On Optimal Power Allocation and Relay Assignment in Multiuser Cognitive Radio Networks

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Abstract—This paper considers the problem of joint spectrum sensing and secondary data transmission in multiuser, multirelay cognitive radio networks. An optimization framework has been developed for the selection of source nodes, relay assignment and power allocation with an aim to maximize the sum throughput of secondary transmission under the constraints of sum transmission power for secondary source nodes, number of relays and probability of detection of primary user’s signal. A suboptimal scheme is then suggested to get rid of the high complexity of the optimal relay assignment scheme. Numerical results are presented to validate the proposed power allocation, optimal and sub-optimal relay assignment schemes for various values of probability of detection and total number of relays constraints.

Keywords—cognitive relay networks; source selection; relay assignment; power allocation; end-to-end throughput.

I. INTRODUCTION

The usage of radio spectrum resources and the regulation of radio emissions are coordinated by regulatory bodies like the Federal Communications Commission (FCC), International Telecommunication Union (ITU) and others. These regulatory bodies assign spectrum to licensed holders (also known as primary users) on a long-term basis for large geographical regions. However, a recent study reveals a large portion of the assigned spectrum remains underutilized [1]. This inefficient usage of the limited spectrum necessitates the development of dynamic spectrum access techniques, where users who have no spectrum licenses, also known as secondary users (SUs), are allowed to use the unused licensed spectrum temporarily. This necessity of proper utilization of the frequency spectrum has lead to the concept of cognitive radio.

In recent times, joint spectrum sensing for detecting primary user’s (PU’s) signal and data transmission of SU has become an important research topic in cognitive radio networks (CRNs). To increase sensing reliability, multiple SUs or a single SU and relay(s) can participate in cooperative spectrum sensing (CSS) by exploiting the spatial diversity of the received PU signal [2]. At the same time, cooperative communication using relays has been shown not only to improve the throughput of the system, but also to conserve transmission power and to reduce interference to PU in a significant manner [3].

Majority of the existing works on joint spectrum sensing for PU detection and SU transmission maintain a time slot in each medium access control (MAC) frame for periodic spectrum sensing, while the remaining slot is used for SU data transmission [4]-[7]. In [8], authors don’t consider provision of any sensing slot in the MAC frame of SU transmission. Instead, the possible presence of PU is monitored throughout the entire frame by SU and its cooperative relay. They propose an optimal power allocation strategy for maximizing SU throughput under a sensing and sum power constraints.

However, this problem is also important in a multiuser multi-relay assisted CRN. To that aim, a multiuser multi-relay assisted CRN (MMCRN) is considered where a set of sources is selected to meet the target sensing performance under a sum transmission power constraint. This is accompanied by an optimal source-relay assignment. A closed form expression for the transmit power allocation (TPA) of the SUs is derived over Rayleigh fading channels. An optimal and a suboptimal algorithm are then proposed to highlight the performance complexity trade-off. Numerical results show that the proposed algorithm performs close to the optimal approach at lower computational complexity with the increase in the number of relays.

II. SYSTEM MODEL AND PRELIMINARIES

A. System Model

An MMCRN consisting of a PU, a fusion center (FC), M secondary source-destination (also referred as cognitive sources CS and cognitive destinations CD, respectively) pairs and K (K<M) amplify-and-forward (AF) relays is considered and is shown in Fig. 1. Each frame consists of sensing phase followed by SU transmission. CSs first monitor the PU signal, thereby, a set of CSs is selected that amplify and forward their received signals to FC, where detection of PU presence/absence is done. If PU is found absent, selected CSs then transmit data to their respective relays, which then forward the same to the respective destinations.

