargaining externality effect. The next theorem characterizes the signs of these effects.

**Theorem 7 (Effects of Upstream Cost Advantage)** The product profit effect $\frac{dT_i}{dc_S} < 0$, the downstream bargaining effect $\frac{dD_i}{dc_S} > 0$, the upstream bargaining effect $\frac{dd_i}{dc_S} < 0$, and the bargaining externality effect $\frac{dE_j}{dc_S} < 0$. When $c_S$ decreases,

i) the supplier’s profit increases, and
ii) there exists a \( \tilde{\theta}_j \in [0, 1] \) such that manufacturer \( i \)'s profit increases (decreases) for \( \theta_j < (>) \tilde{\theta}_j \).

Moreover, \( \tilde{\theta}_j \) is quasiconvex in \( \theta_i \), weakly decreasing in \( \Delta_i \) and weakly increasing in \( \Delta_j \).

Theorem 7 confirms that in the one-to-two channels, the product profit and downstream bargaining effects have the same signs as their counterparts in the one-to-one channels. In addition, an upstream cost reduction enhances the supplier’s bargaining position and increases the bargaining externality. As the supplier’s cost decreases, his disagreement payoff from serving only one manufacturer increases, and thus his bargaining position strengthens. Meanwhile, his profit from each trade increases, which increases the bargaining externality (recall the expression of the bargaining externality in Eq.14). Combining these effects, we find that the supplier always benefits from his own cost reduction and the manufacturers may gain or lose, as we have demonstrated earlier.

6 Concluding Remarks

Low cost outsourcing has become a norm in many manufacturing industries. While the cost reduction benefit of outsourcing is intuitive, the strategic implications of upstream cost advantage on outsourcing negotiations are not well understood. In contrast to the previous outsourcing literature, which has largely ignored the cost differential and negotiations between suppliers and manufacturers, we study competing manufacturers’ low cost outsourcing behavior in a multiunit bilateral bargaining framework. Our analysis highlights the strategic impacts of upstream cost advantage on the outcomes of outsourcing negotiations—upstream cost advantage weakens a manufacturer’s bargaining position and may enhance a supplier’s bargaining position. The bargaining effects have a profound impact on manufacturers’ profitability. We find that manufacturers with low bargaining power are more likely to lose from outsourcing although competitive market pressure forces them to outsource to acquire a low cost position.

We also identify an interesting bargaining externality between competing manufacturers when they outsource to a common supplier. This bargaining externality benefits the manufacturers and hurts the supplier because the supplier’s profit from the trades with both manufacturers have to be included in the negotiation with each manufacturer. We find that a decrease in a manufacturer’s bargaining power increases the disagreement payoff of the supplier in his negotiation with the competing manufacturer, and thus increases the bargaining externality. As a result, a counterintuitive outcome may arise in which a manufacturer’s profit may be decreasing in her bargaining power. In addition, a manufacturer with small bargaining power may be more likely to outsource than one with large bargaining power, contrasting the behavior of a monopolist manufacturer characterized in the previous literature.

In summary, our results highlight the importance of modeling firm negotiations in competing supply chains. Following the footsteps of some recent papers in the operations management literature that model firm negotiations (e.g., Lovejoy 2010, Nagarajan and Bassok 2008), we find it
a fruitful area of research to apply bargaining-based frameworks to supply chain problems. Our work demonstrates that modeling firm negotiations is of particular importance when competition is present. This is because firms’ disagreement payoffs (reservation profits) are no longer fixed, unlike those in the bilateral monopoly setting.

**Appendix: Proofs**

**Lemma 1 (Product Competition)** Let \( \bar{c}_i \) denote the cost of producing product \( i \), \( i = 1, 2 \). Then the product competition equilibrium satisfies

\[
\begin{aligned}
p_i^e(\bar{c}_i, \bar{c}_j) &= \frac{a-\gamma(a-\bar{c}_i)}{2}, \quad q_i^e(\bar{c}_i, \bar{c}_j) = 0, \quad \text{if } \frac{a-\bar{c}_i}{a-\bar{c}_j} \leq \frac{\gamma}{2}, \\
p_i^e(\bar{c}_i, \bar{c}_j) &= \bar{c}_i + \frac{2(a-\bar{c}_i)-\gamma(a-\bar{c}_j)}{4\gamma^2}, \quad q_i^e(\bar{c}_i, \bar{c}_j) = \frac{2(a-\bar{c}_i)\gamma(a-\bar{c}_j)}{b(4-\gamma^2)}, \quad \text{if } \frac{\gamma}{2} < \frac{a-\bar{c}_i}{a-\bar{c}_j} < \frac{2}{\gamma}, \\
p_i^e(\bar{c}_i, \bar{c}_j) &= \frac{a+\bar{c}_i}{2}, \quad q_i^e(\bar{c}_i, \bar{c}_j) = \frac{a-\bar{c}_i}{2b}, \quad \text{if } \frac{a-\bar{c}_i}{a-\bar{c}_j} \geq \frac{2}{\gamma}.
\end{aligned}
\]  

