On an Approach to Visualizing Data of Flying Objects
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Abstract—The concept of temporal GIS utilizes a space-time cube based on a three-dimensional Cartesian coordinate system to represent spatio-temporal and moving objects. For a space-time cube, the elevation of object is not represented. This paper proposes an approach to visualizing data of flying objects. The proposed approach is a four-dimensional cube (4D cube) that is constituted by integrating a three-dimensional cube into a space-time cube so that they share the ground plane. In a 4D cube, a flying object is indicated by the ground coordinates, the elevation, and the present time point. The visualization of a flying object in a 4D cube enables user to cognize or monitor the movement of the object through its space trajectory, ground trajectory, and space-time path. The consideration of different flying objects visualized in a 4D cube based on space trajectories, ground trajectories, and space-time paths results in the information on the interaction between them. Following the movement of flying objects on the fly, user can estimate their possibility of collision. The 4D cube also assists user in making plan for a space trajectory of a new flying object in an area available several trajectories of different objects.

I. INTRODUCTION

Since the concept of temporal GIS (geographic information science) was proposed, spatio-temporal and moving objects have been represented on a three-dimensional Cartesian coordinate system as a space-time cube. According to the temporal geography, a space position is a position on the Earth, without elevation, where the ground position is indicated by two-dimensional coordinates as in geographic coordinate systems. Hence, a space-time cube may not represent flying objects. The problem to reveal is how to represent a flying object in space over time.

The objective of this study is to propose a model of four-dimensional cube to represent visually flying objects in space over time, where the space trajectory is a path time-ordered connecting space positions associated with ground coordinates and elevations. The main idea is to integrate a three-dimensional cube displaying the position of object in space into a space-time cube (STC) displaying the ground position associated with the time position. The paper also provides with the basic applications of 4D cube for studying the interaction between flying objects.

This paper includes five sections. In the next section, we regard to the concepts and related works by several authors, which are used in this paper. In the subsequent item, the approach of four-dimensional cube (4D cube) to representing flying objects in space over time are proposed. In the follow-up section, the applications of 4D cube for representing space trajectories of flying objects are presented to study the collision possibility of two flying objects. The main results of the study is recap in the final item as a conclusion. At the final section, a new problem is also revealed to discuss.

II. CONCEPTUAL FRAMEWORK AND RELATED WORKS

The cartography has been developing traditional maps based on two-dimensional Cartesian coordinate systems applied on the Earth such as the longitude-latitude coordinate, UTM, WGS84, and so on, to indicate space positions on the Earth, also called ground positions in this study [1], [2]. The two-dimensional maps has been developed to depict visually spatial objects in various themes. In 1861, Charles Joseph Minard employed a two-dimensional map to portray visually the 1812 Napoleon’s campaign on Moscow, where the march, the retreat of troops, as well as the disastrous defeat of Napoleon’s army are showed visually [3]. Innumerable maps and atlases of various countries and organizations edited with diverse themes on two-dimensional Cartesian coordinate systems have been published with hard copies or digital copies. The development of graphics techniques on computer nowadays diversifies the visual representation of spatial objects on two-dimensional maps.

According to the concept of temporal geography proposed by Hargerstrand in 1970, the position of an object is indicated by one position on ground associated with the time point when it exists. Mathematically, he used a three-dimensional Cartesian coordinate system as a space-time cube (STC) to display the position of the object, where two axes are utilized to show a map indicating ground position, another indicates the time position [4], [5], [6]. Though time is a complicated concept, it may be understood as a data variable, each value of which is a time point. In reality, there is no a geometrical point of time. A time point is time interval of which duration is defined so that it is convenient for the consideration [7], [8].

The applications of STC for objects moving on the Earth are studied by several authors. Kraak employed a STC to
depict again the 1812 Napoleon’s operation to Russia, he traced the march and retreat of Napoleon’s troops passing each battlefield associated with the day the combat happened [9]. Nguyen et al. used a STC to represent a bus route as a poly-line passing bus stops and a bus trip as a poly-line time-ordered connecting the points, each of which associates a bus stop with the time point when a bus reaches the stop. The authors applied this representation for the problem finding bus paths suitable for user’s need, especially the time expected to come to an arrival location [10]. Tran et al. also applied the concept of STC for tracking moving objects [11]. For a STC, the path traced by the movement of an object on a two-dimensional map is termed ground trajectory, and the path representing the movement of the object in three-dimensional spatio-temporal domain is termed space-time path [12]. Meanwhile, the concept of 3D Geographic Information System (3D GIS) employs a three-dimensional Cartesian coordinate system to determine the ground position of an object and its height such as the height of bridge, building, etc. [13], [14].