B. Signal Modeling

The signal received by CS, can be expressed as

$$y_s(n) = \lambda \sqrt{P} |h_{s_p}|^2 x_p(n) + u_s(n), \quad n = 1, \ldots, N$$  \hspace{1cm} (1)

where binary valued $\lambda$ indicates PU’s presence ($\lambda=1$) and $\lambda=0$ otherwise. Here $P$ is the total transmit power, $|h_{s_p}|^2$ is the squared channel gain between source node $s_p$ and CS $s$, $x_p(n)$ is the discrete-time data symbol on a channel, and $u_s(n)$ is the additive white Gaussian noise.
absence ($\lambda=0$). The symbol $h_{x_{CP}}$ indicates fading channel coefficient of the link between PU-CS$_i$. Similar symbols used in the subsequent discussions indicate the same between FC-CS$_i$, CS$_i$-CR$_j$ ($h_{x_{CR}}$) and CR$_j$-CD$_i$ ($h_{x_{CD}}$). The symbol $P_o$ denotes the transmission power of the PU, $P_i$ and $P_j$ similarly indicates the power of the CS$_i$ and the relays, respectively. Primary signal $s_P(n)$ is assumed to have zero mean and unit variance, i.e. $E[x_P(n)^2]=1$. The noise $u_{CP}(n)$ at CS$_i$ and $u_{C}(n)$ at FC in (2) are additive, independent and identically distributed (i.i.d.), circularly symmetric, complex Gaussian (CSCG) random sequences with zero mean and variance $P_o$. The symbol $N$ denotes the number of samples.

The signal received by the FC can be expressed as follows

$$y_{rf}(n) = \sum_{i=1}^{M} a_i \sqrt{r_i} h_{x_{rf}} y_{srf}(n-N) + u_{rf}(n), \quad n=N+1,...,2N$$

where binary valued $a_i$ indicates ($a_i=1$) the selection of CS$_i$. If CS$_i$ is selected, it transmits data to a predetermined relay (say CR$_j$), CR$_j$ forwards the data to CD$_i$. Received signal $y_{x_{rf}}(n)$ at CR$_j$ is

$$y_{x_{rf}}(n) = a_i \sqrt{r_i} h_{x_{rf}} x_{ci}(n) + u_{rf}(n), \quad n = 1,...,N$$

where $x_{ci}(n)$ denotes the signal sent from CS$_i$, which is having zero-mean and unit variance. The noise $u_{rf}(n)$ is like $u_{ci}(n)$ with variance $N_{rf}$.

The signal transmitted from CR$_j$ after normalization by a factor of $\sqrt{E[|y_{x_{rf}}(n)|^2]}$ (where $E[\cdot]$ is the expectation operator) can be written as

$$\hat{y}_{x_{rf}}(n) = \frac{y_{x_{rf}}(n)}{\sqrt{E[|y_{x_{rf}}(n)|^2]}} = \frac{a_i \sqrt{r_i} h_{x_{rf}} x_{ci}(n) + u_{rf}(n)}{\sqrt{E[|y_{x_{rf}}(n)|^2]}} + \hat{u}_{rf}(n)$$

Along the lines of (4), the signal received by CD$_i$ from CR$_j$ is

$$y_{rf}(n) = \sqrt{r_j} h_{x_{rf}} \hat{y}_{x_{rf}}(n-N) + u_{rf}(n), \quad n=N+1,...,2N$$

$$= \frac{a_i \sqrt{r_i} r_j h_{x_{rf}} h_{x_{rf}} x_{ci}(n-N) + \hat{u}_{rf}(n)}{\sqrt{E[|y_{x_{rf}}(n)|^2]}}$$

where the noise $u_{rf}(n)$ is like $u_{ci}(n)$, with variance $N_{rf}$. The symbol $\hat{u}_{rf}(n)$ is the equivalent noise term in $y_{rf}(n)$. It can be easily shown that $\hat{u}_{rf}(n)$ is also additive i.i.d., CSCG with zero mean and variance being $\hat{N}_{rf} = N_{rf} + P_o |h_{x_{rf}}|^2 + N_{rf}$. It is assumed that $N_{rf} = N_{rf} = P_o$ and the gains of all available channels are known at the source before the transmission begins. We denote $G_{ij} = |h_{ij}|^2$, $G_{ic} = |h_{ic}|^2$, $G_{rf} = \frac{\hat{h}_{rf}}{\sqrt{r_j}} + |h_{rf}|^2$, and $G_{rf} = |h_{rf}|^2$, which represent the channel power gains.

