Retailer’s Ordering Policy in a Supply Chain when Demand is Price and Credit Period Dependent

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ABSTRACT

Trade credit is a well established promotional tool in the present competitive world and its impact on demand cannot be ignored. Businesses often use trade credit to increase their market share and, in turn, the profit. Undoubtedly, trade credit plays a great role in increasing the demand but it also involves a great risk of non-payment. In order to reduce the risk of non-payment, businessman at times use a partial trade credit policy in which they demand a certain percentage of the total amount from the customer at the time of purchase and offers the credit for the remaining amount. Furthermore, it is also observed that the demand of FMCG is highly price sensitive. In order to see the effect of credit and price together, on demand, the retailer’s demand is taken as a function of price and credit period. Moreover it is assumed that the supplier offers the full credit to the retailer but the retailer passes a partial credit to customers. The inventory model, determines the optimal replenishment time, credit period, and price for the retailer that maximizes profit. Numerical examples have been provided to support the model followed by the comprehensive sensitivity analysis.

Keywords: Inventory, Partial Trade Credit, Permissible Delay in Payments, Price and Credit Period Linked Demand, Two-Level of Trade Credit

INTRODUCTION

In today’s competitive business environment, trade credit plays an important role in capturing maximum market share. Supplier offers trade credit to the retailers to encourage selling, promote market share, and reduce on hand inventory stock levels. On the other hand, retailer gains capital, materials and services without any payment during the credit period. Hence both the retailer and the supplier can take advantage of the trade credit policy. As evidence by previous literature including Peterson and Rajan (1997), Nilsen (2002), Fisman and Love (2003), Love et al. (2007), and Ge and Qiu (2007) trade credit is important source of finance for the retailers. The impact of trade credit policy on EOQ was first developed by Goyal (1985). Aggarwal and Jaggi (1995) generalized Goyal’s (1985) model

In all the mentioned models it was assumed that the supplier would offer the trade credit to the retailer but the retailer does not extend this same to his customers. This is termed as one level of trade credit. But, in most business transactions, this assumption is unrealistic. As it is observed that sometimes the retailer tend to also extend this benefit to his customers by offering a delayed payment period, known as two level trade credit. Huang (2003) and Biskup et al. (2003) developed the retailer’s optimal ordering policy under a two-level trade credit policy in which the retailer receives a favorable delayed payment period from the supplier and subsequently provides a delayed payment period to customers. Later, Huang (2006) modified Huang (2003) to incorporate a retailer’s storage space limitation into the model. Huang (2007) extended Huang (2003) to an economic production quantity (EPQ) model with two-level trade credit. In all of the abovementioned articles, it is assumed that the trade credit period offered by the supplier is longer than the trade credit period offered by the retailer. Teng and Goyal (2007) released this assumption and established an EOQ model with two-level trade credit. Chung and Huang (2007) continued to amend Huang (2006) to propose an EOQ model for deteriorating items under the two-level trade credit policy. Liao (2008) developed an EOQ model for exponentially deteriorating items under the two-level trade credit policy. In contrast, Teng and Chang (2009) modified Huang (2007) and developed an EPQ model. Jaggi et al. (2008) developed an EOQ model under a two-level trade credit policy with credit-linked demand. Thangam (2009) extends Jaggi (2008) model for perishable items when demand depends on both selling price and credit period.

To reduce default risks, in practice, a retailer sometimes, offers a partial trade credit to its customers who must pay a portion of the purchase amount at the time of placing an order as a collateral deposit, and then gets a credit on the rest of the outstanding amount. Huang (2005) developed an optimal retailer’s policy for one level where the supplier offers partial credit to the retailer. Adding to this, Huang and Hsu (2008) proposed an EOQ model with a two-level trade credit policy in which the retailer obtained the full trade credit from the supplier, but only offered a partial trade credit to end customers. However he has ignored the fact that the retailer offers his customer a permissible delay period N hence the retailer receives his revenue till $T+N$ and not till $N$. Recently, Teng (2009) developed an optimal ordering policies for a retailer who offers distinct trade credits to its good and bad credit customers. But, in his paper he has considered demand to be constant whereas, in real practice it has been observed that the demand does get influenced with the credit period and the price. Here, in this paper we extend Teng (2009) for price as well as credit sensitive demand. Numerical example is presented to illustrate the theoretical result followed by some managerial applications.