The data of an object visualized in a space-time cube enables to basically answer the questions in the relation What - Where - When developed by Peuquet [7]. The approach of space-time cube can respond all questions resulting from the What - Where - When triad. The What - Where - When relation equivalent to the Object - Location - Time relation indicates that a certain component of the triad can be inferred from two others [15]. Indeed, a certain object may be determined by its existing time associated with location, the location of an object can be known if its existing time is given, and the existing time of an object is determined if its location is indicated.

Several documents on geographic information science (GIS) have used the word “space” for the significance of “on the Earth”. In this study, a position on the Earth is considered as a ground position, meanwhile, a space position is defined by two components, ground position and elevation. In the temporal geography, a space-time position of an object is indicated by a ground position associated with the time point when the object exists. In this study, an object is considered as a geometrical point, of which the position is unique in a coordinate system at a time point. Basically, the position of an object is indicated by a space position associated with a time position, where each space position is determined by a position on the Earth associated with an elevation position.

The position of an object in space is indicated by its ground position, and elevation position. The movement of an object is the continuous displacement over time. As moving, the object traces a path in space associated with time, called space trajectory, of which the projection on the Earth is ground trajectory. Technically, the movement is recorded discretely based on time, location, change, event, or miscellaneous approach [12]. At each recording time point, the object is considered as a spatio-temporal object of which data comprises recording time, ground position, and elevation. On a three-dimensional Cartesian coordinate system, the position of a spatial object is defined by the ground position and elevation position, where the ground position is indicated on the $X \times Y$ plane of a geographic map, the elevation on another axis, termed $E$ axis. The model of 3D GIS does not display the time when the object exists at a space position.

III. THE REPRESENTATION OF OBJECT MOVEMENTS

A. The analysis of cases of movements

The ground position is determined by a geographic coordinate system $X \times Y$ on the Earth. The subset $G$ of $X \times Y$ is the set of the projections $g$ of an object on the Earth, $g \equiv (x, y), g \in G \subset X \times Y$. A space position of the object is defined by the ground position and elevation. The subset $S$ of $X \times Y \times E$, $S \subset X \times Y \times E$, is the set of the space positions of the object, $s \equiv (x, y, e), s \in S \subset X \times Y \times E$, where $E$ is the set of the elevation values of the object, $e \in E$. The time is considered as an independent variable, where $T$ is the time interval of observation, each value of which is a time point or time position $t$, $t \in T$.

An object occupies one and only one space position at a time position. Each space position $s$ of an object associates with one time position $t$, $t \mapsto s$, to constitute a tuple $(x, y, e, t) \in S \times T \subset X \times Y \times E \times T$. Based on the relation $t_i \mapsto s_i$ and $s_j \mapsto t_j$ of an object at two time points $t_i$ and $t_j$, we analyze different states of a fly object as follows.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.jpg}
\caption{The object moves, does not stop, and does not come back any previous location.}
\end{figure}

- Case 1: If $g_j \neq g_i$ and $e_j \neq e_i$ for $t_j \neq t_i$, with $\forall (i, j)$, the object flies, does not stop, and does not come back any previous location. In this case, the following functions $f_1 : t \mapsto e$, $f_2 : t \mapsto g$, $f_3 : e \mapsto g$ have one-
one relationships. They therefore have the inverse functions \( f^{-1}_1 : e \mapsto t \), \( f^{-1}_2 : g \mapsto t \), \( f^{-1}_3 : g \mapsto e \). This result may be portrayed as a triad of the ground \( G \), elevation \( E \), and time \( T \). As a result of these functions, from a value of one of the sets \( G \), \( E \), \( T \), the values of other sets associated with it are inferred, i.e. from a value \( g \) of \( G \), a value \( e \) of \( E \) associated with \( g \) may be inferred by \( f^{-1}_1 : g \mapsto e \); from a value \( t \) of \( T \), a value \( e \) of \( E \) associated with \( t \) may be inferred by \( f^{-1}_2 : t \mapsto e \); and so on. (Fig. 1)