**Proof.** The channel profit \( \Pi_i = (a - bq_i - \gamma bq_j - \bar{c}_i)q_i \) is concave in \( q_i \). The first-order condition of \( q_i \) gives \( a - 2bq_i - \gamma bq_j - \bar{c}_i = 0 \), for \( i = 1, 2 \). Solving \( (q_i, q_j) \) leads to the second relation in Eq. (11). In this case, each manufacturer sells a positive quantity. Setting \( q_i \geq 0 \) in this solution, we obtain the condition \( \frac{a-\bar{c}_i}{a-\bar{c}_j} \geq \frac{\gamma}{2} \). When \( \frac{a-\bar{c}_i}{a-\bar{c}_j} < \frac{\gamma}{2} \), manufacturer \( j \) becomes a monopolist and \( q_i = 0 \). Substituting \( q_i = 0 \) into manufacturer \( j \)’s first-order condition gives rise to the first and the third relations in Eq. (11). \( \square \)

**Remark 1** Lemma 1 suggests that manufacturer \( i \) may be priced out of the market by her rival (i.e., \( q_i^e(\bar{c}_i, \bar{c}_j) = 0 \)) if her cost \( \bar{c}_i \) is significantly higher than the rival’s cost \( \bar{c}_j \). This is possible, for example, under \( II \) where \( \bar{c}_i = c_i \) and \( \bar{c}_j = c_j \). Whenever this happens, outsourcing always weakly dominates insourcing for manufacturer \( i \) because her supplier’s cost is lower than her own cost. Therefore, it is without loss of generality that we focus only on the case when both manufacturers sell positive quantities. This requires \( \frac{\gamma}{2}\Delta_j \leq \Delta_i \leq \frac{2}{\gamma}, j = 3 - i \) and \( i = 1, 2 \).

**Proof of Theorem 1.** Based on Lemma 1, we can compute manufacturer 1’s profits under different sourcing structures as:

\[
\begin{aligned}
\Pi_1^{II} &= \frac{(a-cs)^2(2\Delta_1^{-1} - \gamma\Delta_2^{-1})^2}{b(4-\gamma^2)^2}, \\
\Pi_1^{IO} &= \frac{(a-cs)^2[\theta_1(2 - \gamma\Delta_1^{-1})^2 + (1 - \theta_1)(2\Delta_1^{-1} - \gamma\Delta_2^{-1})^2]}{b(4-\gamma^2)^2}, \\
\Pi_1^{IO} &= \frac{(a-cs)^2(2\Delta_1^{-1} - \gamma)^2}{b(4-\gamma^2)^2}, \\
\Pi_1^{OO} &= \frac{\theta_1(a-cs)^2}{b(2+\gamma)^2} + \frac{(1-\theta_1)(a-cs)^2(2\Delta_1^{-1} - \gamma)^2}{b(4-\gamma^2)^2}.
\end{aligned}
\]

Without loss of generality, we can normalize \( \frac{(a-cs)^2}{b} = 1 \) and compare the profits:

\[
\begin{aligned}
\Pi_1^{OO} - \Pi_1^{IO} &= \frac{4\theta_1}{(4-\gamma^2)^2}(1 - \Delta_1^{-1})(1 + \Delta_1^{-1} - \gamma), \\
\Pi_1^{OI} - \Pi_1^{II} &= \frac{\theta_1}{(4-\gamma^2)^2}[2 + (2 - \gamma)\Delta_1^{-1} - \gamma\Delta_2^{-1}]\{2 + \gamma\Delta_2^{-1} - (2 + \gamma)\Delta_1^{-1}\}.
\end{aligned}
\]
From the first equation, we deduce that given manufacturer 2 outsources, manufacturer 1 should outsource. The sign of the second equation is the same as that of \(2 + \gamma \Delta_2^{-1} - (2 + \gamma) \Delta_1^{-1}\). Hence, given that manufacturer 2 insources, manufacturer 1 should outsource if and only if \(\Delta_1 \geq \frac{2 + \gamma}{2 + \gamma \Delta_2^{-1}}\).