ASSUMPTIONS AND NOTATIONS

The assumptions and notations throughout this study are as follows:

1. The discussion and analysis in this paper is restricted to the case of a single-supplier, single-retailer and many customers of a specific product.
2. Demand is credit as well as selling price dependent.
3. Shortages are not allowed.
4. Time horizon is infinite.
5. Replenishments are instantaneous.
6. The retailer gets full trade credit ($M$) from the supplier.
7. The retailer just offers a partial trade credit ($N$) to each customer i.e., the customer has to make an initial payment at the time of purchase and the remaining payment at the end of credit period ($N$).
8. During the credit period offered by the supplier, the retailer sells the items and uses the sales revenue to earn interest at a rate $I_e$. At the end of the permissible delay period, the retailer pays the purchasing cost to the supplier and incurs a capital opportunity cost at a rate $I_p$ for the items in stock or for the items already sold but not yet paid for by the customers.

In addition, following notation are used to develop the model:

- $D(N, P) \approx D$, demand per unit of time
- $Q$, order quantity
- $T$, inventory cycle length
- $I(t)$, the inventory level at time $t$
- $A$, ordering cost per order
- $C$, unit purchase price of the item
- $P$, unit selling price of the item
- $I$, inventory carrying charge per $ per unit time (excluding interest)
- $I_e$, interest rate that can be earned
- $I_p$, interest rate payable per $ per unit time
- $\alpha$, fraction of total amount payable by the customers to the retailer at the time of placing an order, $0 \leq \alpha \leq 1$
- $M$, credit period offered by the supplier to the retailer
- $N$, credit period offered by the retailer to his customers
- $Z(T, P, N)$, retailer’s profit per unit time

**MATHEMATICAL MODEL**

The demand rate is taken as the function of price and credit period offered by the retailer to his customers. For a given selling price $P > 0$, under the assumption, that the marginal effect of credit period on sales is proportional to the unrealized potential of the market demand without any delay; the demand $D(N, P)$ can be represented as a partial differential equation

$$\frac{\partial D(N, P)}{\partial N} = r[\mu(P) - D(N, P)]$$  \hspace{1cm} (1)

where:

- $\mu(P) = KP^e$, maximum market demand over the planning horizon when the selling price is $P$ and $r$ is the saturation rate of demand, $0 \leq r < 1$ (It can be estimated using the past data).

The solution of the equations (1) can be found using the initial condition that, at $N = 0$, $D(N, P) = \lambda(P) = kP^e$ (there is a minimum demand when there is no permissible delay period offered to the customers from the retailer).

The solution of (1) is

$$D \equiv D(N, P) = \mu(P) - [\mu(P) - \lambda(P)]e^{-rN}$$  \hspace{1cm} (2)

where $\lambda(P) \leq \mu(P)$ for any value of $P$.

Further, we assume that the retailer’s gets full credit from his supplier but he just offers a partial credit to his customer. The retailer sells items at time $t \in [0, T]$ and receives the partial payment from his customer at the time of selling of the items and the remaining payment from all his customers till $T+N$.

The total profit per unit time for the retailer is composed of sales revenue, purchasing cost, ordering cost, inventory carrying cost, interest earned and interest paid. The last two components will depend on the values of $N, M$ and $T+N$.

(a) Sales revenue $= PD$  \hspace{1cm} (3)
(b) Cost of placing orders $= A/T$  \hspace{1cm} (4)
(c) Cost of purchasing units $= CD$  \hspace{1cm} (5)
(d) Cost of carrying inventory $= \frac{ICDT}{2}$  \hspace{1cm} (6)

Depending on the value of $N$ and $M$ two cases arises:

Case 1. $N \leq M$ and Case 2: $N \geq M$.

