Minimizing Interferences in Wireless Ad Hoc Networks through Topology Control

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Abstract—This paper investigates minimizing mutual interferences in wireless ad hoc networks by means of topology control. Prior work defines interference as a relationship between link and node. This paper attempts to capture the physical situation more realistically by defining interference as a relationship between link and link. We formulate the pair-wise interference condition between two links, and show that the interference conditions for the minimum-transmit-power strategy and the equal-transmit-power strategy are equivalent. Based on the pair-wise definition, we further investigate the “typical” interference relationship between a link and all other links in its surrounding. To characterize the extent of the interference between a link and its surrounding links, we define a new metric called the interference coefficient. We investigate the property of interference coefficient in detail by means of analysis and simulation. Based on the insight obtained, we propose a topology control algorithm - minimum interference algorithm (MIA) – to minimize the overall network interference. Simulation results indicate that the network topologies produced by MIA show good performance in terms of network interference and spanner property compared with known algorithms such as LIFE, Gabriel Graph and k-NEIGH.

I. INTRODUCTION

In recent years, much attention has been given to wireless ad hoc networks, thanks to their flexibility and many potential applications. Energy is a precious resource in wireless ad hoc networks when the battery life of nodes is limited. To conserve energy, topology control (TC) has been proposed to reduce transmit powers while keeping the network connected.

The prior studies of TC [1-8] have mostly focused on the conservation of transmit powers without regard to minimizing mutual interferences among links. The implicit assumption is that “sparse” networks with few links, a usual consequence to minimizing transmission power, results in low mutual interferences. This assumption, however, is not always true [9]. There has also been selected work that deals with mutual interferences directly [9-13]. In [9], it considers the interference from the viewpoint of links, for which the term “edge coverage” of a link is defined to be the number of nearby nodes that may be affected by the transmission on that link. However, the definition of edge coverage therein (also adopted in [10-12]) turns out to omit many nodes that can potentially be affected by the link. Our paper adopts a definition which captures the fact that mutual interference is a phenomenon between links rather than that between a link and a node, thus more accurately reflecting the physical situation. Ref. [13] defines interference from the viewpoint of nodes, which again does not reflect the physical situation accurately.

In this paper, we consider networks with symmetric links: if nodes a and b form a link in which node a can transmit to node b, then there is a corresponding link in the reverse direction from node b to node a. A pair of directional links, a→b and b→a, is modeled as an edge (a, b) in our graph. For simplicity, we will use the term “link” and “edge” interchangeably in this paper. If we mean a link to be directional, we will state so explicitly.

Based on this, we define the interference between two edges as follows. There is no interference between two edges, (a, b) and (c, d), if and only if each directional link of edge (a, b) (i.e., a→b or b→a) does not interfere with any directional link of edge (c, d) (i.e., c→d and d→c), and vice versa. Hereinafter, two edges are said to “have an interference relationship” if there is an interference relationship as defined above.

With this definition, we find an interesting equivalence relationship between two power-control strategies as explained below. In the minimum-power strategy, nodes a and b use the minimum power required for successful transmission between them; likewise for nodes c and d [14-16]. In the equal-power strategy, all nodes use the same transmission power [17-18]. We prove in this paper that according to our interference condition above, there is no interference between (a, b) and (c, d) under the minimum-power strategy if and only if there is no interference between them under the equal-power strategy.

We then define the mean partial interference coefficient K to be the average interference coefficient of nodes located in a particular area, which will be described in the following paper. Using K, the overall edge interference I(a,b) can be defined. The “global” measure for network interference of G=(V,E) can then be defined as $I(G) = \max_{(a,b) \in E} I(a,b)$ [9], which is then used as the “utility function” in our TC algorithm.

We present a particular TC algorithm, the minimum interference algorithm (MIA), which aims to minimize the network interference while maintaining good spanner property. We prove the network-interference minimization property of MIA theoretically. Through simulation, it is verified that MIA not only can minimize the network interference, but can also maintain good spanner property with respect to other algorithms: specifically, LIFE [9], Gabriel Graph [2] and k-NEIGH [10].

