2 dB better than CP-OFDM with OFDM/OQAM for preamble-based channel estimation

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Abstract—OFDM/OQAM is a special type of multi-carrier modulation that can be considered as an alternative to conventional OFDM with cyclic prefix (CP) for transmission over multi-path fading channels. Indeed, as it requires no CP, it has the advantage of a theoretically higher spectral efficiency. Furthermore, efficient pulse shaping can also be easily implemented with OFDM/OQAM. However, the classical channel estimation methods used for OFDM cannot be directly applied to OFDM/OQAM. In this paper we present an analysis of this problem and we introduce a new preamble-based channel estimation method. The performance results are obtained either by considering an IEEE802.22 channel model or regarding to the channel delay spread variation of a two-tap channel. The proposed OFDM/OQAM channel estimation method is evaluated, in both scenarios, using different pulse shaping and taking conventional CP-OFDM as reference.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is an efficient Multi Carrier Modulation (MCM) to fight against multi-path fading channels. Its robustness to multi-path propagation effects comes from the insertion of a CP and is therefore obtained at the price of a reduced spectral efficiency. As furthermore the rectangular shape of OFDM symbols lead to a $\sin(x)/x$ frequency spectrum, several alternatives have been researched to find better MCM schemes w.r.t. the frequency and/or time-frequency localization criteria.

As suggested in [1]–[3], OFDM/OQAM is a MCM scheme which may be the appropriate alternative. In OFDM/OQAM each subcarrier is modulated with an Offset Quadrature Amplitude Modulation (OQAM). This principle has been introduced at first in [4], [5]. Compared to OFDM that transmits complex-valued symbols at a given symbol rate, OFDM/OQAM transmits real-valued symbols at twice this symbol rate. Therefore a similar spectral efficiency is achieved by both systems. In practice, OFDM/OQAM may provide a higher useful bit rate, since it operates without CP. Furthermore, with a pulse shaping that can be optimized according to given channel characteristics, its performance can be improved. However all the nice features of OFDM/OQAM come at the price of a relaxation of the orthogonality conditions that only hold in the real field. Consequently, existing OFDM channel estimation (CE) methods cannot be directly applied to the case of OFDM/OQAM signals. Indeed, a specific problem of intrinsic interference has to be solved either for perfect, preamble-based and scattered-based CE [6], [7], [9]. Here, we focus on one of the preamble-based CE method in [9]. We introduce an improved variant of the proposed Interference Approximation Method (IAM). We show that this variant may provide higher gains when compared to CP-OFDM.

In section II, we give a short description of the continuous-time OFDM/OQAM modulation that, for concision, is named OQAM in the rest of the paper. Then, in section III, we provide an overview of the specific problem related to CE for the OQAM modulation. Section IV is devoted to the description of the IAM. Finally, in section V, we compare our new preamble structure performance with those given in [9] and with CP-OFDM. We use two channel models, the first one is issued from IEEE802.22 studies, the second one is a simple 2-tap delay channel. Conclusion and perspectives are in section VI.

II. THE OFDM/OQAM MODULATION

We can write the baseband equivalent of a continuous-time OFDM/OQAM signal as follows [1]:

$$s(t) = \sum_{m=0}^{M-1} \sum_{n \in \mathbb{Z}} a_{m,n} g(t - n\tau) e^{j2\pi mF_0 t} e^{j\phi_{m,n}}.$$  

(1)

with $M$ an even number of sub-carriers, $F_0 = 1/T_0 = 1/2\tau_0$ the subcarrier spacing, $g$ the pulse shape and $\phi_{m,n}$ an additional phase term. The transmitted symbols $a_{m,n}$ are real-valued. They are obtained from a $2^2K$-QAM constellation, taking the real and imaginary parts of these complex-valued symbols of duration $T_0 = 2\tau_0$, where $\tau_0$ denotes the time offset between the two parts [1]–[3], [8]. The rule to take, for a given subcarrier $m$ and symbol time $n$, the real and imaginary parts are driven by the phase term $\phi_{m,n}$ given by

$$\phi_{m,n} = \phi_0 + \frac{\pi}{2} (n + m) \quad (\text{mod} \quad \pi)$$  

(2)

where $\phi_0$ can be arbitrarily chosen.