Case 1. $N \leq M$

In this case the retailer sells the products and uses the sales revenue to earn interest at a rate of $I_e$ in the interval $[0, M]$. At the end of the permissible delay period $M$, the retailer pays all
of the purchasing cost to the supplier and incurs a capital opportunity cost at a rate of \( I_p \) for the items still in stock and for the items sold but which have not yet been paid full by customers. Based on the values of \( T \) (replenishment cycle time), \( M \) (the time at which the retailer must pay the supplier to avoid interest charge) and \( T+N \) (the time at which the retailer receives the payment from the last customer), we have the following three possible Situations:

Situation 1: \( N \leq T \leq T+N \leq M \),
Situation 2: \( T \leq N \leq M \leq T+N \),
Situation 3: \( N \leq M \leq T \leq T+N \).

Now let us discuss the detailed formulation in each situation.

**Situation 1: \( N \leq T \leq T+N \leq M \)**

In this Situation the retailer receives the partial payments in the interval \([0, T]\) and the remaining payment from \([N, T+N]\). The retailer accumulates interest from the partial payment as well as the remaining payment of his customer at the rate \( I_e \) (Figure 1).

Interest earned per unit time by the retailer is:

\[
\frac{I_e P}{T} \left[ \frac{\alpha DT^2}{2} + \alpha DT(M - T) + \frac{(1 - \alpha)DT^2}{2} + (1 - \alpha)DT(M - T - N) \right] = \frac{I_e PD}{2T} \left[ T + 2\alpha(M - T) + 2(1 - \alpha)(M - T - N) \right]
\]

(7)

Since \( M \geq T+N \) the retailer does not incur any opportunity cost. At \( M \) the retailer pays off all units sold and keep his profits.

**Situation 2: \( T \leq N \leq M \leq T+N \)**

During the interval \([0, T]\) the retailer receives the partial payment from his customers \( T \). Further, from time \( N \), the retailer starts receiving the remaining payment and earns interest on it till \( M \) (Figure 2).

Consequently, the interest earned per unit time is:

\[
\frac{I_e P}{T} \left[ \frac{\alpha DT^2}{2} + \alpha DT(M - T) + \frac{(1 - \alpha)DT^2}{2} + (1 - \alpha)DT(M - T - N) \right]
\]

(8)

Furthermore, without receiving all the payments from the customers but paying off the supplier at the due date \( M \), the retailer has to bear a capital opportunity cost at the rate \( I_p \) for the items sold but which have not yet been
Interest earned per unit time is:

\[
\frac{I_{CD}T}{N} \left[ \frac{(1-\alpha)D(T+N-M)^2}{2} \right] = \frac{I_{PD}T}{M} \left[ \frac{\alpha M^2 + (1-\alpha)(M-N)^2}{2} \right]
\]

(9)

The capital opportunity cost per unit time for the items in stock as well as for the items sold but which have not yet been paid full by customers is:

\[
\begin{align*}
&\left[ \frac{I_{CD}T}{N} \left( \frac{(1-\alpha)D(T+N-M)^2}{2} \right) + \frac{\alpha D(T-M)^2}{2} \right] = \\
&\left[ \frac{I_{PD}T}{M} \left( \frac{\alpha M^2 + (1-\alpha)(M-N)^2}{2} \right) + \frac{\alpha D(T-M)^2}{2} \right]
\end{align*}
\]

(10)

Situation 3: \( N \leq M \leq T \leq T+N \)

The retailer receives the partial payments during the interval \([0, M]\) and the remaining payment in the interval \([N, M]\). The retailer accumulates interest from the partial payment as well as the remaining payment of his customer at the rate \( I_{c} \) (Figure 3).