By symmetry, given that manufacturer 1 insources, manufacturer 2 should outsource if and only if \(\Delta_2 \geq \frac{2 + \gamma}{2 + \gamma \Delta_1^{-1}}\). Therefore, \(II\) is an equilibrium when both relations hold, which implies \((2 + \gamma) \Delta_2^{-1} \geq 2\) and \(\frac{2 + \gamma}{2 + \gamma \Delta_2^{-1}} \leq \Delta_1 \leq \frac{2 + \gamma}{2 + \gamma \Delta_1^{-1}}\). Both inequalities hold only when \(\Delta_1 = \Delta_2 = 1\). Therefore, \(II\) cannot be an equilibrium whenever \(\Delta_1 > 1\) or \(\Delta_2 > 1\).

**Proof of Theorem 2.** From the proof of Theorem 1, we have (when normalizing \(\frac{(a-c_S)^2}{b}\) to 1)

\[
(4 - \gamma^2)(\Pi_{OO}^i - \Pi_{II}^i) = -\gamma(1 - \Delta_j^{-1})(4\Delta_i^{-1} - \gamma - \gamma \Delta_j^{-1}) + 4(1 - \Delta_i^{-1})(1 + \Delta_i^{-1} - \gamma)\theta_i.
\]

Setting the right-hand side to be zero and solving for the threshold \(\theta_i\) give the result.  \(\square\)

**Proof of Theorem 3.** From Lemma 1, \(q_{OO} = \frac{a-c_S}{b(2+\gamma)} < \frac{a-c_S}{\gamma}\) and thus \(\frac{\partial q_{OO}}{\partial c_S} = -\frac{1}{b(2+\gamma)} < 0\). Also, \(T_{OO} = q_{OO}(a - bq_{OO} - \gamma bq_{OO} - c_S)\). We have

\[
\frac{dT_{OO}}{dc_S} = \frac{\partial T_{OO}}{\partial c_S} + \frac{\partial T_{OO}}{\partial q_{OO}} \frac{\partial q_{OO}}{dc_S} = -q_{OO} + [a - c_S - 2b(1 + \gamma)q_{OO}] \frac{-1}{b(2 + \gamma)} = \frac{-bq_{OO} - a - c_S}{b(2 + \gamma)} < 0.
\]

Given that manufacturer \(j\) outsources, then manufacturer \(i\)'s insourcing profit computed in the proof of Theorem 1 constitutes his disagreement point, i.e., \(D_i(c_S) = \frac{[2(a-c_i)-\gamma(a-c_S)]^2}{b(4-\gamma^2)}\). It is easy to check that \(\partial D_i/\partial c_S > 0\). It follows that \(\frac{\partial \Pi_{OO}^i}{\partial c_S} > 0\) and

\[
\frac{(a-c_S)^2}{b(4-\gamma^2)^2} \cdot \frac{d\Pi_{II}^i}{dc_S} = 2\gamma \Delta_i^{-1} - \gamma^2 - 2[2(1 - \gamma) + \gamma \Delta_i^{-1}]\theta_i.
\]

Hence, there exists a threshold \(\bar{\theta}_i\) such that \(\frac{d\Pi_{OO}^i}{dc_S} > (\theta_i < (\theta_i)\bar{\theta}_i\). It is also straightforward to check that \(\bar{\theta}_i\) is weakly decreasing in \(\Delta_i\). \(\square\)

**Proof of Theorem 4.** Given the negotiation outcome of bargaining unit \(j\), the negotiation between manufacturer \(i\) and supplier \(i\) must settle on an order quantity \(q_i\) that maximizes their joint profit \(\Pi_i + \pi_i\). It is also straightforward to see that \(\Pi_i + \pi_i = T_i + T_j - \Pi_j\). Therefore, \(q_i\) must maximize \(T_i + T_j = q_i(a - bq_i - \gamma bq_j - c_S) + q_j(a - bq_j - \gamma bq_i - c_S)\). Thus \(q_i = \frac{a-c_S - 2b\theta_i q_j}{2b}\). Solving for the equilibrium \((q_i, q_j)\) leads to \(q_{OO} = \frac{a-c_S}{2b(1+\gamma)}\). Then, \(T_i = T_j = q_{OO}(a - bq_{OO} - \gamma bq_{OO}) = \frac{(a-c_S)^2}{4b(1+\gamma)}\).

