Orthogonal Space-Time Block Codes over Semi-Identical Channels with Channel Estimation

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Abstract—Assuming the channel gains associated with different receive antennas are not identically distributed, we study the orthogonal space-time block codes over non-identical channels with channel estimation. It is shown that the non-identical statistics lead to non-identical channel estimation errors, which make the conventional optimum decoder sub-optimum in this case. A new optimum decoder is derived. We show that it can be simplified to a symbol-by-symbol decoder under certain conditions. Analytical and simulation results show that our new decoder substantially outperforms the conventional decoder.

I. INTRODUCTION

Orthogonal space-time block codes (OSTBC) [1] is a well known technique in multiple-input multiple-output (MIMO) systems, due to its elegant code structure and simple decoding complexity. The receiver structure and the performance of OSTBC have been extensively studied in many works with both perfect and estimated channel state information (CSI) at the receiver; see [2]–[4] and the references therein. These works, however, are based on the assumption that the channels are independent and identically distributed (i.i.d.), but it does not always hold in real environments, especially in MIMO systems, where the antennas are usually placed far enough to ensure non-correlation between the channels.

OSTBC over non-identical channels first appeared in cooperative diversity scenarios [5]–[7], where the distributed nodes normally experience non-identical statistics. The performance of OSTBC over non-identical channels was also implicitly discussed in [8]–[10], as the issue of non-identical channels can be viewed as the special case of the correlated channels. More recently, we have investigated the receiver structure and the performance of OSTBC over non-identical channels with both coherent detection [11] and differential detection [12].

However, all the works on OSTBC over non-identical channels assume that the CSI is perfectly known at the receiver, where the non-identical statistics actually do affect the variances of channel estimation. Generally, non-identical channels will result in non-identical channel estimation errors and consequently affect the performance of the current systems, and even the structure of the existing receiver. Therefore, we investigate here the performance of OSTBC over semi-identical channels with channel estimation. The semi-identical channels refer to the case where the channel gains associated with a common receive antenna are identically distributed, but the ones associated with a common transmit antenna are not, as in [12]. This assumption applies to channel distributions in the uplink of a cellular system, or in ground-to-air communications [12].

We show that the conventional symbol-by-symbol (SBS) decoder [13] for OSTBC is no longer optimum in this case. The optimum decoder is obtained and a new SBS decoder is derived by weighting the received signals and estimated channels. To the best of our knowledge, our work here is the first to consider the optimum decoder for OSTBC over non-identical channels with channel estimation. Our analytical and simulation results show that our new SBS decoder provides a much better performance compared to the conventional SBS decoder in this semi-identical case.

The rest of the paper is organized as follows. In Section II, we describe the system model. Section III examines the structure of the optimum receiver. Performance analysis is given in Section IV. Sections V and VI are numerical examples and conclusion, respectively.

II. SYSTEM MODEL

We consider a communication system with $M_T$ transmit and $N_R$ receive antennas. The transmit/receive antennas can be co-located in one communication unit, or distributed in several units. If the antennas are not co-located, we assume the synchronization is perfect. The space-time block code $\mathbf{S}$ is a $P \times M_T$ matrix, where each row of $\mathbf{S}$ is transmitted through $M_T$ transmit antennas at one time, and the transmission covers $P$ symbol periods. It has a linear complex orthogonal design, and can be represented as [14]

$$\mathbf{S} = \sum_{k=1}^{K} (s_k \mathbf{A}_k + s_k^* \mathbf{B}_k).$$

(1)

Here, $\mathbf{A}_k$ and $\mathbf{B}_k$ are $P \times M_T$ matrices with constant complex entries, and $K$ is the number of information symbols transmitted in one block. Therefore, each entry of $\mathbf{S}$ is a linear combination of the symbols $s_k$, $k = 1, \ldots, K$, and their conjugates $s_k^*$, where each $s_k$ is from a certain complex signal constellation. The rate of the OSTBC is defined as $K/P$.