**Case 2:** If \( g_j \not= g_i \) and \( e_j = e_i \) for \( t_j \not= t_i \), with \( \forall (i,j) \), the object has the same elevation at two different time points, \( t_j \) and \( t_i \). In this case, the one-one function \( f_2 : t \mapsto g \) has the inverse \( f^{-1}_2 : g \mapsto t \). Meanwhile, the functions \( f_1 : t \mapsto e \) and \( f^{-1}_3 : g \mapsto e \) have not the inverse functions. A time position \( t \) may not be inferred from an elevation \( e \), but it may be resulted from \( g \). A ground position \( g \) is referred from a value of \( t \). An elevation position \( e \) may be resulted from a time position \( t \) or a ground position \( g \). (Fig. 2)

**Case 3:** If \( g_j = g_i \) and \( e_j \not= e_i \) for \( t_j \not= t_i \), with \( \exists (i,j) \) where \( j = i + N, N \not= 1 \), the object has the same ground position corresponding to two different elevations, \( e_j \) and \( e_i \), at two different time points, \( t_j \) and \( t_i \) respectively. In this case, the one-one function \( f_1 : t \mapsto e \) has the reverse \( f^{-1}_1 : e \mapsto t \). Meanwhile, the functions \( f_2 : t \mapsto g \) and \( f_3 : e \mapsto g \) have not the inverse functions. A time position \( t \) may not be inferred from the ground position \( g \), but it may be resulted from \( e \). An elevation \( e \) is only referred from a time point \( t \). A ground position \( g \) may be resulted from a time point \( t \) or an elevation position \( e \). (Fig. 3)

**Case 4:** If \( g_j = g_i \) and \( e_j = e_i \), for \( (t_j,t_i) \), where \( j = i + n \), with \( n = 1, 2, \ldots, N \), the object stops at \( (x_i, y_i, e_i) \) from \( t_i \) to \( t_{i+N} \). In this case, the functions \( f_1 : t \mapsto e \), \( f_2 : t \mapsto g \), \( f_3 : e \mapsto g \) have not the inverse functions. There is only one space position \( s \equiv (x, y, e) \equiv (g, e) \) during \( [t_i, t_{i+N}] \). Therefore, a space position \( s \equiv (x, y, e) \equiv (g, e) \) may be resulted from \( t \), but it is impossible to infer a time position \( t \) from a space position \( s \equiv (x, y, e) \equiv (g, e) \). (Fig. 4)

**Case 5:** If \( g_j = g_i \) and \( e_j = e_i \), for \( (t_j, t_i) \), where \( j = i + m \), with \( m \not= 1 \), the object returns back the space location \( s \) associated with \( t_i \), at the time point \( t_{i+m} \). In this case, the functions \( f_1 : t \mapsto e \), \( f_2 : t \mapsto g \), \( f_3 : e \mapsto g \) have not the inverse functions. There is one space position \( s \equiv (x, y, e) \equiv (g, e) \) associated with two time points, \( t_i \) and \( t_{i+m} \). Similar to the case 4, the space position \( s \equiv (x, y, e) \equiv (g, e) \) may be resulted from \( t \), but a time position \( t \) may not to be inferred from \( s \equiv (x, y, e) \equiv (g, e) \). (Fig. 5)

From the above analyses, the relations among the variables of \( G \), \( E \), and \( T \) may be summarized in a triad of \( G.E.T \). As a summary, for the \( GET \) triad, a value of \( t \in T \) may be inferred from the pair of value of \( (g, e) \in G \times E \), a value of \( g \in G \) may be inferred from a value of \( t \in T \) or \( e \in E \), and a value of \( e \in E \) may be inferred from a value of \( t \in T \).