Interest earned per unit time by the retailer is:
Therefore, the retailer’s profit per unit time for the case when \( N \leq M \) is \( Z(T, P, N) \)

\[
Z(T, P, N) = \begin{cases} 
Z_1(T, P, N) & \text{if } N \leq T \leq T + N \leq M \\
Z_2(T, P, N) & \text{if } N \leq T \leq M \leq T + N \\
Z_3(T, P, N) & \text{if } N \leq M \leq T \leq T + N 
\end{cases}
\]

where:

\[
Z_1(T, P, N) = (P-C)D \frac{A}{T} - \frac{ICDT}{2} + \frac{I_P PD}{2T} \left[T + 2\alpha(M - T) + (1 - \alpha)(M - T - N)\right]
\]

(12)

\[
Z_2(T, P, N) = (P-C)D \frac{A}{T} - \frac{ICDT}{2} + \frac{I_P PD}{2T} \left[\alpha T^2 + 2\alpha T(M - T) + (1 - \alpha)(M - N)^2\right] - \frac{I_P CD}{2T} \left[(1 - \alpha)(T + N - M)^2\right]
\]

(13)

\[
Z_3(T, P, N) = (P-C)D \frac{A}{T} - \frac{ICDT}{2} + \frac{I_P P}{T} \left[\frac{\alpha DM^2}{2} + \frac{(1 - \alpha)D(M - N)^2}{2}\right] - \frac{I_P CD}{2T} \left[\frac{(1 - \alpha)D(T + N - M)^2}{2} + \alpha D(T - M)^2\right]
\]

(14)

Case 2. \( N \leq M \)

In this case, the length of credit period \( M \) offered by the supplier is less than the length of credit period \( N \) offered by the retailer. The total profit per unit time for the retailer is composed of sales revenue, purchasing cost, ordering cost, inventory carrying cost, interest earned and interest paid. Among these components the first four components are the same as in Case 1. According to the values of \( T \) and \( T+N \) the interest earned and the interest paid is evaluated under the two possible situations:

\( T \leq M \) and \( M \leq T \).

**Situation 1:** \( T \leq M \leq N \leq T+N \). During the interval \([0, M]\) the retailer earns the interest on the partial payment received from the customer at the rate \( I_e \) (Figure 4).

The interest earned per unit time by the retailer is:

\[
\left\{ \frac{I_p PD \alpha}{2} \left( T + 2(M - T)\right) \right\}
\]

(15)

At \( M \) the retailer has to pay the cost of all the units purchased without receiving the full payment from his customer. Therefore he has to bear the opportunity cost at the rate \( I_p \), hence the opportunity cost per unit time is:

\[
\left\{ \frac{I_p C}{T} \left[ (1 - \alpha)DT(N - M) + \frac{(1 - \alpha)DT^2}{2}\right] \right\} = \left\{ \frac{I_p CD(1 - \alpha)}{2} (2(N - M) + T) \right\}
\]

(16)

**Situation 2:** \( M \leq N \leq T \leq T+N \) during the interval \([0, M]\) the retailer earns the interest on the partial payment received from the customer at the rate \( I_e \) (Figure 5).

The interest earned per unit time by the retailer is:

\[
\left\{ \frac{I_p P}{T} \left[ \frac{\alpha DM^2}{2} \right] \right\}
\]

(17)

At \( M \) the retailer has to bear an opportunity cost for the stock in hand as well as for the items sold yet not been paid full by the customer. The interest payable per unit time is given by:

\[
\left\{ \frac{I_C}{T} \left[ \frac{\alpha D(T - M)^2}{2} + (1 - \alpha)DT(N - M) + \frac{(1 - \alpha)DT^2}{2}\right] \right\} = \left\{ \frac{I_C CD(1 - \alpha)}{2} \left( (T - M)^2 + 2(1 - \alpha)T(N - M) + (1 - \alpha)T^2\right) \right\}
\]

(18)
Consequently, the retailer’s profit per unit time in this case is $Z(T, p, N)$

$$Z(T, P, N) = \begin{cases} Z_s(T, P, N) & \text{if } T \leq M \leq N \leq T + N \\ Z_s(T, P, N) & \text{if } M \leq N \leq T \leq T + N \end{cases}$$

Where:

$$Z_s(T, P, N) = (P-C)D \left( \frac{A}{T} - \frac{ICD}{2} + \frac{I \cdot PD \alpha}{2}(T + 2(M - T)) - \frac{I \cdot CD(1-\alpha)}{2}(2(N - M) + T) \right)$$

$$Z_5(T,P,N) = (P-C)D \left( \frac{A}{T} - \frac{ICD}{2} \right) + \frac{I \cdot P \alpha}{T} \left( \frac{\alpha DM^2}{2} \right) - \frac{I \cdot CD}{2T} \left( \frac{\alpha(T - M)^2}{2} + \frac{2(1 - \alpha)T(N - M)}{(1 - \alpha)T^2} \right)$$

**Solution Procedure**

The objective is to determine the optimum value of $T$, $P$, and $N$ which maximizes the total profit $Z(T,P,N)$.