For OSTBC, we have [1]

$$\mathbf{S}^H \mathbf{S} = \text{diag} \left[ \sum_{k=1}^{K} \lambda_{1,k} |s_k|^2, \ldots, \sum_{k=1}^{K} \lambda_{M_T,k} |s_k|^2 \right] = \mathbf{D}$$

(2)
\[
\begin{align*}
\lambda_{i,k}^{M_T} & \text{ are non-negative numbers. For an arbitrary signal constellation, it requires that} \\
A_k^H A_k + B_k^H B_k & = \delta_{k,l} \text{diag}[\lambda_{1,k}, \ldots, \lambda_{M_T,k}], \\
A_k^H B_k + A_l^H B_l & = 0.
\end{align*}
\]

We assume here \(M\)-ary phase-shift keying (MPSK) modulation and a constant transmitted energy per information bit as \(E_b\). Therefore, the total energy assigned to one block is \(E_b K \log_2 M\). From the orthogonality condition \((2)\), it can be seen that the total energy for one block is given by \(\sum_{m}^{M_T} \sum_{k}^{K} \lambda_{m,k} |s_k|^2\). Thus, the transmitted energy per MPSK symbol is given by

\[
E_s = \frac{E_b K \log_2 M}{\sum_{m}^{M_T} \sum_{k}^{K} \lambda_{m,k}}
\]

The received signal at \(t\)-th block is a \(P \times N_R\) matrix, which is given by

\[
R(t) = S(t)H(t) + N(t).
\]

Here, \(N(t)\) is a \(P \times N_R\) noise matrix, whose entries are i.i.d., complex, Gaussian random variables with mean zero and variance \(N_0/2\) per dimension. \(H(t)\) is a \(M_T \times N_R\) channel matrix, where each entry \(h_{mn}\) is the channel gain of the link from \(m\)-th transmit antenna to \(n\)-th receive antenna. We assume \(h_{mn}\) is circularly complex Gaussian random variable with mean zero and variance \(2\sigma^2_{mn}\). It is also assumed that the channels are all block-wise constant, i.e., they remain constant for \(P\) symbols time. The autocorrelation function of each channel is given as \(E[|h_{mn}(t)h_{mn}^*(t')|^2] = 2\sigma^2_{mn} R(t-t')\), where \(R(t-t') = J_0(2\pi f_d T_s |t-t'|)\) for Jakes’ model [15], and it is identical for all channels.

We apply pilot-symbol assisted modulation (PSAM) [16], such that a pilot block is inserted into data stream after every \(L_f\) blocks. The estimation of channel matrix \(\hat{H}(t)\) is based on the information set \(\Lambda(t)\), which contains \(2L_p\) pilot blocks nearest in time to the \(t\)-th block. The estimated channel matrix \(\hat{H}(t)\) is the conditional mean or MMSE estimate of the channel matrix \(H(t)\) given \(\Lambda(t)\). Therefore, given \(\Lambda(t)\), each \(h_{mn}(t)\) is a conditional Gaussian with mean \(\hat{h}_{mn}(t)\) and variance \(2\sigma^2_{mn}\).

For the semi-identical channels case considered in this paper, it is assumed that the channel gains related to a common receive antenna are identically distributed, but the gains associated with different receive antennas are non-identical. Consequently, the variances of channel estimation related to different receive antennas are also different in general. Therefore, the variance of \(h_{mn}(t)\) reduces to \(2\sigma^2_{mn}\), and the variance of estimation error reduces to \(2\sigma^2_{on}(t)\). Here, notice that the variances of channel gains are constant, but the variances of the estimation errors depend on the position of code block.