The rest of this paper is organized as follows. Section II defines our network graph model and the notation used. Section
III presents our interference model and establishes the equivalence of the minimum-power and equal-power strategies. Section IV establishes the $K \approx 0.5$ result. Section V presents the MIA algorithm, and Section VI investigates its performance. Finally, Section VII concludes the paper.

II. NETWORK GRAPH MODEL AND NOTATION

We consider $n$ nodes randomly and uniformly distributed in a fixed square area. An ad hoc network is modeled as a Euclidian graph $G=(V, E)$, in which the vertices in $V$ represent the nodes, and the edges in $E$ represent the links. And $|V|=n, |E|=m$.

In the graph $G$, all nodes have the same maximum transmission power $p_{\text{max}}$, the same maximum transmission range $r_{\text{max}}$ and the same received power threshold $p_{\text{th}}$ for proper signal detection. Nodes can adjust their transmit power continuously from zero to $p_{\text{max}}$.

For nodes $a$ and $b$ in the graph $G$, $a \rightarrow b$ denotes the directional link from node $a$ to node $b$, and $d_{ab}$ is the distance between them. A common path loss model is adopted, that for $a \rightarrow b$ the received power is $p_{a,b} = f \cdot p_a / d_{ab}^\alpha$, where $p_a$ is the transmission power used by node $a$, $f$ is a constant, and $\alpha \geq 2$ is the path-loss exponent. In addition, $e_i = (u_i, v_i)$, $i = 1, 2, \ldots, m$, denotes the $i$th edge in the graph, and correspondingly $d_i = |u_i - v_i|$ represents its distance.

III. INTERFERENCE CONDITION AND DEFINITION

A. Interference Condition

Consider simultaneous transmissions on two directional links, $a \rightarrow b$ and $c \rightarrow d$, and ignore the effect of noise. We assume that no collision at receiver $b$ will happen if and only if $p_{a,b} / p_{c,d} \geq \beta$ holds, where $\beta$ is the minimum signal-to-interference ratio necessary for successful receptions [19].

Interference between two symmetric edges, $e_i = (u_i, v_i)$ and $e_j = (u_j, v_j)$, is defined as follows. No mutual interference exists between them if all the following inequalities (1)-(8) hold [16]:

\[
\begin{align*}
&\frac{p_{u_i,v_i}}{p_{u_j,v_j}} \geq \beta \quad (1); \quad \frac{p_{u_i,v_i}}{p_{u_j,v_j}} \geq \beta \quad (2); \quad \frac{p_{u_i,v_i}}{p_{u_j,v_j}} \geq \beta \quad (3); \quad \frac{p_{u_i,v_i}}{p_{u_j,v_j}} \geq \beta \quad (4); \\
&\frac{p_{v_i,u_i}}{p_{v_j,u_j}} \geq \beta \quad (5); \quad \frac{p_{v_i,u_i}}{p_{v_j,u_j}} \geq \beta \quad (6); \quad \frac{p_{v_i,u_i}}{p_{v_j,u_j}} \geq \beta \quad (7); \quad \frac{p_{v_i,u_i}}{p_{v_j,u_j}} \geq \beta \quad (8).
\end{align*}
\]

Under the minimum-power strategy, for $a \rightarrow b$, $p_a = p_{\text{max}} \cdot (a-b)^\alpha / f$ and $p_{a,b} = p_a$. Thus, inequalities (1)-(8) can be rewritten as:

\[
\begin{align*}
&\frac{|u_i - u_j|}{d_{ij}} \geq \beta \quad (1); \quad \frac{|v_i - v_j|}{d_{ij}} \geq \beta \quad (2); \quad \frac{|v_i - v_j|}{d_{ij}} \geq \beta \quad (3); \quad \frac{|u_i - u_j|}{d_{ij}} \geq \beta \quad (4); \\
&\frac{|v_i - v_j|}{d_{ij}} \geq \beta \quad (5); \quad \frac{|v_i - v_j|}{d_{ij}} \geq \beta \quad (6); \quad \frac{|v_i - v_j|}{d_{ij}} \geq \beta \quad (7); \quad \frac{|u_i - u_j|}{d_{ij}} \geq \beta \quad (8).
\end{align*}
\]