Assuming a distortion-free channel, perfect reconstruction of real symbols is obtained owing to the following real orthogonality condition:

$$\Re \{ \langle g_{m,n} | g_{p,q} \rangle \} = \Re \left\{ \int g_{m,n}(t) \overline{g_{p,q}(t)} dt \right\} = \delta_{m,p} \delta_{n,q},$$

where, $\delta_{m,p} = 1$ if $m = p$ and $\delta_{m,p} = 0$ if $m \neq p$. For concision purpose we set $\langle g \rangle_{m,n}^p = -j \langle g_{m,n} | g_{p,q} \rangle$, with $\langle g_{m,n} | g_{p,q} \rangle$ a pure imaginary term for $(m, n) \neq (p, q)$. 

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However, in practice, for transmission over a realistic channel, the orthogonality property is lost, leading to inter symbol and inter carrier interferences.

In previous studies [6], [7], [9], it has been shown for ideal, preamble and scattered based CE, that if the prototype filter $g$ has good time and frequency localization properties, a simple one tap equalization process could be sufficient to restore the real orthogonality. However, this equalization requires channel estimates that are complex-valued. As the orthogonality is limited to the real field, for all these CE techniques, a specific estimation procedure has to be carried out.

III. PROBLEM STATEMENT

Generally the CP-OFDM system is dimensioned in order to get a flat channel on each sub-carrier leading to the possibility of a simple zero forcing (ZF) equalization. Here we place ourselves in a similar context for OQAM.

A. The channel model

Assuming a flat fading on each sub-carrier, the received OQAM signal can be written as [9]:

$$y(t) = \sum_{n}^{M-1} \sum_{m=0}^{n} a_{m,n} H^{(c)}_{m,n} g_{m,n}(t) + \eta(t),$$

(3)

where $\eta(t)$ is an additive noise component and $H^{(c)}_{m,n}$ a complex-valued number that represents the channel for sub-carrier $m$ at symbol time $n$.

B. Demodulation and ZF equalization

Using the real orthogonality property of OQAM, it can be shown [9] that, noise taken apart, for any demodulated signal of index $(m_0, n_0)$ in the time-frequency plane, the estimated symbol is given by

$$\hat{a}_{m_0,n_0} = a_{m_0,n_0} + \Re\{I_{m_0,n_0}\},$$

where

$$I_{m_0,n_0} = j \sum_{(p,q)\neq(0,0)} a_{m_0+p,n_0+q} \left| \frac{H^{(c)}_{m_0+p,n_0+q}}{H^{(c)}_{m_0,n_0}} \right| \Re\{g^{m_0,n_0}_{m_0+p,n_0+q}\}.$$  \quad (4)

is a complex-valued number. $\Re\{I_{m_0,n_0}\}$ can be considered as the residual, or intrinsic [6], interference due to the channel spreading. [9] shows that, if the prototype filter is well localized in time and frequency, we can get: $\Re\{I_{m_0,n_0}\} \approx 0$, leading to reliable estimation of $a_{m_0,n_0}$. Therefore we have an accurate detection of $a_{m_0,n_0}$ when knowing the channel coefficient $H^{(c)}_{m_0,n_0}$ at the receiver side.

IV. THE INTERFERENCE APPROXIMATION METHOD (IAM)

A. Basic principle of the IAM

Assuming a locally time and frequency invariant channel and a prototype function well localized in time and frequency, the received signal can be accurately approximated by [9]:

$$y^{(c)}_{m,n} \approx H^{(c)}_{m,n} (a_{m,n} + ja^{(i)}_{m,n}).$$  \quad (5)

with $a^{(i)}_{m,n}$ the residual interference that occurs in the vicinity of any symbol of index $n$, and for any subcarrier of index $m$. The IAM proposed in [9] is a preamble-based CE where the receiver uses an approximation of $a^{(i)}_{m,n}$. In this case, considering a neighborhood of size $\Delta m \times \Delta n$, denoted $\Omega_{\Delta m,\Delta n}$, around a given time-frequency position $(m_0, n_0)$, but not containing it, the imaginary component in (5) can be approximated by

$$a^{(i)}_{m_0,n_0} \approx \sum_{(p,q)\in \Omega_{\Delta m,\Delta n}} a_{m_0+p,n_0+q} \left| \frac{g^{m_0,n_0}_{m_0+p,n_0+q}}{H^{(c)}_{m_0,n_0}} \right|^2.$$  \quad (6)