**Determination of Optimal Replenishment Time $T$ for Any Given Value of $P$ and $N$**

In order to maximize the total profit, taking the first and second order derivatives of $Z_s(T,P,N)$,
\[
Z_2(T, P, N), Z_3(T, P, N), Z_4(T, P, N), \text{ and } Z_5(T, P, N)
\]
with respect to \( T \) (keeping \( N \) and \( P \) fixed), we get

\[
\frac{\partial Z_i(T, P, N)}{\partial T} = \frac{A}{T^2} - \frac{ICD}{2} - \frac{IPD}{2}
\]

\[
\frac{\partial^2 Z_i(T, P, N)}{\partial T^2} = -\frac{2A}{T^3}
\]  
\[
\frac{\partial^2 Z_3(T, P, N)}{\partial T^2} = \frac{-2A - (I_p C - I_e P)(1-\alpha)D(M-N)^2}{2T^2}
\]
\[
\frac{\partial^2 Z_i(T, P, N)}{\partial T^2} = \frac{\alpha D M^2 (1-\alpha) D(M-N)^2}{2T^2}
\]  
\[
\frac{\partial^2 Z_i(T, P, N)}{\partial T^3} = \frac{-2A - (I_p C - I_e P)(1-\alpha)D(M-N)^2}{T^3}
\]  
\[
\frac{\partial Z_5(T, P, N)}{\partial T} = \frac{A}{T^2} + \frac{D[IC + CI_p]}{2}
\]

For fixed \( N \), equations (22) and (28) imply that, \( Z_1(T, P, N) \) and \( Z_4(T, P, N) \) are concave on \( T > 0 \). However, \( Z_2(T, P, N), Z_3(T, P, N) \) and \( Z_5(T, P, N) \) are concave on \( T > 0 \) if \( C_l p > P_l e \).

Thus, there exists a unique value of \( T \) (say \( T_1^* \)) which maximizes \( Z_1(T) \) as

\[
T_1^* = \sqrt{\frac{2A}{(IC + I_e P)D}}
\]

\( T_1^* \) would satisfy the condition:

\[
0 \leq T \leq (M - N) \quad \text{provided:}
\]

\[
\Delta_1 = 2A - D(M - N)^2[IC + I_e P] \leq 0
\]  
\[
\Delta_1 = 2A - D(M - N)^2[IC + I_e P] \geq 0
\]  

Substituting equation (32) into (12), we get the optimal value of \( Z_1(T) \) (say \( Z_1^* \)).

Similarly, there exists a unique value of \( T \) (say \( T_2^* \)) which maximizes \( Z_2(T) \) as

\[
T_2^* = \frac{\sqrt{2A + (1-\alpha) D(M-N)^2(C_l p - P_l e)}}{(I_p - I_e)CD + (I_p - I_e)C_0 D}
\]

\( T_2^* \) would satisfy the condition:

\[
0 \leq (M - N) \leq T \quad \text{provided:}
\]

\[
\Delta_1 = 2A - D(M - N)^2[IC + I_e P] \geq 0
\]  

Substituting equation (33) into (13), we get the optimal values of \( Z_2(T) \) (say \( Z_2^* \)).

Likewise, we can obtain the optimal value of \( T \) (say \( T_3^* \)) which maximizes \( Z_3(T) \) as

\[
T_3^* = \sqrt{\frac{2A + (CL_p - PL_e)(\alpha D M^2 + (1-\alpha) D(M-N)^2)}{(I_p)CD}}
\]
Substituting equation (35) into (14), we get the optimal values of $Z_3(T)$ (say $Z_3^*$).