From (10) and the proof of Theorem 1, we can compute manufacturer \(i\)'s profit as

\[
\Pi_{OO}^i = \frac{1}{1-\theta_i} \left[ \theta_i(1-\theta_j) \left( \frac{(a-c_S)^2}{b(4b(1+\gamma))} - \frac{[2(a-c_S) - \gamma(a-c_j)]^2 - 2(a-c_j) - \gamma(a-c_j)]^2}{b(4-\gamma^2)^2} \right) 
+ (1-\theta_i) \left( \frac{(a-c_S)^2}{b(4-\gamma^2)^2} - \frac{[2(a-c_S) - \gamma(a-c_S)]^2}{b(4-\gamma^2)^2} \right) 
+ \theta_i (1-\theta_j) \left( \frac{(a-c_S)^2}{b(4b(1+\gamma))} - \frac{[2(a-c_j) - \gamma(a-c_S)]^2}{b(4-\gamma^2)^2} \right) 
+ \theta_i \theta_j (1-\theta_i) \left( \frac{[2(a-c_S) - \gamma(a-c_S)]^2}{b(4-\gamma^2)^2} - \frac{[2(a-c_j) - \gamma(a-c_j)]^2}{b(4-\gamma^2)^2} \right) \right].
\]
We normalize $\frac{(a-c_S)^2}{\theta} = 1$ and obtain
\[ \bar{\Pi}_i = 4(1 + \gamma)(4 - \gamma^2)^2\pi_i^{O}\]
\[ = \frac{1}{(1 - \theta_i \theta_j)} \left[ \theta_i (1 - \theta_j) [(4 - \gamma^2)^2 - 4(1 + \gamma)(2 - \gamma \Delta_i^{-1})^2 + 4(1 + \gamma)(2 \Delta_i^{-1} - \gamma \Delta_i^{-1})^2] \\
+ 4(1 - \theta_i)(1 + \gamma)(2 \Delta_i^{-1} - \gamma^2 + \theta_i (1 - \theta_j) [(4 - \gamma^2)^2 - 4(1 + \gamma)(2 \Delta_i^{-1} - \gamma^2)] \\
+ 4\theta_i \theta_j (1 - \theta_i)(1 + \gamma)((2 - \gamma \Delta_i^{-1})^2 - (2 \Delta_i^{-1} - \gamma \Delta_i^{-1})^2) \right] . \]

We have
\[ \frac{2(1 - \theta_i \theta_j)^2}{\gamma(1 - \theta_i)} \theta_i \frac{\partial \bar{\Pi}_i}{\partial \theta_j} = 16 - 16(1 + \gamma)(\Delta_i^{-1} + \Delta_j^{-1} - \Delta_i^{-1} \Delta_j^{-1}) + \gamma(2 + \gamma)(6 - \gamma), \]
\[ 2(1 - \theta_i \theta_j)^2 \frac{\partial \Pi_i}{\partial \theta_i} = C - 2A\theta_i \theta_j + A\theta_i^2 \theta_j^2, \]
where
\[ A = 2(1 + \gamma)[2 + (2 - \gamma) \Delta_i^{-1} - \gamma \Delta_j^{-1}][2 + \gamma \Delta_j^{-1} - (2 + \gamma) \Delta_i^{-1}], \]
\[ C = A + \gamma(1 - \theta_j)[-16 + 16(1 + \gamma)(\Delta_i + \Delta_j^{-1} - \Delta_i^{-1} \Delta_j^{-1}) - \gamma(2 + \gamma)(6 - \gamma)]. \]

Note that if $C - 2A\Theta + A\Theta^2 = 0$ has two real roots in $\Theta$, one of them must be above one and the other below one. Also, $C - 2A\Theta + A\Theta^2$ is convex (concave) when $A > ( < ) 0$ or $\Delta_i > ( < ) \frac{2 + \gamma}{2 + \gamma \Delta_j^{-1}}$.

Part i) follows immediately.

Now we compute the supplier’s profit.
\[ \pi_i^{O} = \frac{1}{1 - \theta_i \theta_j} \sum_{i = 1, 2, j \neq i} \left[ (1 - \theta_j)(1 - \theta_i) \left( \frac{(a - c_S)^2}{4(1 + \gamma)} - \frac{[2(a - c_i) - \gamma(a - c_S)]^2}{(4 - \gamma^2)^2} \right) \\
+ (1 - \theta_j)\theta_i (1 - \theta_j) \left( \frac{[2(a - c_S) - \gamma(a - c_j)]^2}{(4 - \gamma^2)^2} - \frac{[2(a - c_j) - \gamma(a - c_i)]^2}{(4 - \gamma^2)^2} \right) \right]. \]

We normalize $\frac{(a-c_S)^2}{\theta} = 1$ and obtain
\[ \hat{\pi} = 4(1 + \gamma)(4 - \gamma^2)^2\pi_i^{O}\]
\[ = \frac{1}{(1 - \theta_i \theta_j)} \sum_{i = 1, 2, j \neq i} \left[ (1 - \theta_j)(1 - \theta_i)[(4 - \gamma^2)^2 - 4(1 + \gamma)(2 \Delta_i^{-1} - \gamma^2)] \\
+ 4(1 - \theta_j)\theta_i (1 - \theta_j)(1 + \gamma)((2 - \gamma \Delta_j^{-1})^2 - (2 \Delta_j^{-1} - \gamma \Delta_j^{-1})^2) \right] . \]