### III. Optimum Receiver

One important advantage of OSTBC is that the ML decoder can reduce to a SBS decoder, which greatly reduces the decoding complexity. This conventional SBS decoder is optimum when channels are identical with perfect CSI [1] or with imperfect CSI [4]. It is also an optimum receiver for the case of non-identical channels with perfect CSI [11]. However, in our case of semi-identical channels with imperfect CSI, the conventional receiver is no longer optimum. Therefore, we need to investigate the structure of optimum decoder first.

For optimum ML decoding, we compute the likelihood \(p(R(t), \Lambda(t) | S(t))\) for each possible value of the signal block \(S(t)\). Since we have

\[
p(R(t), \Lambda(t) | S(t)) = p(R(t) | S(t), \Lambda(t))p(\Lambda(t) | S(t))
\]

and the information set \(\Lambda(t)\) is independent of \(S(t)\), the ML decoding rule simplifies to

\[
\hat{S}(t) = \arg \max_{S(t)} p(R(t) | S(t), \Lambda(t))
\]

where \(R(t)\) is conditionally Gaussian with mean \(S(t)\hat{H}(t)\), given \(S(t)\) and \(\Lambda(t)\).

The column vectors of \(R(t)\) are independent of one another and each has covariance matrix of

\[
C_n(t) = S(t)V_n(t)S^H(t) + N_0 I_{P \times P}, \quad n = 1, \ldots, N_R.
\]

where

\[
V_n(t) = \text{diag}[2\sigma_{on}^2(t)]_{m=1}^{M_T}, \quad n = 1, \ldots, N_R.
\]

The probability density function of the received signal is now given by

\[
p(R(t) | S(t), \Lambda(t)) = \left( \prod_{n=1}^{N_R} \det(\pi C_n(t)) \right)^{-1} e^{-\frac{1}{2} \sum_{n=1}^{N_R} (r_n(t) - S(t)\hat{h}_n(t))^H C_n^{-1}(t) (r_n(t) - S(t)\hat{h}_n(t))}.
\]

Therefore, the ML block-by-block receiver becomes

\[
\hat{S}(t) = \arg \min_{S(t)} \left( \sum_{n=1}^{N_R} (r_n(t) - S(t)\hat{h}_n(t))^H C_n^{-1}(t) (r_n(t) - S(t)\hat{h}_n(t)) \right).
\]

If the OSTBC employed satisfies

\[
S(t)S^H(t) = \beta I_{P \times P}
\]

where \(\beta\) is a constant, then \(C_n(t)\)'s become constants proportional to an identity matrix. Therefore, the ML receiver simplifies to

\[
\hat{S}(t) = \arg \min_{S(t)} \left\| \hat{R}(t) - S(t)\hat{H}(t) \right\|^2
\]

where

\[
\hat{R}(t) = \left[ \frac{1}{2\sigma_{on}^2(t) + N_0} r_n(t) \right]_{n=1}^{N_R} = R(t) \text{diag} \left[ \frac{1}{2\sigma_{on}^2(t) + N_0} \right]_{n=1}^{N_R}.
\]
Applying equations (3) and (4) to (14), the receiver can be further simplified to a SBS decoder, given by

\[ \hat{s}_k(t) = \arg \max_{k'} \Re[z_{k'}(t)s_k(t)], \quad (17) \]

where \( z_{k'}(t) = \text{Tr} \left[ R^H(t)B_{k'} \hat{H}(t) + \hat{H}^H(t)A_{k'}^H R(t) \right]. \quad (18) \)

Therefore, in the case of semi-identical channels with channel estimation, the ML decoding can also be achieved by a SBS decoder, under the condition that the received signal matrix \( R(t) \) and the estimated channel matrix \( \hat{H}(t) \) are properly weighted column by column, according to the variances of the channel estimation errors.

\[ \text{IV. PERFORMANCE ANALYSIS} \]

In this section, we will examine the performance of the optimum SBS decoder proposed above. For the sake of simplicity, we drop the block index \( t \) hereafter, but note that the results obtained do depend on the positions of blocks.