Thus, there exists a unique value of $T$ (say $T_4^*$) which maximizes $Z_4(T)$ as

$$T_4^* = \frac{2A}{(IC + \alpha I, P + (1-\alpha)CI_p)D}$$

Substituting equation (37) into (19), we can get the optimal value of $Z_4(T)$ (say $Z_4^*$).

Similarly, there exists a unique value of $T$ (say $T_5^*$) which maximizes $Z_5(T)$ as:

$$T_5^* = \frac{2A + \alpha DM^2(Cl_p - PI_p)}{(I + I_p)CD}$$

$T_5^*$ would satisfy the condition $0 \leq M \leq T$ provided

$$\Delta_5 = 2A - DM^2[IC + \alpha I_pP + (1-\alpha)CI_p] \geq 0$$

(40)

Substituting equation (39) into (20), we can get the optimal values of $Z_5(T)$ (say $Z_5^*$).

Combining the possible cases, we obtain the following theorem.

**Theorem 1.** For a fixed value of $N$ and $p$,

(a) If $\Delta_1 \leq 0$ then $T^* = T_1^*$.

(b) If $\Delta_1 \geq 0$ then $T^* = T_2^*$.

(c) If $\Delta_1 \geq 0$ and then $T^* = T_3^*$.

Proof: It immediately follows from (34) and (36).

**Theorem 2.** For a fixed value of $N$ and $p$.

(d) If $\Delta_3 \leq 0$ then $T^* = T_4^*$.

(e) If $\Delta_3 \geq 0$ then $T^* = T_5^*$.

Proof: It immediately follows from (38).

**Determination of Optimal Value of $P$, for a Fixed Value of $T$ and $N$**

For a fixed value of $T$ and $N$, optimal value of $P$ can be obtained by solving the first order necessary condition (i.e., $\frac{\partial Z_i}{\partial P} = 0$) and examining the second order sufficient condition for concavity (i.e., $\frac{\partial^2 Z_i}{\partial P^2} \leq 0$).

For the given values of $T$ and $N$, the first order and second order partial derivatives of $Z_i(T,P,N), i=1, 2, 3, 4, 5$ with respect to $P$ are given by:

$$\frac{\partial Z_i}{\partial P} = \frac{D}{2} \left[ 1 + \frac{I_e}{2} \left[ 2M - T - 2N(1-\alpha) \right] \right] +$$

$$\frac{\partial D}{\partial P} \left[ (P-C) - \frac{I_eT}{2} + \frac{I_eP}{2} \left[ 2M - T - 2N(1-\alpha) \right] \right]$$

$$\frac{\partial Z_i}{\partial P} = 0 \Rightarrow P =$$

$$\frac{(2C + ICT)e}{2 + 2I_eM + 2I_eN(1-\alpha) - I_eT + 2I_eN\alpha(e-1)}$$

$$\frac{\partial^2 Z_i}{\partial P^2} =$$

$$\frac{1}{2} \frac{eP^{-2-e}}{(e-1) + (2C + ICT)(1+e)}$$
As the equations are mathematically intractable the concavity of the function has been established graphically (Figure 6).

In order to find the optimal solution, Algorithm 1 is used.

**Algorithm 1**

Step 1. Set \( N = 1 \).

Step 2. Determine the optimal values of \( T \) (i.e., \( T_1^* \), \( T_2^* \), or \( T_3^* \)) using Theorem 1

Step 3. If \( M \geq T \) and \( (M-N) \geq T \) then calculate \( Z_1(T, P, N) \) else go to step 5.
Step 4. If \( Z_1(T, P, N) > Z_1(T, P, N-1) \), increment the value of \( N \) by 1 and go to step 2; else the current value of \( N \) is optimal and the corresponding value of \( T \) and \( Z(T, P, N) \) can be calculated.

Step 5. If \( M \geq T \) and \( (M-N) \leq T \) then calculate \( Z_2(T, P, N) \) else go to step 7.