We derive
\[ (1 - \theta_i \theta_j)^2 \frac{\partial \hat{\pi}}{\partial \theta_i} = -\hat{C} + 2A\theta_i \theta_j - A\theta_i^2 \theta_j^2, \]
where $\hat{C} = A - \gamma(1 - \theta_j)^2[16 - 16(1 + \gamma)(\Delta_i^{-1} + \Delta_j^{-1} - \Delta_i^{-1} \Delta_j^{-1}) + \gamma(2 + \gamma)(6 - \gamma)]$. The rest of the proof can be carried out similarly as that for the manufacturer’s profit.
Finally, we check the case when $\Delta_1 = \Delta_2 = \Delta$ and $\theta_1 = \theta_2 = \theta$. We have $(1 - \theta^2)^2 \partial \Pi_i / \partial \theta = c(\Delta^{-1}) + 2a(\Delta^{-1}) \theta + a(\Delta^{-1}) \theta^2$, where

$$a(Y) = 8(1 + \gamma)(1 - Y)[1 + (1 - \gamma)Y]$$

$$c(Y) = 8 - 8\gamma - 12\gamma^2 - 4\gamma^3 + 4\gamma^4 + 24\gamma(1 + \gamma)Y - 8(1 + \gamma)^2 Y^2.$$ 

Because $c(1) = \gamma^2 (2 - \gamma)^2 > 0$ and $c(\gamma/2) = (2 - \gamma)^2(2 - \gamma^2) > 0$, we have $\partial \Pi_1^O / \partial \theta > 0$. Finally, because the system profit is independent of bargaining power, we have $(1 - \theta^2)^2 \partial \pi / \partial \theta < 0$. \hfill \Box

**Proof of Theorem 5.** The value of $\Pi_1^O$ and $\Pi_1^{\circ}$ are computed in the proofs of Theorem 1 and Theorem 4 respectively. We normalize $\frac{(a - c\bar{\theta})^2}{b} = 1$ and obtain

$$\Phi(\theta_1, \theta_2, \Delta_1, \Delta_2) = \frac{2(1 + \gamma)(4 - \gamma^3/2)(1 - \theta_1 \theta_2)}{\theta_1} (\Pi_1^O - \Pi_1^{\circ})$$

$$= -2(1 + \gamma)(4 - \gamma^3/2)(1 - \theta_1 \theta_2) \Delta_1^{-1} + 8(1 - \Delta_2^{-1})\gamma(1 + \gamma)[1 - (2 - \theta_1 \theta_2)] \Delta_1^{-1} 
- 2\gamma^2(1 + \gamma)(1 - \theta_1 \theta_2) \Delta_2^{-2} + 16\gamma(1 + \gamma)(1 - \theta_2) \Delta_2^{-1} + 8(1 - \gamma) 
- \gamma^2(2 + \gamma)(6 - \gamma)(1 - \theta_2) + 16\gamma \theta_2 - 8(1 + \gamma) \theta_1 \theta_2.$$ 

Likewise, we can show

$$\pi^O - \pi_1^O = \frac{1 - \theta_1}{2(1 + \gamma)(4 - \gamma^3/2)(1 - \theta_1 \theta_2)} \Phi(\theta_1, \theta_2, \Delta_1, \Delta_2).$$

In other words, given that product 2 is outsourced, manufacturer 1 is willing to outsource product 1 whenever the supplier is willing to produce product 1.

Note that $\Phi$ is a concave function of $\Delta_2^{-1}$. Also $\Phi(\theta_1, \theta_2, \Delta_1, 1) = \gamma^2(2 - \gamma)^2(1 - \theta_2) + 2(1 - \Delta_1^{-2})(1 + \gamma)(4 - \gamma^3/2)(1 - \theta_1 \theta_2) > 0$. This suggests a threshold of $\tilde{\Delta}_2$ such that $\Phi(\theta_1, \theta_2, \Delta_1, \Delta_2) > (<) 0$ for $\Delta_2 < (> \tilde{\Delta}_2$. We also note that

$$\frac{\partial \Phi}{\partial \Delta_1^{-1}} = -4(1 + \gamma)(4 - \gamma^3/2)(1 - \theta_1 \theta_2) \Delta_1^{-1} + 8(1 - \Delta_2^{-1})\gamma(1 + \gamma)[1 - (2 - \theta_1 \theta_2)].$$

The above is nonpositive if $1 - (2 - \theta_1 \theta_2) \leq 0$. Otherwise, we have

$$\frac{\partial \Phi}{\partial \Delta_1^{-1}} \leq -4(1 + \gamma)(4 - \gamma^3/2)(1 - \theta_1 \theta_2) (\gamma/2) + 8(1 - (\gamma/2))\gamma(1 + \gamma)[1 - (2 - \theta_1 \theta_2)] 
- 2\gamma(1 + \gamma)(2 - \gamma) [4(1 - \theta_1 \theta_2) + \gamma(1 - \theta_1 \theta_2)] \leq 0.$$ 

In other words, $\Phi$ is increasing in $\Delta_1$. Hence, there exists a $\tilde{\Delta}_1$ such that $\Phi(\theta_1, \theta_2, \Delta_1, \Delta_2) < (> 0$ for $\Delta_1 < (> \tilde{\Delta}_1$. This gives part ii).