The end-to-end signal-to-noise ratio (SNR) from CS$_i$ to its corresponding destination via CR$_j$ is

$$\gamma_{end}(i,j) = \frac{P_o G_{rf} \gamma_{rf}(i,j)}{1 + P_o G_{rf} + \frac{G_{rf} \gamma_{rf}(i,j)}{P_o}}$$

The end-to-end throughput for the corresponding link can be evaluated as follows

$$R(i,j) = \log_2\left(1 + \gamma_{end}(i,j)\right)$$

C. Cooperative Spectrum Sensing

The FC adopts an energy detection scheme for spectrum sensing by utilizing all the received signals from the selected CS$_i$. The test statistic received by the FC is expressed as $Y = \sum_{n=N+1}^{2N} y(n)^2$. According to the central limit theorem, for a sufficiently large value of $N$, the test statistic $Y$ approximately follows a Gaussian distribution under both hypotheses $H_1$ (i.e., $\lambda = 1$) and hypothesis $H_0$ (i.e., $\lambda = 0$). Mean values of the test statistic $Y$ under $H_1$ and $H_0$ can be expressed as $E(Y_1) = NW_0$ and $E(Y_0) = NW_0$, respectively where $\Omega_1 = \sum_{a} a_i$, $\Omega_0 = N\Omega_0$, and $\Omega_1 = \sum_{a} a_i |P_o r_j + P_o| r_j = \Omega_0$. Variance of the test statistic $Y$ under $H_1$ and $H_0$ can be expressed as $D(Y_1) \approx N \Omega_1^2$ and $D(Y_0) \approx N \Omega_0^2$, respectively.
Let $\varepsilon$ be the decision sensing threshold. Then, the probability of detection $p_d$ and probability of false alarm $p_f$ calculated at FC are given by [8]

$$p_d = Q \left( \frac{\varepsilon - E(Y_1)}{\sqrt{D(Y_1)}} \right) \approx Q \left( \frac{\varepsilon - N\Omega_d}{\sqrt{\Omega_d}} \right), \quad (9)$$

$$p_f = Q \left( \frac{\varepsilon - E(Y_0)}{\sqrt{D(Y_0)}} \right) \approx Q \left( \frac{\varepsilon - N\Omega_f}{\sqrt{\Omega_f}} \right), \quad (10)$$

where $Q(.)$ is the complementary cumulative distribution function of a standard Gaussian random variable.

Our objective is to maximize the sum of the end-to-end throughput (referred as sum throughput in the subsequent discussion) of the MMCRN by optimizing the source selection, relay assignment and sharing of the available power budget $P_T$ so that the probability of detection $p_d$ is maintained above a minimum threshold $p_d^\text{th}$. Hence, the optimization problem can be formulated as

$$\max \quad P(H_0)(1 - p_f) \sum_{i=1}^{M} \sum_{j=1}^{K} a_i b_{ij} R(i,j) \quad (11)$$

subject to

- (C1: Source sum power constraint)
  $$\sum_{i=1}^{M} a_i P_{s_i} \leq P_T \quad (11a)$$

- (C2: Probability of detection constraint)
  $$p_d \geq p_d^\text{th} \quad (11b)$$

- (C3: Source selection constraint)
  $$\sum_{i=1}^{M} a_i = K \quad (11c)$$

- (C4: Relay assignment constraint)
  $$\sum_{i=1}^{M} b_{ij} = 1, \quad j = 1,2, ..., K \quad (11d)$$

where C3 ensures the selection of $K$ out of $M$ sources. Additionally, $b_{ij}$ is the relay assignment indicator, i.e., $b_{ij}=1$ if the $i$th CS is linked with $CR_j$, or zero, otherwise. The constraint C4 ensures that at any point of time, a single relay is assigned to a single selected CS.