In the presence of a noise $\eta$, the estimate of $\hat{H}$ becomes:

$$\hat{H}^{(c)}_{m_0,n_0} = H^{(c)}_{m_0,n_0} + \frac{\eta_{m_0,n_0}}{(a_{m_0,n_0} + ja^{(i)}_{m_0,n_0})}.$$  \quad (7)

The goal here is to find a preamble structure which makes the power of $(a_{m_0,n_0} + ja^{(i)}_{m_0,n_0})$ as high as possible without increasing the overall power of the preamble at the transmitter side, i.e. to introduce a virtual boosting. Indeed, from (7) the higher the power of $(a_{m_0,n_0} + ja^{(i)}_{m_0,n_0})$, the better the estimation will be. To get a fair comparison with CP-OFDM, we have to define similar setups for both modulation schemes. Otherwise said, if the i.i.d. real-valued data of duration $\tau_0$ generated by OQAM have a variance equal to $\sigma^2_a$, then the complex data of duration $2\tau_0$ for CP-OFDM must have a variance equal to $2\sigma^2_a$. However, if the OFDM preamble length can be limited to $2\tau_0$, with IAM a length of $3\tau_0$ is required. But, in practice, assuming a frame-by-frame transmission mode, with each frame containing as usual tens of symbols, the resulting loss in spectral efficiency can be neglected. Let us assume that no boosting is used for pilot symbols for both modulation schemes and that the OQAM/IAM preamble is composed of i.i.d pilot values. Then we have a “pseudo pilot” sequence, $b_{m_0,n_0} = a_{m_0,n_0} + ja^{(i)}_{m_0,n_0}$, such that:

$$E\{|b_{m_0,n_0}|^2\} = \sigma^2_a \left(1 + \sum_{(p,q)\neq(0,0)} \left| \frac{g^{m_0,n_0}_{m_0+p,n_0+q} H^{(c)}_{m_0,n_0}}{H^{(c)}_{m_0,n_0}} \right|^2 \right).$$

In [9], it is shown a real orthogonal prototype satisfies:

$$\sum_{(p,q)\neq(0,0)} \left| \frac{g^{m_0,n_0}_{m_0+p,n_0+q} H^{(c)}_{m_0,n_0}}{H^{(c)}_{m_0,n_0}} \right|^2 = 1.$$  \quad (8)

Consequently, we have $E\{|b_{m,n}|^2\} \approx 2\sigma^2_a$, showing that the pilot power is nearly the same for OQAM and OFDM.

In [9] the IAM variant that uses a random sequence for preamble is named IAM1 and it is shown that if instead we select an appropriate deterministic preamble sequence, denoted IAM2, we can get a pseudo-pilot with higher power.

B. Analysis of the deterministic preamble structure in [9]

Several preamble structures have already been proposed in [9] that have shown the superiority of the IAM2 technique. Here, the purpose is to compare the difference between a preamble only using, as usual for OQAM, purely real pilots to the new CE method proposed in subsection IV-C. In the rest of
shown in Fig. 3, only the sign of each triplet is random, i.e. to be modified accordingly. In any case, the only constraint is that for all the variants of IAM presented in [9], including this variant of IAM, which will be designated as IAM-I. Note that in IAM-R, the phase term $\phi_m$ is designed for the particular case where $\phi_0 = 0$. If, as in [3], we set $\phi_0 = -\pi mn$, the preamble has to be modified accordingly. In any case, the only constraint we have is to build a preamble by triplet of subcarriers. As shown in Fig. 3, only the sign of each triplet is random, i.e. the sign of one third of the non zero pilots, $\text{sgn}(a_{3m,1})$, is selected randomly.

At frequencies where a pure imaginary is transmitted, the pseudo-pilot, given by $ja_{m,1} + ja_{m,1}^{(i)}$, has a power equal to:

$$E \left\{ j a_{m,1} + j a_{m,1}^{(i)} \right\}^2 = 2 \sigma_a^2 (1 + \left| \langle g \rangle_{m+1,1} \right| + \left| \langle g \rangle_{m-1,1} \right|)^2.$$  
(9)

Then, for a real-valued prototype filter, we get

$$E \left\{ j a_{m,1} + j a_{m,1}^{(i)} \right\}^2 = 2 \sigma_a^2 (1 + 4 \alpha + 4 \alpha^2).$$  
(10)

So, this new preamble structure has a pseudo-pilot power that is at least superior or equal to the one we get with IAM-R preamble (9). Then, we can say that the pseudo-pilot power obtained with IAM-R and IAM-I is essentially dependent upon the quantity $\alpha$. Thus, performance is in direct relation with the value of the ambiguity function of $g$, see e.g. [6] for a definition, at a particular location of the time-frequency plane. Furthermore, as the IAM-I preamble sequence is partly random, its instantaneous power is comparable to the one resulting from the useful i.i.d. data. At the contrary, the deterministic preamble of IAM-R can produce significantly higher peaks of power.