B. The Ground-Elevation-Time Cube for Representing the Movement of an Object

The movement of an object is portrayed by the relation...
between space and time positions, where the space position is indicated by two ground variables \( X \) and \( Y \) of the plane \( XY \times E \) and the elevation variable \( E \), the time position is indicated by the time variable \( T \). Traditionally, a 3D cube represents a space trajectory of 3 variables \( X,Y,E \) and a ground trajectory of 2 variables \( X,Y \), and does not display the time.

In this study, the 3D cube is modified to represent the movement of an object, termed Ground-Elevation-Time cube (GET Cube). For a ground-elevation-time cube, an axis represents the ground trajectory \( G \), an axis indicates the variable of elevation \( E \), and the rest indicates the variable of time \( T \). In other words, the axis termed \( G \) is the extension of ground trajectory, where each ground position is determined by the mileage from the previous ground position according to the following expression:

\[
[g_{i-1}, g_i] = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}
\]  

where \( g_i \) is the position of the object at the time point \( t_i \) on the axis \( G \) of GET cube, and \((x_i,y_i)\) is the position of the object at the time point \( t_i \) on the plane \( X \times Y \).

C. The 4-dimension Cube for Representing Flying Objects

In this study, a four-dimensional cube is proposed to represent flying objects. A four-dimensional cube, called 4D cube, consists of a time axis \( T \), an elevation axis \( E \), and two axes \( X \) and \( Y \) indicating the locations of objects on the Earth depicted by the \( X \times Y \) plane. A 4D cube is constituted by integrating a space-time cube (STC) into a 3-dimensional cube (3D cube), where they share the \( X \times Y \) plane to indicate the ground position of objects, the time and elevation axes are parallel and in reverse directions.

In a 4D cube, the movement of an object is represented with a space trajectory in the 3D cube, a ground trajectory on the \( GX \equiv X \times Y \) plane, and a space-time path on the STC. The ground trajectory is the projection of the space trajectory, also the projection of the space-time path, on the \( X \times Y \) plane. The position of an object flying in space is defined by: the elevation \( e \) is the projection of the space position \( s \) on the elevation axis \( E \); the ground position \( g \) is the projection of the space position \( s \) of the 3D cube on the \( G \) plane and also the projection of the spatio-temporal position \( p \), \( p \equiv (x,y,t) \), of the space-time cube; the time point \( t \) is the projection of \( p \) on the time axis \( T \) of the space-time cube (Fig. 1).

The relationships between a position \( s \) of \( S \) in the 3D cube as well as a point \( p \) of \( P \) on the STC and a ground position \( g \) on the \( G \) plane are described in the cases mentioned above. Viewing a ground position on the ground trajectory of an object, human can find out the time point associated with this ground position, it is just the intersection.
point of the line parallel to the time axis with the space-time path in the space-time cube. In special cases, if the intersection is a segment, the object stops during the time interval corresponding to the segment; if there are two or more intersection points, the object returns back this ground position one or several times. Regarding a ground position on the ground trajectory of an object, user is able to find out its space position, it is just the intersection point of the line parallel to the elevation axis with the space trajectory, if there are two or more intersection points, it is understood that the object turns back the associated ground position many times. The return of an object at a space position is not discovered if the associated time points in the space-time cube are not related.

IV. THE VISUALIZATION OF TWO FLYING OBJECTS

The possible collision of two flying objects in the same space may be studied visually with a 4D cube. Two flying objects A and B collides if and only if they exist at the same space position at the same time point, i.e. they collide if and only if:

\[ \exists t: s_A(t) = s_B(t) \]  

where, \( s_A \) is a space position of the object A, \( s_A = (x_A, y_A, e_A) \), \( s_A(t) \) is the space position of the object A at the time point \( t \), \( s_A(t) = (x_A, y_A, e_A, t_A) \); \( s_B \) is a space position of the object B, \( s_B = (x_B, y_B, e_B) \), \( s_B(t) \) is the space position of the object B at the time point \( t \), \( s_B(t) = (x_B, y_B, e_B, t_B) \).