When $\Delta_1 = \Delta_2 = \Delta$, we have $\frac{\partial \Phi}{\partial \theta_1} = -2(1 + \gamma)\theta_2 [2 + (2 - \gamma) \Delta^{-1} - \gamma \Delta^{-1}] [2 + \gamma \Delta^{-1} - (2 + \gamma) \Delta^{-1}] \leq 0$. Solving $\Phi(\theta, \theta, \Delta, \Delta) = 0$, we obtain $\theta_1 = 1 + \frac{8(1 + \gamma)(1 - \Delta^{-1})}{8(1 + \gamma)(1 - \Delta^{-1})[1 + \gamma \Delta^{-1}]}$, where $\varpi(Y) = -8(1 + \gamma)^2 Y^2 + 24\gamma(1 + \gamma)Y + 8 - \gamma(8 + 12\gamma + 4\gamma^2 - \gamma^3)$ is a concave function in $Y$ with $\varpi(1) = \gamma(2 + \gamma)^2 \geq 0$ and $\varpi(\gamma/2) = (2 - \gamma)^2(2 - \gamma^2) \geq 0$. We deduce $\varpi(\Delta^{-1}) \geq 0$ and thus $\theta_1 \geq 1$, which gives part i). \hfill \Box
Proof of Theorem 6: Comparing $\Pi_1^{IO}$ and $\Pi_1^{I}$, we have

\[
\Psi(\theta_1, \theta_2, \Delta_1, \Delta_2) = 2(1 + \gamma)(1 - \theta_1 \theta_2)(\Pi_1^{IO} - \Pi_1^{I})
\]

\[
= 2 \gamma^2(1 + \gamma) + [8 - 8\gamma - \gamma^2(2 + \gamma)(6 - \gamma)]\theta_1 + [16\gamma + 10\gamma^2 + 2\gamma^3 - \gamma^4 - 8(1 + \gamma)\theta_1]\theta_2
\]

\[-2(2 - \gamma)(1 + \gamma)(2 + \gamma)\theta_1(1 - \theta_1 \theta_2)\Delta_1^{-1} - 8\gamma(1 + \gamma)(1 - \theta_1)(1 + \theta_1 \theta_2)\Delta_1^{-1}
\]

\[-2\gamma^2(1 + \gamma)(1 + \theta_1)(1 - \theta_1 \theta_2)\Delta_2^{-2} + 8\gamma(1 + \gamma)[2\theta_1(1 - \theta_2) + (1 - \theta_1)(1 + \theta_1 \theta_2)\Delta_1^{-1}]\Delta_2^{-1}.
\]

The right-hand side is a concave and quadratic function of $\Delta_2^{-1}$. Also, $\Psi(\theta_1, \theta_2, \Delta_1, 1) = (2 - \gamma)\theta_1[\gamma^2(2 - \gamma)(1 - \theta_2) + 2(1 + \gamma)(2 + \gamma)(1 - \theta_1 \theta_2)(1 - \Delta_1^{-2})] \geq 0$. Thus, there exists a $\hat{\Delta}_2$ such that $\Psi > (0)\Delta_2 < (>)\hat{\Delta}_2$. It is also easy verify that $\Psi$ is a concave and quadratic function of $\Delta_1^{-1}$ which is maximized at some $\Delta_1^* \leq 0$. Therefore, there exists a $\hat{\Delta}_1$ such that $\Psi < (0)\Delta_1 < (>)\hat{\Delta}_1$. This gives part iii).