Since $P(H_0)$ is constant and $p_f$ is taken as an input, the factor $P(H_0)(1 - p_f)$ is nothing but a constant and the optimization problem can be rewritten as

$$\max \quad \sum_{i=1}^{M} \sum_{j=1}^{K} a_i b_{ij} R(i,j) \quad (12)$$

subject to

- (C1)-(C4)

### III. POWER ALLOCATION AND RELAY ASSIGNMENT

The value of $\varepsilon$ is calculated from (10) for a given set of $p_d$, $P_p$, $P_u$, $N$, $G_{s_i,c}$ and $G_{s_j,p}$. Equation (11b) then becomes

$$\sum_{i=1}^{M} a_i P_{s_i} (G_{s_i,c} G_{s_j,p} P_p + G_{s_o,c} P_o) \leq \frac{\varepsilon}{\sqrt{N(\Omega_f^* (p_d^\text{th})^2 / \sqrt{\Omega_f})}} \quad (13)$$

As the RHS is constant for a particular value of $p_d$, $p_d^\text{th}$, $\varepsilon$ and $N$, it is expressed as a constant $\eta$. Since $P_{s_i} = P_{s_i,c} = P_{s_i,r_j}$ and taking into account that $CS_i$ is connected with $CR_j$, the optimization problem is finally expressed as

$$\max \quad \sum_{i=1}^{M} \sum_{j=1}^{K} a_i b_{ij} \log_2 (1 + \frac{p_{s_i,c} G_{s_i,c} G_{s_j,p} P_p + G_{s_o,c} P_o}{\eta}) \quad (14)$$

subject to

- (C3), (C4)

### A. Source Selection and Power Allocation

Initially $K$ out of $M$ CSs are randomly selected and an estimated power allocation is done. Then this set of selected CSs is matched against (11a) and (11b). If both (11a) and (11b) are satisfied, then only the set is considered as a valid one and the next step is to follow optimal power allocation and relay assignment using sub-gradient method. Until both (11a) and (11b) are satisfied simultaneously, the above process goes on for various combinations of $K$ out of the $M$ CSs.

The objective function is convex in nature and also, the constraints are linear. Thus, it is obvious that (14) falls under the category of convex optimization problems, which can be efficiently solved using modern convex programming tools. In order to develop the primal-dual iteration [9] to solve (14) for power allocation, we need the Lagrangian function. Thus,

$$L\left(P_{s_i,r_j}, \alpha, \beta \right) = \sum_{i=1}^{M} \sum_{j=1}^{K} a_i b_{ij} \log_2 \left( 1 + \frac{p_{s_i,c} G_{s_j,c} G_{s_j,p} P_p + G_{s_o,c} P_o}{\eta} \right)$$

subject to

$$a_i b_{ij} + \beta \frac{p_{s_i,r_j}}{\eta} + \frac{p_{s_i,c} G_{s_j,c} G_{s_j,p} P_p + G_{s_o,c} P_o}{\eta} \leq 1, \quad j = 1,2,...,K \quad (15)$$

where $\alpha, \beta$ are the Lagrange multipliers.

The Lagrange dual function is now defined as

$$f(\alpha, \beta) = \max \left\{ L\left(P_{s_i,r_j}, \alpha, \beta \right) \right\}$$

subject to

$$a_i b_{ij} + \beta \frac{p_{s_i,r_j}}{\eta} + \frac{p_{s_i,c} G_{s_j,c} G_{s_j,p} P_p + G_{s_o,c} P_o}{\eta} \leq 1, \quad j = 1,2,...,K \quad (16)$$

where $\alpha, \beta$ are the Lagrange multipliers.
Taking into consideration the low SNR regime, (16) is solved for the optimal power and we get,

\[
S \left( P_{s,i'^{j}} \right) = \log_2 \left( 1 + \frac{P_{u} G_{s,i'^{j}} P_{r} G_{r,j} a_i}{P_u P_r} + \frac{\eta P_{u} G_{s,i'^{j}} G_{s,j} P_{p} + G_{s,j} c_{p}}{P_u P_r} \right) - \alpha P_{s,i'^{j}} - \beta P_{s,i'^{j}}
\]