Under the equal-power strategy, inequalities (1)-(8) are equal to:

\[
\begin{align*}
&\frac{|u_i - u_j|}{d_{ij}} \geq \beta \quad (1); \quad \frac{|v_i - v_j|}{d_{ij}} \geq \beta \quad (2); \quad \frac{|v_i - v_j|}{d_{ij}} \geq \beta \quad (3); \quad \frac{|u_i - u_j|}{d_{ij}} \geq \beta \quad (4); \\
&\frac{|v_i - v_j|}{d_{ij}} \geq \beta \quad (5); \quad \frac{|v_i - v_j|}{d_{ij}} \geq \beta \quad (6); \quad \frac{|v_i - v_j|}{d_{ij}} \geq \beta \quad (7); \quad \frac{|u_i - u_j|}{d_{ij}} \geq \beta \quad (8).
\end{align*}
\]

THEOREM 1: The interference conditions in the minimum-power strategy and the equal-power strategy are equivalent.

PROOF: From above, (1)’ is the same as (8)*, (2)’ is the same as (7)*…, and so on. So, the interference occurs under the minimum-power strategy if and only if the interference occurs under the equal-power strategy. □

DEFINITION 1: The distance $d_{uv}$ between two edges, $e_i = (u_i, v_i)$ and $e_j = (u_j, v_j)$, is defined as follows:

\[
d_{ij} := \min(d_{u_i,v_i}, d_{u_j,v_j}, d_{v_i,u_i}, d_{v_j,u_j})
\]

Assuming $\sqrt{\beta} = \max(1 + \Delta, 1)$, under two power-control strategies discussed above, the interference condition between $e_i$ and $e_j$ is:

\[
d_{ij} < (1 + \Delta) \max(d_i, d_j)
\]

B. Interference Definition

Consider $e_i = (u_i, v_i)$. The area around $e_i$ is divided into three regions, $A$, $B$ and $C$:

\[
A = \{ c \in \mathbb{R} \mid |c - u_i| < (1 + \Delta)d_i, \text{ or } |c - v_i| < (1 + \Delta)d_i \}
\]

\[
B = \{ c \in \mathbb{R} \mid |c - u_i| < (1 + \Delta)p_{\text{max}} , \text{ or } |c - v_i| < (1 + \Delta)p_{\text{max}} \text{ and } c \notin A \}
\]

\[
C = \{ c \in \mathbb{R} \mid c \notin A \text{ and } c \notin B \}
\]

$N_A(e_i)$ and $N_B(e_i)$ denote the sets of nodes in $A$ and $B$ respectively. $S_A$ and $S_B$ are their areas respectively.

The interference coefficient of a node around $e_i$ is defined as follows:

DEFINITION 2: The interference coefficient of node $h$ with respect to $e_i$ is defined to be $I_h(e_i) = q_h / p_h$, where $p_h$ is the number of edges incident to node $h$, and $q_h$ is the number of edges having an interference relationship with $e_i$ among the $p_h$ edges. We assume that when $p_h = 0, I_h(e_i) = 0$.

The interference coefficients of nodes in $A$, $B$ and $C$ are 1, within the interval [0, 1], and 0, respectively. Correspondingly, $A$, $B$ and $C$ are respectively called the whole interference area, the partial interference area and the no-interference area.

DEFINITION 3: The mean partial interference coefficient $K(e_i)$ of $e_i$ is defined as follows:

\[
K(e_i) := \frac{1}{|N_A(e_i)|} \sum_{h \in N_A(e_i)} I_h(e_i)
\]

Correspondingly, the interference of $e_i$ is given by the following definition.