With PSK modulation, i.e., \( s_k = \sqrt{E_s} e^{j\phi_k} \), the decoding rule (17) is equivalent to

\[ \hat{s}_k = \arg \max_{k'} \Re[z_{k'}e^{-j\phi_k}] \quad (19) \]

\[ z_{k'} = \text{Tr} \left[ R^H B_{k'} \hat{H} + \hat{H}^H A_{k'}^H R \right] = x_{k'} + \mu_{k'}, \quad (20) \]

\[ x_{k'} = \sum_{k=1}^{K} \left[ s_k \text{Tr}[\hat{H}^H A_{k'}^H B_{k'} \tilde{H}] + s_k \text{Tr}[\hat{H}^H A_{k'}^H B_{k'} \tilde{H}] \right], \quad (21) \]

\[ \mu_{k'} = \text{Tr} \left[ \hat{H}^H B_{k'} \tilde{H} + \hat{H}^H A_{k'}^H \tilde{N} \right]. \quad (22) \]

For equally likely symbols, we can assume \( s_{k'} = \sqrt{E_s} \) without loss of generality, thus the bit error probability (BEP) depends on the probability \( P_\alpha(e) = P(|R[z_{k'}e^{-j\phi}]| < 0 | s_{k'} = \sqrt{E_s}) \), where \( \alpha \) is some angle depending on modulation order [17]. For BPSK modulation, the BEP is obviously given by \( P_b = P_{\alpha=0}(e) \). For QPSK modulation with Gray mapping, the BEP is given by \( P_b = P_{\alpha=\pm \pi}(e) \) [17].

Conditioning on \( \Lambda \) and \( s_{k'} \), and substituting (3) and (4) into (21), we can find that \( x_{k'} \) is a Gaussian random variable, which is given by

\[ (x_{k'}|s_{k'}, \Lambda) \sim \mathcal{CN} \left( \frac{N_R}{V_n} \sum_{n=1}^{N_R} \mathcal{H}, \frac{N_T}{V_n} \sum_{n=1}^{N_T} 2\nu_{on}^2 |s_{k'}|^2 \right), \quad (23) \]

where

\[ \mathcal{H} = \sum_{m=1}^{N_T} \lambda_{m,k} |\hat{h}_{mn}|^2, \quad (24) \]

\[ \mathcal{V}_n = \frac{2\nu_{on}^2 + \lambda_{m,k}^2}{V_n} \frac{\nu_{on}^2}{V_n}, \quad (25) \]

and

\[ \xi_{mn} = \sum_{k=1}^{K} \sum_{i=1}^{M_T} \left( (\mathbf{a}_{k,m}^H \mathbf{b}_{k,i} + \mathbf{b}_{k,m}^H \mathbf{b}_{k,i}^H) \hat{h}_{mn} + (\mathbf{a}_{k,i}^H \mathbf{b}_{k,m} + \mathbf{b}_{k,i}^H \mathbf{a}_{k,m}^H) \hat{h}_{mn}^* \right). \quad (26) \]

Here, \( \mathbf{a}_{k,i} \) and \( \mathbf{b}_{k,i} \) are the \( i \)-th column vectors of matrices \( \mathbf{A}_k \) and \( \mathbf{B}_k \), respectively. Similarly, the noise term \( \mu_{k'} \) in (22) is also a conditional Gaussian random variable, which is given by

\[ (\mu_{k'}|s_{k'}, \Lambda) \sim \mathcal{CN} \left( 0, \frac{N_o}{2} \sum_{n=1}^{N_R} \mathcal{H} \right). \quad (27) \]

Therefore, conditioning on the set \( \Lambda \), the probability \( P_\alpha(e) \) is given by

\[ P_\alpha(e|\Lambda) = Q \left( \sqrt{\frac{2E_s}{N_o}} \frac{\cos^2 \alpha}{\mathcal{V}_n} \sum_{n=1}^{N_T} \sum_{m=1}^{N_R} \lambda_{m,k} |\hat{h}_{mn}|^2 \right). \quad (28) \]