To show part ii), we derive

\[
\Psi(\theta_1, \theta_2, \Delta, \Delta) = 2\gamma(1 + \gamma)(1 - \Delta^{-1})[\gamma - (4 - \gamma)\Delta^{-1}] + [24\gamma(1 + \gamma)\Delta^{-1} - 8(1 + \gamma)^2\Delta^{-2} + 8
\]

\[-8\gamma - 12\gamma^2 - 4\gamma^3 + \gamma^4]\theta_1 + \theta_1 \theta_2[16\gamma + 10\gamma^2 + 2\gamma^3 - \gamma^4 - 24\gamma(1 + \gamma)\Delta^{-1}
\]

\[+2\gamma(1 + \gamma)(4 + \gamma)\Delta^{-2} - 8(1 + \gamma)(1 - \Delta^{-1})[1 + (1 - \gamma)\Delta^{-1}]\theta_1].
\]

It is easy to see that $\Psi(\theta_1, \theta_2, \Delta, \Delta)$ is a concave function in $\theta_1$. Also, $\Psi(0, \theta_2, \Delta, \Delta) = -2\gamma(1 + \gamma)(\gamma - \Delta^{-1})(\Delta^{-1} - 2 - \gamma - \gamma^2) \leq 0$ and $\Psi(1, \theta_2, \Delta, \Delta) = (1 - \theta_2)\lambda(\Delta^{-1})$, where $\lambda(Y) = 8 - 8\gamma - 10\gamma^2 - 2\gamma^3 + \gamma^4 + 16\gamma(1 + \gamma)Y - 2(1 + \gamma)(4 + \gamma^2)Y^2$. We note that $\lambda(1) = (2 - \gamma)^2 \geq 0$ and $\lambda(\gamma/2) = (1 - \gamma)(4 - \gamma^2)/2 \geq 0$. Hence, $\lambda(\Delta^{-1}) > 0$. This suggests a threshold $\hat{\theta}_1$ such that $\Psi(\theta_1, \theta_2, \Delta, \Delta) < (>)0$ when $\theta_1 < (>)\hat{\theta}_1$, leading to part ii).

Finally, we prove part i). We have

\[
\frac{\Psi(\theta, \theta, \Delta, \Delta)}{1 - \theta} = 2\gamma^2(1 + \gamma) - 8(1 + \gamma)\Delta^{-1} + 2\gamma(1 + \gamma)(4 - \gamma)\Delta^{-2}
\]

\[+ [8 - 8\gamma - 10\gamma^2 - 2\gamma^3 + \gamma^4 + 16\gamma(1 + \gamma)\Delta^{-1} - 2(1 + \gamma)(4 + \gamma^2)\Delta^{-2}]\theta
\]

\[+ 8(1 + \gamma)(1 - \gamma\Delta^{-1} - (1 - \gamma)\Delta^{-2})\theta^2.
\]

It is easy to verify that the coefficients of $\theta$ and $\theta^2$ are both positive. Therefore, $\Psi$ is a convex function in $\theta$ and whenever $\Psi = 0$ has real solutions, at least one of them is below zero. We further note that when $\theta = 1$, the right-hand side of the above reduces to $(2 - \gamma)^2[2 + (2 + \gamma) - 4(1 + \gamma)\Delta^{-2}] > 0$. Hence, there exists a threshold $\hat{\theta}$ such that $\Psi < (>)0$ for $\theta < (>)\hat{\theta}$. \hfill \Box

Proof of Theorem 7: The downstream bargaining effect is the same as that in Theorem 4 and the product profit effect can be derived similarly. Note that $d_1 = \pi_2^{IO}$ and $d_2 = \pi_1^{IO}$. We can derive from the proof of Theorem 4

\[
\frac{d\pi_i}{dc_S} = \frac{-4(1 - \theta_j)[2(a - c_S) - \gamma(a - c_j)]}{b(4 - \gamma^2)} < 0.
\]
It follows that $\frac{dF}{dc} \leq 0$ is negative.

To show part i), we compute
\[
\frac{\sigma_i}{dc} = \frac{a - c S}{2b(1 + \gamma)(4 - \gamma^2)(1 - \theta_i, \theta_j) S_i(\theta_i, \theta_j, \Delta_i^{-1}, \Delta_j^{-1}),}
\]
where
\[
\Xi_i(\theta_i, \theta_j, Y_i, Y_j) = -(1 - \theta_i)[(4 - \gamma^2)^2 + 4\gamma (1 + \gamma)(2Y_i - 1)] + (1 - \theta_i, \theta_j)(1 + \gamma)(2 - \gamma Y_i) + \theta_i (1 - \theta_j) \gamma [16(1 + \gamma) Y_j - (16 + 12\gamma + 4\gamma^2 - \gamma^3)].
\]

Let
\[
\Xi(\theta_i, \theta_j, Y_i, Y_j) = \frac{[\Xi_i(\theta_i, \theta_j, Y_i, Y_j) + \Xi_i(\theta_j, \theta_i, Y_j, Y_i)]}{2}
\]
\[
= -4\gamma (1 + \gamma)(1 - \theta_i)(1 - 2\theta_j + \theta_i \theta_j) Y_i - 4\gamma (1 + \gamma)(1 - \theta_j)(1 - 2\theta_i + \theta_i \theta_j) Y_j
\]
\[
-8(1 + \gamma)(1 - \theta_j^2 \theta_i + (1 - \theta_j)\theta_i) Y_j - [16 - \gamma^2 (1 + \gamma)(6 - \gamma)](1 - \theta_i)(1 - \theta_j).
\]

Note that $2(1 - \theta_j^2 \theta_i + (1 - \theta_j)\theta_i) Y_j = (1 - \theta_j)(1 - \theta_i) Y_j \geq 0$. It follows that $\Xi \leq 0$.