To obtain the solution, initial values for \( \alpha, \beta \) are assumed and also that \((i, j)\) is a valid pair. Towards the solution of (15), the next step is to consider the inner maximization, that is to solve the corresponding sub-problem for every \((i, j)\) pair

\[
\max S \left( P_{s,i'^{j}} \right)
\]

s.t. \( P_{s,i'^{j}} \geq 0 \) \hspace{1cm} (17)

Taking into consideration the low SNR regime, (16) is solved for the optimal power and we get,

\[
P_{s,i'^{j}} = \frac{P_{u} G_{s,i'^{j}} \sqrt{\frac{\log_2 e}{P_u a + \beta (G_{s,i'^{j}} G_{s,j} P_{p} + G_{s,j} c_{p})}} - 1}{G_{s,j}}
\]

where \([x]^+ = \max(0, x)\). \hspace{1cm} (19)

### B. Optimal Relay Allocation Scheme

Substituting the optimal power allocation (19) into (17), the obtained corresponding dual function is

\[
f(\alpha, \beta) = \max \left[ \sum_{i=1}^{M} \sum_{j=1}^{K} a_i b_{ij} S \left( P_{s,i'^{j}} \right) + \alpha P_{r} + \beta \eta \right]
\]

s.t. \( P_{s,i'^{j}} > 0, a_i, b_{ij} \) \hspace{1cm} (20)

Now the \( K \times K \) SNR matrix \( S = \{ S \left( P_{s,i'^{j}} \right) \} \) of SNR is defined for every \((i,j)\) pair. The objective function defined in (20) is maximized by picking the elements from \( S \) such that the sum of the SNRs is the maximum. This problem is mathematically modeled as a weighted bipartite matching problem.

The weighted bipartite graph is modeled as follows. Firstly, the set of selected SUs are represented by a graph of \( K \) vertices. Another set of \( K \) vertices represent the \( K \) relays. A link between \( CS_i \) and \( CR_j \) exists if and only if \( CS_i \) is connected to \( CR_j \). Cost of the link has been assigned (7) as the end-to-end SNR from the \( i^{th} \) SU to its corresponding destination via \( j^{th} \) relay. Hence, this is clearly a linear assignment problem and it is solved by implementing the Hungarian method [10] with a complexity of \( O(K^3) \).

The sub-gradient method [11] is now implemented to solve the dual problem with guaranteed convergence. After finding the optimal solution of the dual function at given values of \( \alpha \) and \( \beta \), the \((i+1)^{th}\) iteration updates the \( \alpha \) and \( \beta \) values as:

\[
\alpha^{(i+1)} = \alpha^{(i)} - \delta \left( P_r - \sum_{i=1}^{M} \sum_{j=1}^{K} a_i P_{s,i'^{j}} \right)
\]

\[
\beta^{(i+1)} = \beta^{(i)} - \delta \left( \eta - \sum_{i=1}^{M} \sum_{j=1}^{K} a_i P_{s,i'^{j}} (G_{s,i'^{j}} G_{s,j} P_{p} + G_{s,j} c_{p}) \right)
\]

where \( \delta \) is the step size. With the updated values of \( \alpha \) and \( \beta \), the optimal power allocation and relay assignment are reevaluated until convergence occurs. (14) being a convex optimization problem, the duality gap between the primal and the dual problem is zero.

### C. Suboptimal relay assignment scheme

A suboptimal relay allocation scheme is proposed, as follows, to reduce complexity:

1. For the first selected SU, the end-to-end SNR is calculated for all the relays and that particular relay is assigned to the SU, for which the SNR value is the highest.
2. For the next selected SU, the search space for selecting the relay gets reduced as the relay selected in (Step 1) is excluded from the set and the same procedure is followed for this selected SU too.
3. This process is repeated until all the selected SUs are assigned relays and the complexity reduces to \( O(K^2) \).