If the CSI is perfect, such that \( \nu_{on}^2 = 0 \) for all \( n \), we have \( \hat{h}_{mn} = h_{mn} \). The above conditional BEP can be simplified to

\[ P_\alpha(e|\Lambda) = Q \left( \sqrt{\frac{2E_s}{N_o}} \frac{\cos^2 \alpha}{\mathcal{V}_n} \sum_{n=1}^{N_T} \sum_{m=1}^{N_R} \lambda_{m,k} |h_{mn}|^2 \right). \quad (29) \]

Averaging over the channel gains \( \{h_{mn}\} \), the average probability is given by [11]

\[ P_\alpha(e) = \frac{1}{\pi} \int_0^{\pi} \prod_{m=1}^{N_T} \prod_{n=1}^{N_R} \left( 1 + 2\frac{\nu_{on}^2 \mathbf{E}_s \lambda_{m,k} \cos^2 \alpha}{N_o \sin^2 \theta} \right)^{-1} d\theta. \quad (30) \]

If the channels are estimated, however, the exact average BEP is difficult to obtain. Therefore, performance bounds and approximation will be applied. In the following section, we will use Alamouti’s code [18] as an example to show how to analyze the average BEP. The method used in this paper can similarly be extended to other OSTBC’s.

Using Alamouti’s code [18], the code matrix and \( \mathbf{A}_k \) and \( \mathbf{B}_k \) are given by