DEFINITION 4: The interference of $e_i$ is defined as:

\[
I(e_i) := \sum_{h \in N_A(e_i)} I_h(e_i) + \sum_{h \in N_B(e_i)} I_h(e_i) = |N_A(e_i)| + K(e_i) \times |N_B(e_i)|
\]

The edge level interference defined as such can then be extended to the global network interference:

DEFINITION 5: The network interference of $G=(V, E)$ is defined as follows:

\[
I(G) := \max_{e_i \in E} (I(e_i))
\]
IV. THE VALUE OF $K$  

In the section, we estimate the value of $K(e)$ both theoretically and through simulation.  

A. Theoretical Analysis  

We will analyze the expected value of $K(e)$ here. The expected $K(e)$ is the expectation of the interference coefficient of a node $u_i$ randomly placed in the partial interference area of $e_i=(u_i, v_i)$. That is:

$$\hat{K}(e_i) = E(K(e_i)) = E(I_{u_i}(e_i))$$  

(14)

where the expectation is taken all possible positions of node $u_i$ within $B$.

Assume node $v_j$ to be an arbitrary neighbor of node $u_i$, node $v_j$ is equally likely to be located within a circle of radius $r_{\text{max}}$ around node $u_i$. The interference area (IA) of node $u_i$ is defined as a part of its neighborhood, within which node $v_j$ can cause $e_i=(u_i, v_i)$ and $e_i$ to have an interference relationship. Denote the area of IA by $S_{\text{IA}}(u_i)$. We have:

$$I_{u_i}(e_i) = \frac{S_{\text{IA}}(u_i)}{\pi r_{\text{max}}^2}$$  

(15)

Note that the expected value of $K(e_i)$ has no relationship with network density.

For illustration, in the following we consider two cases of $S_{\text{IA}}(u_i)$. And $S_I$ to $S_{\text{IV}}$ are used to denote the areas of I, II, III and IV respectively in the analysis.

1) When $d_i \rightarrow 0$

Under this situation, $\max(d_i, d_j) = d_j$, (10) is reduced to:

$$d_{\text{max}} < (1+\Delta)d_j \quad \text{or} \quad d_{\text{min}} < (1+\Delta)d_j$$  

(16)

For convenience, we assume $(-c, 0)$ and $(c, 0)$ being the respective positions of $u_i$ and $u_j$, where $c = d_{\text{min}} / 2$. Denote the position of $v_j$ by $(x, y)$. Then we get Fig. 1, where $r_1 = 2c^2 / (1+\Delta)^2$, $d = (1+\Delta)^2 + 1 - c^2$, $r_2 = d^2 - c^2$ and $S_{\text{IA}}(u_i) = S_I + S_{\text{II}}$.

And $d_{\text{max}}$ obeys the following probability density function:

$$f_{d_{\text{max}}}(l) = \frac{2l}{(1+\Delta)r_{\text{max}}} = 0.63l, \quad 0 < l < (1+\Delta)r_{\text{max}}$$  

(17)

Let $\Delta = 0.78$. Then we get:

$$\hat{K}(e_i) = E(I_{u_i}(e_i)) = 0.59$$  

(18)

2) When $d_i \rightarrow r_{\text{max}}$

Under this situation, $\max(d_i, d_j) = d_j \rightarrow r_{\text{max}}$. The partial interference region $B$ where $u_j$ can be located actually disappears. However, we are interested in the asymptotic limit of $K(e_i)$ as $d_i \rightarrow r_{\text{max}}$. In this limit, region $B$ lies on the line of symmetry between the two halves of region $A$. Let us denote the right half of region $B$ by $\overline{MN}$. We are interested in the positions of $v_j$ that will induce an interference relationship between $e_i$ and $e_j$ as $u_j$ varies along $\overline{MN}$.

In Fig. 2, $C_1$ and $C_2$ are the points on $\overline{MN}$ such that $|C_1M| = r_{\text{max}}$ and $|C_2N| = r_{\text{max}}$.