V. SIMULATION RESULTS

A. Simulation results with an IEEE802.22 channel

Our first simulations have been carried out with a channel model and modulation parameters that are borrowed from the IEEE802.22 standard. This standard aims at constructing Wireless Regional Area Network (WRAN) utilizing free TV bands. The channel profile and the main parameters of the system used are given below:

- Sampling frequency: 9.14 MHz
- Number of paths: 6
- Power profile (in dB): -6.0, 0.0, -7.0, -22.0, -16.0, -20.0.
- Delay profile ($\mu$s) : -3, 0, 2, 4, 7, 11
- FFT size: 2048
- CP composed of 130 samples (14.22 $\mu$s)
- QPSK and 16-QAM modulation and convolutional channel coding ($K = 7$ with $g_1 = (133)_o$, $g_2 = (171)_o$ and code rate=$\frac{1}{2}$)
- Frame length: 41 OFDM symbols

In order to accurately satisfy the approximations presented in section III, we only use at first prototype filters being well-localized in time and frequency. The simulations are carried out with a discrete-time signal model [9] and prototype filters
of finite length, denoted by \( L \). A first prototype filter is obtained from the Isotropic Orthogonal Transform Algorithm (IOTA) presented in [1]. A truncation of the IOTA prototype function, limiting its duration to \( 4T_{0} \), leads here to a prototype filter containing \( L = 4M = 8192 \) taps. It will be designated as IOTA4. We also use another prototype filter that results from a direct optimization, with \( L = M = 2048 \) coefficients, of the time-frequency localization (TFL) criterion [3]. We designate it by TFL1. As explained in section IV the preamble is perfectly known at the receiver side and its content depends on the CE method under consideration.

For OQAM, we have compared the two CE variants, IAM-R and IAM-I, with the CE method used in CP-OFDM. Since the CE methods are preamble-based, we consider that the channel coefficients estimated thanks to the preamble are kept constant afterwards over the whole frame. The preambles have different lengths: \( 2T_{0} \) for OFDM (see Fig. 2) and \( 3T_{0} \) for OQAM/IAM-R and IAM-I, as shown in Figs. 1 and 3, respectively.

As usual, the performance of the different estimation methods are evaluated by a comparison of the Bit Error Rate (BER) as a function of the \( E_b/N_0 \) ratio, with \( E_b \) the useful bit energy and \( N_0 \) the monolateral noise density.

The results obtained for the preamble-based CE methods are reported in Figs. 4 and 5 for the coded QPSK and 16-QAM cases, respectively. For OQAM systems, to designate the results obtained with a given estimation method and a given prototype, both acronyms are combined. For instance, IOTA4/IAM-R corresponds to the association of IAM-R with the IOTA4 prototype.

The performance results are compared at BER=\( 10^{-5} \). It can be noted that IOTA4/IAM-R and TFL1/IAM-R give performance that are 1 dB and 1.3 dB better than the OFDM one, respectively. Also, we can note that the best results are obtained with IAM-I and outperforms IAM-R of 1 dB, i.e. is approximatively 2 dB better than CP-OFDM whatever the modulation order. IAM-I outperforms IAM-R because the power of its pseudo pilot is higher. The \( \alpha \) value which has a direct impact on the pseudo-pilot power is equal to 0.538 and 0.441 for TFL1 and IOTA4, respectively.

### B. Impact of the delay spread duration

In a second step, we have carried out simulations with a 2-tap channel model with the second path varying in delay duration. Indeed, since OQAM has no CP, the delay spread may induce intrinsic interference, see section III.B. The system is identical to the previous one, except that the channel has 2 paths with power (in dB): 0, -6 and delay profile (\( \mu \)s) : 0, \( \Delta \).