\[ \mathbf{S} = \begin{bmatrix} s_1 & s_2^* \\ -s_2 & s_1^* \end{bmatrix}, \quad \mathbf{A}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{B}_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}. \quad (31) \]
respectively. Thus, \( \lambda_{i,k} = 1 \) for all \( i \) and \( k \). Substituting (31) into (26), we have
\[
\begin{bmatrix}
\xi_{11} & \xi_{12} \\
\xi_{21} & \xi_{22}
\end{bmatrix} =
\begin{bmatrix}
\hat{h}_{11} & \hat{h}_{12} \\
-\hat{h}_{21} & -\hat{h}_{22}
\end{bmatrix} +
\begin{bmatrix}
\hat{h}_{11} & \hat{h}_{12} \\
\hat{h}_{21} & \hat{h}_{22}
\end{bmatrix}
\]
\[
= \begin{bmatrix}
2R[\hat{h}_{11}] & 2R[\hat{h}_{12}] \\
-23[\hat{h}_{21}] & -23[\hat{h}_{22}]
\end{bmatrix}.
\]
(32)
Since the channels related to the same transmit antenna are identically distributed, and each \( \hat{h}_{mn} \) is circularly Gaussian, we make the approximation here that
\[
E_s \sum_{m=1}^{M_T} \sum_{n=1}^{N_R} \frac{v_{2n}^2|\hat{h}_{mn}|^2}{\nu_n^2} \approx 2E_s \sum_{m=1}^{M_T} \sum_{n=1}^{N_R} \frac{v_{2n}^2|\hat{h}_{mn}|^2}{\nu_n^2}. \tag{33}
\]
Now equation (28) can be rewritten as
\[
P_a(\epsilon|\Lambda) \leq Q \left( \frac{E_s \sum_{n=1}^{N_R} \sum_{m=1}^{M_T} \frac{|\hat{h}_{mn}|^2}{\nu_n} \cos^2 \alpha}{2E_s \max \left[ \frac{v_{2n}^2}{\nu_n} \right] + \frac{N_t}{2} \max \left[ \frac{1}{\nu_n} \right]} \right).
\]
(34)
where the terms \( I = \sum_{n=1}^{N_R} \sum_{m=1}^{M_T} \frac{|\hat{h}_{mn}|^2}{\nu_n} \) and \( N = \sum_{n=1}^{N_R} \sum_{m=1}^{M_T} \frac{|\hat{h}_{mn}|^2}{\nu_n} \)
are upper bounded by
\[
I \leq \sum_{n=1}^{N_R} \sum_{m=1}^{M_T} \frac{|\hat{h}_{mn}|^2}{\nu_n} \left( \max_{n=1,\ldots,N_T} \left[ \frac{v_{2n}^2}{\nu_n} \right] \right),
\]
\[
N \leq \sum_{n=1}^{N_R} \sum_{m=1}^{M_T} \frac{|\hat{h}_{mn}|^2}{\nu_n} \left( \max_{n=1,\ldots,N_T} \left[ \frac{1}{\nu_n} \right] \right).
\]
(35)
Consequently, the probability (34) can be upper bounded as
\[
P_a(\epsilon|\Lambda) \leq Q \left( \frac{E_s \sum_{n=1}^{N_R} \sum_{m=1}^{M_T} \frac{|\hat{h}_{mn}|^2}{\nu_n} \cos^2 \alpha}{2E_s \max \left[ \frac{v_{2n}^2}{\nu_n} \right] + \frac{N_t}{2} \max \left[ \frac{1}{\nu_n} \right]} \right).
\]
(36)
Averaging over the estimated channel gains \( \{\hat{h}_{mn}\} \), we have
\[
P_a(\epsilon) \leq \frac{1}{\pi} \int_0^{\pi} \prod_{n=1}^{N_R} \left( 1 + \frac{(2\sigma_{m,n}^2 - 2v_{2n}^2)\mu_n}{\nu_n \sin^2 \theta} \right)^{-2} d\theta
\]
(37)
where \( \mu_n = \frac{E_s \cos^2 \alpha}{4E_s \max \left[ \frac{v_{2n}^2}{\nu_n} \right] + N_t \max \left[ \frac{1}{\nu_n} \right]} \). Similarly, the lower bound is given by
\[
P_a(\epsilon) \geq \frac{1}{\pi} \int_0^{\pi} \prod_{n=1}^{N_R} \left( 1 + \frac{(2\sigma_{m,n}^2 - 2v_{2n}^2)\mu_n}{\nu_n \sin^2 \theta} \right)^{-2} d\theta.
\]
(38)
where \( \mu_n = \frac{E_s \cos^2 \alpha}{4E_s \min \left[ \frac{v_{2n}^2}{\nu_n} \right] + N_t \min \left[ \frac{1}{\nu_n} \right]} \).
In order to better approximate the accurate performance, we propose two more approximations, namely geometric approximation and arithmetic approximation. The terms \( I \) and \( N \) can be closely approximated as
\[
\sum_{m=1}^{M_T} \sum_{n=1}^{N_R} \frac{|\hat{h}_{mn}|^2}{\nu_n} \approx I \approx \sum_{m=1}^{M_T} \sum_{n=1}^{N_R} \frac{|\hat{h}_{mn}|^2}{\nu_n} \left[ \frac{v_{2n}^2}{\nu_n} \right]_g
\]
(39)
\[
\text{Fig. 1. BEP results for conventional and optimum receivers, 2Tx and 2Rx Alamouti’s code with QPSK modulation, } f_dT_b=0.1, \text{ channel variances are 0.5 and 5 respectively.}
\]
and
\[
\sum_{n=1}^{N_R} \sum_{m=1}^{M_T} \frac{|\hat{h}_{mn}|^2}{\nu_n} \left[ \frac{1}{\nu_n} \right]_a \approx N \approx \sum_{n=1}^{N_R} \sum_{m=1}^{M_T} \frac{|\hat{h}_{mn}|^2}{\nu_n} \left[ \frac{1}{\nu_n} \right]_g
\]
(40)
respectively. Here, \( \left[ \frac{v_{2n}^2}{\nu_n} \right]_g \) and \( \left[ \frac{1}{\nu_n} \right]_g \) \( \left[ \frac{v_{2n}^2}{\nu_n} \right]_a \) and \( \left[ \frac{1}{\nu_n} \right]_a \) are the geometric and arithmetic means of all \( \frac{v_{2n}^2}{\nu_n} \) and \( \frac{1}{\nu_n} \). Following the same steps as above, the approximations of the probability \( P_a(\epsilon) \) are given by
\[
P_a(\epsilon) \approx \frac{1}{\pi} \int_0^{\pi} \prod_{n=1}^{N_R} \left( 1 + \frac{(2\sigma_{m,n}^2 - 2v_{2n}^2)\mu_n}{\nu_n \sin^2 \theta} \right)^{-2} d\theta,
\]
(41)
\[
P_a(\epsilon) \approx \frac{1}{\pi} \int_0^{\pi} \prod_{n=1}^{N_R} \left( 1 + \frac{(2\sigma_{m,n}^2 - 2v_{2n}^2)\mu_n}{\nu_n \sin^2 \theta} \right)^{-2} d\theta
\]
(42)
where
\[
\mu_g = \frac{E_s \cos^2 \alpha}{4E_s \max \left[ \frac{v_{2n}^2}{\nu_n} \right] + N_t \max \left[ \frac{1}{\nu_n} \right]} \]
\[
\mu_a = \frac{E_s \cos^2 \alpha}{4E_s \min \left[ \frac{v_{2n}^2}{\nu_n} \right] + N_t \min \left[ \frac{1}{\nu_n} \right]}
\]
As we omitted the block index \( t \) here, the BEP results obtained above are based on the \( t \)-th block. The average BEP for all the blocks can be calculated by averaging over the \( L_f \) blocks within two adjacent pilot blocks.