Therefore, the condition that two flying objects collide is:

\[ (x_A, y_A, e_A, t_A) = (x_B, y_B, e_B, t_B) \]

(3)

The condition (3) may be analyzed as follows:

\[ (g_A = g_B) \land (e_A = e_B) \land (t_A = t_B) \]

(4)

The representation of two flying objects, A and B, in a 4D cube enables user to recognize the possibility of collision of the two objects. The following are main applications of 4D cube for recognizing the possible collision of two objects with respect to movements in the past, monitoring movements in real time, and planning a new flying path in an area available several flying paths.

A. Discovering the collision of two flying object in the past

-Step 1: Studying the ground intersection point. If two ground trajectories on the \( X \times Y \) plane of the 4D cube do not intersect, the collision does not happen. If two ground trajectories intersect at a point, termed ground intersection point, \( (g_A = g_B) \). The collision is possible.

-Step 2: Studying the space intersection point. If two space trajectories do not intersect at a point on the line parallel to the elevation axis from the ground intersection point, there is no collision because \( e_A \neq e_B \) at \( g_A = g_B \). If two space trajectories intersect at a point on the line parallel to the elevation axis from the ground intersection point, this intersection point is called space intersection point, i.e. \( (g_A = g_B) \land (e_A = e_B) \). The collision is possible.

-Step 3: Studying the time intersection point. If two space-time paths do not intersect at a point on the line parallel to the time axis from the ground intersection point, the collision does not happen because \( t_A \neq t_B \) at \( g_A = g_B \land (e_A = e_B) \). If the two paths intersect at a point on the line, this intersection point is called time intersection point, i.e.  \( (g_A = g_B) \land (e_A = e_B) \land (t_A = t_B) \), the collision happens.

B. Monitoring two flying objects in real time

Using a 4D cube to predict visually in real time the possibility of the collision of two objects flying in space, user is advised to follow the variation of ground trajectories, space trajectories. Following the change of the ground trajectories of the two objects on the \( X \times Y \) plane and considering the trend in their variation, user estimates whether the two ground trajectories meet. If they are predicted to meet, user continues to study the elevations of the objects, if their elevations are estimated the same at the predicted common ground point, the collision of the two objects may happen. If the ground trajectories of two objects do not meet on the \( X \times Y \) plane though they intersect, or the two objects have not the same elevation at the predicted common ground position on the \( X \times Y \) plane, the collision cannot happen.

C. Planning a new flying path

In a space of several available flying paths, the plan of a new flying path is carried out with respect to each available flying path according to the following process.

-Step 1: Finding ground intersection points. User draws an expected ground trajectory from a place to another on the \( X \times Y \) plan to find out the ground intersection points of the trajectory with an available ground
trajectory. If there is no ground intersection point, the process is continued with the step 4. If there are one or more ground intersection points, the process is continued with the next step.

**Step 2: Finding the elevation intersection points.** User traces the lines parallel to the elevation axis from the ground intersection points to discover the elevation intersection points of the lines with the space trajectory corresponding to the available ground trajectory.

**Step 3: Finding the time intersection points.** User traces the lines parallel to the time axis from the ground intersection points to find out the time intersection points of the lines with the space-time path corresponding to the available ground trajectory.

**Step 4: Repeating for another available ground trajectory.** User repeats the steps from 1 to 3 with respect to each available ground trajectory to find all space and time intersection points.

**Step 5: Indicating flying path.** User traces the space trajectory and space-time path of the new object so that they avoid all elevation intersection points and time intersection points.

V. CONCLUSION

The paper proposed the approach of four-dimensional cube to representing visually flying objects with ground trajectories, space-time paths, and space trajectories which are also flying paths. A 4D cube is formed by the combination of a three-dimensional cube with a space-time cube, where the $X \times Y$ plane is shared to indicate ground positions, the time axis of the space-time cube indicates the time, and the elevation axis of 3D cube indicates the elevation.

The relationship between ground positions, elevation positions, and time positions is analyzed to provide the base for applications of 4D cube. The approach of 4D cube is applied to study visually the flying path of an flying object. It is also used to consider flying paths of various objects in the past. Especially, the 4D cube enables to monitor visually the possible collision of two flying object in real time. In addition, the paper proposed a process of applying 4D cube for planning visually a new flying path in an space area of several available flying paths. In a very large space area, the convenience of Cartesian coordinates in 4D cube for representing flying objects needs be discussed.

REFERENCES