Step 6. If \( Z_2(T, P, N) > Z_2(T, P, N-1) \), increment the value of \( N \) by 1 and go to step 2; else the current value of \( N \) is optimal and the corresponding value of \( T \) and \( Z(T, P, N) \) can be calculated.

Step 7. If \( M \leq T \) then calculate \( Z_3(T, P, N) \).

Step 8. If \( Z_3(T, P, N) > Z_3(T, P, N-1) \), increment the value of \( N \) by 1 and go to step 2; else the current value of \( N \) is optimal and the corresponding value of \( T \) and \( Z(T, P, N) \) can be calculated.

**Computational Algorithm 2**

Step 1. Set \( N = 1 \).

Step 2. Determine the optimal values of \( T \) (i.e., \( T_4^* \) or \( T_5^* \)) using Theorem 2.

Step 3. If \( 0 \leq M \leq T \) then calculate \( Z_4(T, P, N) \) else go to step 5.

Step 4. If \( Z_4(T, P, N) > Z_4(T, P, N-1) \), increment the value of \( N \) by 1 and go to step 2; else the current value of \( N \) is optimal and the corresponding value of \( T \) and \( Z(T, P, N) \) can be calculated.

Step 5. Calculate \( Z_5(T, P, N) \).

Step 6. If \( Z_5(T, P, N) > Z_5(T, P, N-1) \), increment the value of \( N \) by 1 and go to step 2; else the current value of \( N \) is optimal and the corresponding value of \( T \) and \( Z(T, P, N) \) can be calculated.

**NUMERICAL EXAMPLE**

Let \( r = 0.12, C = $30 \) per unit, \( M = 30 \) days, \( A = $1000 \) per order, \( I = 0.1 \) per year, \( I_p = 0.18 \) per year, \( I_e = 0.10 \) per year, \( a = 0.25 \). Using the proposed algorithm, we obtained the optimal results as: Optimal credit period offered by the retailer \( (N^*) = 49.82 \), Optimal cycle length \( (T^* = T_4^*) = 8.25 \) days, \( P^* = $90 \) and profit per day \( Z^* = $69745.21 \). Further, the sensitivity analysis on \( M, A, e \) and \( r \) is shown in Table 1.

**Observations**

- It is observed from Table 1, a higher value of \( M \) causes a higher value of Total Profit, but lower value of \( P \) and \( T \). It indicates the following managerial phenomena: when the supplier provides a longer credit period, the retailer replenishes the goods more often. In other words, the retailer

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Figure 6. Graphical representation of profit function depicting the concavity of the profit function w.r.t. \( p \) and \( N \)
will shorten the cycle time and reduce the selling price in order to take advantage of the longer credit period.

- It is observed that as $r$ increases, $N$ decreases, $P$ decreases and $T$ marginally decreases; but Total Profit increases. It shows that retailer should offer lower credit period to customers when the rate of saturation of demand is high.

- As ordering cost, $A$, increases, the replenishment cycle time $T$ significantly increases; but optimal selling price marginally increases. Keeping the credit period at some threshold, the optimal profit decreases as $A$ increases. It indicates the following managerial effect. If the ordering cost is higher, it is reasonable that the retailer lengths the cycle time to reduce the frequency of replenishment and he marginally increases the selling price.

### Table 1. Sensitivity analysis for various inventory model parameters

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### CONCLUSION AND FUTURE RESEARCH DIRECTIONS

This paper incorporates some pragmatic features namely; partial trade credit, price as well as credit sensitive demand and develop a new inventory model which is very much consistent with the real life. Here we assume that the retailer receives a full trade credit from his supplier, and offers. An easy to use algorithm has been provided for obtaining the optimal cycle length, credit period as well as the price for the retailer. The effect of different parameters on the optimal replenishment time, credit period and price has been investigated. Briefly, findings of this paper, not only provide a valuable reference for decision-makers in planning and controlling the inventory, but also provide a useful platform for many organizations that use the decision rule to improve their total profit in
the real world. In future research, this model can be extended to different forms of credit-linked demand function for deteriorating as well as non-deteriorating items. Further, one could generalize the model to allow for shortages, quantity discounts, inflation rates and others.

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REFERENCES


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