V. NUMERICAL RESULTS

In the numerical examples, we consider a MIMO system with 2 transmit and 2 receive antennas. The Alamouti’s code is applied with QPSK modulation. As we mentioned in Section II, since the channels are block-wise constant, we use the block fade rate \( f_dT_b \) for the BEP computation and simulation. Pilot blocks are inserted after every 9 data blocks, and the 4 nearest pilot blocks are used to estimate the channel using PSAM.
In Figure 1, the variances of the channel gains related to receive antenna 1 and 2 are 0.5 and 5, respectively. The block fade rate is set to 0.1. The simulation results show that our optimum receiver provides a large performance gain compared to the conventional receiver. The irreducible error floor caused by the channel fading is also greatly reduced by the optimum receiver.

The analytical lower (38) and upper (37) bounds in Fig 1 show the same trend as the exact BEP curve, such that they decrease in parallel with the increase of SNR. The three curves converge in the high SNR region. Furthermore, both the geometric (41) and arithmetic (42) approximations can closely approximate the exact BEP performance in all SNR regions, with the latter being a closer approximation, the difference being no larger than 0.5 dB.

In Figures 2 and 3, we change the channel variances and the block fade rate, and similar observations can be made. Notice that the performance gain enjoyed by the optimum receiver comes with little overhead, as it only requires linear processing of the received signal and the estimated channel matrices.

VI. CONCLUSION

This paper analyzes OSTBC over semi-identical channels with channel estimation. It is shown that the conventional SBS decoder is not optimum in this case, and the optimum decoder is derived. It is also shown that this optimum decoder can be simplified to a SBS decoder, under certain conditions. The performance of the optimum decoder is also examined. The upper/lower bounds and two close approximations of the BEP performance are obtained, and both the analytical and simulation results show that our optimum decoder substantially outperforms the conventional SBS decoder.

REFERENCES


Fig. 2. BEP results for conventional and optimum receivers, 2Tx and 2Rx Alamouti’s code with QPSK modulation, $f_dT_b=0.1$, channel variances are 0.9 and 9 respectively.

Fig. 3. BEP results for conventional and optimum receivers, 2Tx and 2Rx Alamouti’s code with QPSK modulation, $f_dT_b=0.06$, channel variances are 0.5 and 5 respectively.