### IV. Numerical Results and Discussions

This section presents performance evaluation for the proposed method. Simulations are done taking the values of the various parameters as follows: number of \( CS-CD \) pairs \( M \) and number of samples \( N \) are taken as 20 and 100, respectively. The values of \( P_p, P_r, \) and \( P_u \) are taken as \( 1 \text{mW} \) and \( 0.01 \text{mW} \), respectively. The probability of false alarm \( (p_f) \) is taken as 0.05. The channel gains are outcomes of independent, identically distributed (i.i.d) exponential random variables (rv’s) \( G_{s,i'^{j}}, G_{r,j}, G_{s,j}, \) and \( G_{s,j} \) with mean equal to 0.05, 0.05, 2.1 and 2.1 respectively. All the results presented here are obtained as an average of 1000 iterations.

In Fig. 2, sum throughput is shown for the proposed optimal, suboptimal and the random allocation schemes. Sum throughput is found to increase with number of relays \( K \) for all three schemes. The increase in throughput with \( K \) is due to the improvement in inherent spatial diversity of the MDCRN. Both the optimal and the suboptimal schemes outperform the random scheme and as the number of relays in the system increases, performance of the suboptimal scheme becomes comparable to that of the optimal scheme. As shown in the figure, maximum difference in the sum throughput between the optimal and the suboptimal schemes is found to be 4.19% for the number of relays \( K = 4 \), available power budget \( P_r = 30 \text{ mW} \) and minimum probability of detection \( P_d^{th} = 0.9 \). This difference in performance of the proposed sub-optimal scheme
gets gradually diminished as the number of relays is increased in the system.

Fig. 2 Sum throughput versus number of relays under available power budget $P_T = 30$ mW and minimum value of probability of detection $p_d^{th} = 0.9$

Fig. 3 is used to depict the increase in achievable sum throughput with $p_d^{th}$ for all three above mentioned schemes. The increase in throughput can be explained using (12), which shows that a higher value of $p_d^{th}$ makes provision for higher transmission power assignment to selected CSs. Furthermore, it also demonstrates that the suboptimal scheme is far better compared to the random scheme and its sum throughput values are near to the optimal scheme for all values of $p_d^{th}$. For example, at $p_d^{th}=0.8$, $K = 15$ and $P_T = 30$ mW, the difference in sum throughput between the suboptimal scheme and the optimal scheme is 2.51% and with that of random allocation scheme is 65.76%. This not only shows the importance of the proposed optimal scheme with respect to the random scheme but also efficiency of the proposed suboptimal scheme in achieving higher throughput with reduced complexity.

Fig. 3 Sum throughput versus minimum value of probability of detection under available power budget $P_T = 30$ mW and number of relays $K = 15$.

Fig. 4 illustrates the joint variation of the sum throughput and probability of detection for different values of available power budget $P_T$. Number of source and relays are set at 10 and 8 respectively. Minimum probability of detection is fixed at 0.62. It is noted that with increase in $P_T$ both achievable sum throughput and probability of detection increases for both of our proposed schemes. It may also be noted that sum throughput of the proposed suboptimal scheme is inferior to the optimal scheme about 14% for all values of $P_T$.

V. CONCLUSIONS

In this paper, we have addressed the problem of joint spectrum sensing and data transmission in multi-user multi-relay-assisted cognitive radio networks. Our proposed approach consists of three steps. Firstly, we follow an iterative procedure in the context of source selection. Secondly, an analytical approach was used in order to derive a closed form solution of the transmission power allocation of the selected secondary sources. Lastly, joint source and relay assignment were considered in order to maximize the end-to-end throughput of the CRN. Moreover, due to the high complexity of the optimal solution, a sub-optimal solution with reduced complexity has also been suggested. Numerical results show that the performance sub-optimal scheme approaches the optimal one when the number of relays in the CRN is large.

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