The Channel Spreading (CS) percentage is measured by the ratio \( \Delta/T_{0} \), with \( \Delta \) varying. In order to show the importance of the prototype filter, in addition to IOTA4 and TFL1, we have added in our comparison another orthogonal prototype which is the Rectangular Window (RW) of length \( T_{0} \), denoted by RW1. Figs. 6 and 7 show the performance results obtained with IAM-I when using QPSK and 16-QAM, respectively.

The results are given in terms of BER versus the percentage of CS, at given values of Signal to Noise Ratio (SNR), i.e. including the CP for OFDM. As usual for OFDM, the CP duration is chosen to be just greater than each given \( \Delta \), leading to variations of the \( E_b/N_0 \) ratio and to a non-constant performance. For CP-OFDM, the performance results for QPSK (resp. 16-QAM) have been obtained either at \( SNR = 7 \) dB (Resp. 14 dB) or \( SNR = 9 \) dB (Resp. 16 dB). Doing so, we can tell what is the delay spread upper limit up to which we can gain 2 dB when using OQAM instead of CP-OFDM.

For all considered modulation schemes, the performance decreases when the CS increases. For all the OQAM modulation schemes, the explanation comes from the residual, or intrinsic, interference \( \mathbb{R}\{I_{m_0,n_0}\} \) that increases with the CS value. Indeed, whatever the pulse shape being used, \( \mathbb{R}\{I_{m_0,n_0}\} \) in (4), is only strictly equal to 0, if the transmission channel is flat, i.e. if \( H^{(c)}_{m_0+p,n_0+q} = H^{(c)}_{m_0,n_0}, \forall (p,q) \neq (0,0) \). When the delay spread increases, the channel becomes more and more selective in frequency and our initial assumption less
accurate. The difference in performance between the pulse shapes can also be explained w.r.t. $\Re \{ I_{m_0,n_0} \}$. Indeed, if the prototype function is well localized in time and frequency, the term $\langle q \rangle_{m_0,n_0}$ in (4) has a fast decay with $p$ and $q$. So the channel variations may only have a significant impact on a small size neighborhood around the location $(m_0, n_0)$. This last feature gives a slight advantage to IOTA4 w.r.t. TFL1, both being significantly better than RW1. The impact of the new CE method we propose, IAM-I, can be observed at low delay spread, i.e. where the approximation $\Re \{ I_{m_0,n_0} \} \approx 0$ is the more valid. Indeed, in Figs. 6 and 7, it can be seen that, when using IOTA4 and TFL1, a gain in SNR of 2 dB is guaranteed. This is no longer true with RW1, since in this case $\alpha = 0$. On the other hand, TFL1, with its higher $\alpha$ value, leads, at low CS percentages, to a lower BER than IOTA. OQAM/TFL1 keeps this advantage w.r.t. OQAM/IOTA4 up to 10% of CS when using QPSK and up to 5% with 16-QAM.

When comparing CP-OFDM and OQAM, it can be seen in Fig. 6 that, with QPSK, the OQAM modulation maintains its 2 dB advantage up to 10% of CS. When using 16-QAM this 2 dB advantage is preserved up to 6% of CS while above 12% of CS, CP-OFDM outperforms OQAM.

The fact that for a given BER the gain obtained with OQAM decreases when going from QPSK to 16-QAM, can be again explained by the intrinsic interference. Indeed, when $\Re \{ I_{m_0,n_0} \}$ cannot be neglected, it can be seen as an additional noise term. For a given channel and pulse shaping, the level of this noise is constant. As, our comparison between QPSK and 16-QAM is carried out in the same range of BER, it is clear that the SNR has to be higher for 16-QAM than for QPSK. Thus, a residual interference that could be neglected at low SNRs becomes significant at higher ones. However, using powerful error coding codes, e.g. turbo-codes, permits to operate at low SNRs and is therefore beneficial for OQAM.

VI. CONCLUSION

We have presented a new preamble structure for channel estimation with OFDM/OQAM. This preamble yields a virtual boosting. It has been shown for a realistic channel model (IEEE802.22) that the most efficient variant of this method, named IAM-I, can provide 2 dB gain compared to CP-OFDM. This result has been analyzed and also confirmed using a 2-tap channel model when the channel delay spread is limited to approximately 10% of the symbol duration. In future work, we will study finer equalization schemes to make OFDM/OQAM still more resistant to larger delay spread.

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