By geometry, we get $S_{\text{III}} < S_{\text{IV}} < 2S_{\text{III}}$. So we can get $S_{\text{III}} < E(S_{\text{IA}}(u_j)) < (1+\Delta)r_{\text{max}}$. For $\Delta=0.78$, we get:

$$0.44 < \hat{K}(e_i) = E(S_{\text{IA}}(u_j)) / (\pi r_{\text{max}}^2) < 0.58$$  

(19)

Considering the asymptotic $\hat{K}(e_i)$ of cases 1) and 2) above, $\hat{K}(e_i)$ does not vary much as $d_i$ varies, and it appears that approximating it with a value of 0.5 is reasonable.

B. Simulation-Based Evaluation  

In our simulation, we distribute $n$ nodes randomly and uniformly in a fixed square area $[0,5]^2$. To study different node densities, the following settings for $n$ are considered: 75, 100, 125, 150, 175, 200, 225, 250, 275, 300, 325, 350 and 375. For each $n$, we generate 1,000 sets of random node placements.

First, we examine the expected value of $K(e_i)$. The distance of $e_i$, $d_i$, is varied from 0.1 to 0.9 by step of 0.1. We find that the
value of $n$ does not affect $E(K(e))$. In addition, as shown in Table 1, $E(K(e))$ depends on $d_i$ only weakly.

Table 1 The expected value of $K(e_i)$ under different distances for all densities

<table>
<thead>
<tr>
<th>$d_i$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>.58</td>
<td>.57</td>
<td>.57</td>
<td>.56</td>
<td>.55</td>
<td>.55</td>
<td>.54</td>
<td>.52</td>
</tr>
</tbody>
</table>

We next investigate the distribution of the value of $K(e_i)$ in the unit disk graph (UDG) under different $n$. Here, we present a typical result in Table 2. In the table, we see that the mean of $K(e_i)$ is approximately 0.5 for all densities.

Table 2 The mean $\mu$ and the variance $\sigma$ of $K(e_i)$

<table>
<thead>
<tr>
<th>$n$</th>
<th>100</th>
<th>125</th>
<th>150</th>
<th>175</th>
<th>200</th>
<th>300</th>
<th>325</th>
<th>350</th>
<th>375</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>.47</td>
<td>.49</td>
<td>.51</td>
<td>.52</td>
<td>.51</td>
<td>.53</td>
<td>.53</td>
<td>.53</td>
<td>.53</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>.15</td>
<td>.13</td>
<td>.12</td>
<td>.11</td>
<td>.09</td>
<td>.06</td>
<td>.03</td>
<td>.03</td>
<td>.03</td>
</tr>
</tbody>
</table>

According to the above analysis, we could approximate $K(e_i)$ as a constant $K=0.5$. Thus, the definitions (12) and (13) can be simplified as:

$$I(e_i) := |N_s(e_i)| + K \times |N_r(e_i)|$$

$$I(G) := \max_{e_i \in E} |N_s(e_i)| + K \times |N_r(e_i)|$$

V. THE MIA ALGORITHM

We now present a topology control algorithm, called the minimum interference algorithm (MIA), which attempts to minimize the network interference as defined in (21). Fig. 3 gives the details of the MIA. We call this sub-graph a “basis”. Another algorithm (see Fig. 4) is then used to prune edges from the basis to reduce the energy cost while maintaining network connectivity.

1. Obtain UDG;
2. Compute the interferences of all edges in UDG by (21);
3. Sort edges in the ascending order of their interferences;
4. Divide edges into groups based on their interferences;
5. Let $S[i]$ denote the group of edges with interference $i$; $I$ denote the total number of groups; $G(i)$ denote the sub-graph including all groups $S[j]$ with $i=j$. Select necessary groups from UDG using binary search:
   1) Set $U=I, L=1$ originally;
   2) If $L=U$, end. The result is $G_T = G(L)$;
   3) If $L<U$, set $L=L/2$;
   If $G(i)$ is connected, then $U=U/L$ else $L=2L$. Go to 2).
6. (PRUNING STAGE) Delete edges from $G_T$ using algorithm in Fig. 4.