To see part ii), we compute
\[
\frac{\Pi_i}{dc} = \frac{a - c S}{b(1 + \gamma)(4 - \gamma^2)(1 - \theta_i, \theta_j) S_i(\theta_i, \theta_j, \Delta_i^{-1}, \Delta_j^{-1}),}
\]
where
\[
\Theta_i(\theta_i, \theta_j, Y_i, Y_j) = \varpi_1(\theta_i, Y_i, Y_j) - \varpi_2(\theta_i, Y_i, Y_j) \theta_i(1 - \theta_j),
\]
\[
\varpi_1(\theta_i, Y_i, Y_j) = 2(1 + \gamma)(1 - \theta_i)[2\gamma Y_i - \gamma^2 - 2(2 - \gamma Y_i) \theta_i],
\]
\[
\varpi_2(\theta_i, Y_i, Y_j) = 8 - 8\gamma - 10\gamma^2 - 2\gamma^3 + \gamma^4 + 8\gamma (1 + \gamma) Y_j - 4(1 + \gamma)(1 - \theta_i)(2 - \gamma Y_i).
\]

It follows that $\partial \Theta_i / \partial Y_i = 4\gamma (1 + \gamma)(1 - \theta_i)(1 + \theta_i \theta_j) \geq 0$ and $\partial \Theta_i / \partial Y_j = -8\gamma (1 + \gamma) \theta_i(1 - \theta_j) \leq 0$.

Hence, $\frac{d\Pi_i}{dc}$ is decreasing in $\Delta_i$ and increasing in $\Delta_j$.

If $\varpi \leq 0$, we have $\Theta_i > (0) 0$ when $\gamma < (\gamma) \gamma = 1 - \frac{\varpi_i(\theta_i, Y_i, Y_j)}{\varpi_2(\theta_i, Y_i, Y_j)}$. If $\varpi > 0$ and $\varpi_1 \leq 0$, we must have $\Theta_i \leq 0$. In this case, we set $\hat{\gamma} = 0$. Therefore, we focus on the case $\varpi > 0$ and $\varpi_1 > 0$, which leads to $\theta_i > \theta_i^b \equiv \frac{\gamma[10 - 4(1 + \gamma)Y_i - 8(1 + \gamma)Y_j + \gamma(10 + 2\gamma - \gamma^2)]}{4(1 + \gamma)(2 - \gamma Y_i)}$ and $\theta_i < \theta_i^b \equiv \frac{\gamma(2Y_i - \gamma)}{2(2 - \gamma Y_i)}$. We must have $\theta_i^b < \theta_i$ so that $8(1 + \gamma)(Y_i + Y_j) - 16 - \gamma(2 + \gamma)(6 + \gamma) > 0$. Now we show that
\[
\Theta_i(\theta_i, 1, Y_i, Y_j) = 2\gamma (1 + \gamma)(2Y_i - \gamma) - [8(1 - \gamma) - \gamma^2 (2 + \gamma)(6 - \gamma) + 4\gamma (1 + \gamma) (Y_i + 2Y_j)] \theta_i
\]
is nonnegative. This is certainly the case if the coefficient of $\theta_i$ is negative. Otherwise, we have
\[
\varpi_1(\theta_i, Y_i, Y_j) - \varpi_2(\theta_i, Y_i, Y_j) \leq \varpi_1(\theta_i^a, Y_i, Y_j) - \varpi_2(\theta_i^a, Y_i, Y_j) \theta_i^a
\]
\[
= \frac{[8(1 + \gamma)(Y_i + Y_j) - 16 - \gamma(2 + \gamma)(6 + \gamma)] [8 - 8\gamma - 10\gamma^2 - 2\gamma^3 + \gamma^4 + 8\gamma (1 + \gamma) Y_j]}{4(1 + \gamma)(2 - \gamma Y_i)} \geq 0.
\]

Note that the second parenthesis in the numerator is positive because $\varpi_2 > 0$. The above relation suggests that $\Theta_i \geq 0$ for any $\theta_i \in [0, 1]$. In this case, we can set $\hat{\gamma} = 1$. Hence, we conclude the proof. \qed
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