Fig. 3. The MIA Algorithm

The pruning stage in Fig. 4 deletes “redundant edges” from the “basis” using Gabriel Graph [2]. It aims at reducing energy cost while maintaining network connectivity.

VI. EVALUATION OF MIA PERFORMANCE

This section presents performance evaluation of MIA. Our results show that MIA not only can achieve minimum interference, but can also reduce energy cost and maintain good spanner property.

A. Simulation Setup

The performance metrics we consider are interference and spanner property. In our simulations, random graphs are generated by randomly and uniformly placing $n$ nodes in a two dimensional square $[0,5]^2$. We assume the path-loss exponent is $\alpha=4$. We consider values of $n$ from 75 to 375 in step of 25.

Three algorithms other than MIA are studied for comparison purposes:

- MIA: MIA without and with the pruning stage (referred to as MIAwoP and MIAwP respectively) are considered.
- LIFE: The centralized TC algorithm in [9].
- $k$-NEIGH: $k$-NEIGH without the pruning stage in [10], with $k=9$.

B. Interference Performance

Fig. 5. Network interference of the topologies generated by MIAwoP, MIAwP, LIFE, GG, UDG and $k$-NEIGH.

The network interferences of the topologies generated by MIAwoP, MIAwP, UDG, LIFE, GG and $k$-NEIGH are shown in Fig. 5. It can be seen that MIAwoP and MIAwP has the best network-interference performance. GG and $k$-NEIGH have relatively bad network-interference performance, which is not surprising considering that they do not aim to minimize interference directly.

C. Spanner Property

To study the spanner property, we calculate the stretch factor of a pair of nodes $a, b$, defined as

$$s(a,b) = \frac{p_G(a,b)}{p(a,b)}$$

where $p_G(a,b)$ ($p(a,b)$) is the sum of the costs of edges along the shortest path in the generated topology (UDG) [6]. The cost of an edge is defined to be either its Euclidean distance or energy cost under $\alpha=4$ (i.e., either $d_i$ or $d_i^4$ for a link of length $d_i$). Correspondingly we call the spanner feature as that “on distance” or “on energy”.

Fig. 6 shows the stretch factor (a) on distance, and (b) on energy. In conclusion, MIA shows good average performance...
on spanner property in terms of both Euclidean distance and energy cost (especially energy cost).

![Graphs showing stretch factor on distance and energy cost for different algorithms.]

Fig. 6. The stretch factor (a) on distance and (b) on energy of the topologies generated by MIAwoP, MIAwP, LIFE, GG and k-NEIGH

VII. CONCLUSION

In this paper, we have tackled the topology control problem with the goal of minimizing network interference. We first establish the “pair-wise” interference condition between two links, $e_i$ and $e_i'$, under two power-control strategies: 1) the minimum-power strategy in which transmitters use the minimum transmit power to transmit to their respective receivers, and 2) the equal-power strategy in which all transmitters use the same common transmit power. An interesting result is that the interference condition is identical for the two power control strategies.

Building on the pair-wise link interference condition, we then consider the interference of a link $e_i$ with respect to all its surrounding links. We divide the surrounding area of a link $e_i$ into three regions: another node 1) in the first region will for sure have an interference relationship with $e_i$; 2) in the second region will have certain probability of having an interference relationship with $e_i$; and 3) in the third region will have no mutual interference relationship with $e_i$. In the second region, the mean partial interference coefficient, $K(e_i)$, has been defined to describe the mean probability of a node in the region having an interference relationship with $e_i$. Through theoretical analysis and numerical simulation, we show that the value of $K(e_i)$ has little relationship with the length of link $e_i$, and it can be approximated as a constant, 0.5 - a convenience that can be used to construct simple topology control algorithms to minimize overall network interference.

We have considered one such particular algorithm, called MIA, to minimize network interference while conserving energy and maintaining good spanner property. Since MIA minimizes network interference, it is optimal and has better performance than other algorithms in that respect. At the same time, compared with Gabriel Graph and $k$-NEIGH algorithms, MIA also has good spanner property.

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