Brief Paper

Adaptive controller using filter banks to reject multi-sinusoidal disturbance

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Abstract

The purpose of this paper is to apply filter banks to the control problem involving the rejection of multi-sinusoidal disturbance from output of slowly time-varying stable systems. The use of filter banks allows to distribute the control effort in many independent adaptive controllers, each of them taking care of a sinusoidal component of the disturbance. By varying the filter banks specifications, the method handles the trade-off between a time-behavior controller, with interesting settling time and a tonal-behavior controller showing the properties of simplified control, reduced computational time and on-line system identification with the system output noise via the feedback loop. Numerical examples are presented to illustrate this trade-off.

Keywords: Adaptive control; Filter banks; Rejection; Sinusoidal signals; Convergence analysis

1. Introduction

The motivation for this work was to design an adaptive feedback controller for active control of pulsed flow. In terms of control, the objective is to reject a multi-sinusoidal disturbance from output of stable system. In the field of active control of sound, Nelson and Elliot (1992) used an adaptive feedforward approach to control the anti-noise with high precision. However, this feedforward approach needs a measured reference signal. Without a reference signal, the problem is to design a feedback loop that is able to give the same attenuation as an adaptive feedforward approach. The feedback loop achieves desirable rejection of sinusoid despite the presence of modeling uncertainty in the plant model and slowly time-varying sinusoidal disturbance. But, in the case of slowly time-varying system, the feedback controller must be adapted to achieve both stability and optimal reduction.

In the field of feedback theory, a well-known Bode's theorem suggests that reduction in one frequency band must be traded off against increase of sensitivity at other frequencies. Indeed, to obtain a perfect attenuation at one frequency, without undesirable property at other frequencies, the feedback controller must be narrow band. To deal with this control problem, different controllers applied to real-world systems have been proposed. Sievers and Flotow (1992) have presented a global comparison of control methods for narrow band disturbance rejection such as higher harmonic control, tonal control, repetitive control, learning control, LMS adaptive feedforward filtering, classical and modern control. The more complex design involving a LQ-based method requires an accurate model of the plant dynamics. On the other hand, the tonal controller is the simplest compensator because it ignores the underlying dynamics, except for the plant gain and the phase at the disturbance frequencies. Micheau, Coirault, Hardouin and Tartarin (1996) experimented with a controller working in a narrow band to reject the preponderant harmonic of a pulsed flow. Good experimental results have motivated extension of the approach to the case of multiple sinusoids. For this purpose, the authors investigated filter banks, which allow analysis and synthesis in many narrow bands. This approach is an adaptation for a control
problem of the subbands echo canceller, using frequency
techniques proposed by Gilloire and Vetterli (1988),
developed by Perez and Amano (1990) and Tang, Camache
and Flores (1995). Micheau, Coirault, Renault and Tar-
tarin (1995) have successfully implemented this new con-
troller in real situations.

The objective of this paper is to show that an adaptive
controller using filter banks is between a time-behavior
controller and a tonal-behavior controller. The paper is
organized in three sections. In Section 2, the problem of
the rejection of a multi-sinusoidal disturbance from
a stable system as well as the signal processing tools are
presented. Section 3 introduces the direct adaptive feed-
back controller. The on-line adaptation needs the identi-
fication of the system and disturbance models in each
subband, which is performed using a complex generalized
recursive estimator with forgetting factor. However,
without extra-signal or dead zone algorithm, this adapt-
tive controller raises the crucial problem of estimator
convergence. Finally, Section 4 presents a numerical
simulation showing the trade-off between interesting
settling time, which needs “time-behavior controller”,
and estimator convergence, which needs tonal-behavior
controller.

2. Problem setting in the time–frequency domain

2.1. Problem setting in the time domain

Assume the system is of finite order and can thus be
described by the following model:

\[ y(t) = h(q^{-1})u(t) + d(t) + b(t), \]

\[ d(t) = CX(t), \]

\[ X(t + 1) = AX(t), \]

\[ b(t) = s(q^{-1})y(t). \]

The discrete real signals \( u(t) \) and \( y(t) \) are, respectively, the
sampled input and output. The stochastic noise \( b(t) \) is
assumed to be a white Gaussian noise \( v(t) \) filtered by
a stable filter \( s(q^{-1}) \). The sub-system \( (C,A) \) models the
disturbance \( d(t) \) as non-stabilizable mode, unstable
(on the unit circle) and non-commandable with
\( A = \text{diag}(a_0, \ldots, a_{L-1}, a_0^{-1}, \ldots, a_{L-1}^{-1}) \in \mathbb{C}^{2L \times 2L} \)
where \( a_n = \exp(2\pi i N/n) / F_o \). \( C = [c_0 \ldots c_{L-1} c_0^{-1} \ldots c_{L-1}^{-1}] \in \mathbb{C}^{2L} \)
where \( c_n = \frac{1}{2} A_n \exp(i p_n) \). The values \( A_n \) and \( p_n \) are,
respectively, the amplitude and the phase of a sinusoid
of frequency \( V_n \). The objective is to reject the first
\( N \) sinusoids of \( d(t) \) from the system output (1).

2.2. Filter banks

To reject the first \( N \) sinusoids of \( d(t) \) we propose to
split the whole frequency domain into many subbands,
and to select the \( N \) ones that include one disturbance
frequency component to reject. Thus, for a sampling
frequency \( F_s \), the frequency domain from 0 to \( F_s/2 \) is
split into \( M \) frequency subbands. Each subband, indexed
\( p \), is located into \([p - 1]F_s/2M; (p + 1)F_s/2M\). The
number of subbands must be chosen such that no
more than one sinusoidal disturbance is present in each
subband.

The analysis consists in transforming the output signal
\( y(t) \) into undersampled complex signals \( Y_p(k) \). With the
down sampling operator \([M] \) and \( w = \exp(-\pi i/M) \),
the analysis implemented as a modulated filter bank is

\[ Y_p(k) = ([M]g(q^{-1})y(k)w^k) \]

Output \( y(t) \) is translated by \(-pF_s/2M\) in the frequency
domain by multiplication with \( w^k \). Using the prototype
low-pass FIR filter \( g(q^{-1}) \) with a cut-off frequency at
\( F_s/2M \), undersampling at the rate \( F_s/M \) is possible without
aliasing.

The analysis filter bank used can be interpreted as a
time–frequency tool where the number \( M \) specifies the
time–frequency sampling. With \( M = 1 \), there is only one
band, no decimation and no low-pass filter; the analysis
filter bank does not provide any frequency analysis. With
\( M \to \infty \), there is an infinity of subbands with infinitely
short frequency support, and the decimation is infinite;
the analysis filter bank computes frequency analysis like
discrete Fourier transform (DFT). For an intermediate
value of \( M \), the analysis filter bank works in the
time–frequency domain, it computes a discrete short time
Fourier transform (STFT). The STFT is one of many
time–frequency representations mapping a time signal
into a function of two variables, corresponding to time
and frequency.

The synthesis consists in generating the input signal
according to the undersampled complex signals \( U_p(k) \).
With the upsampling operator \([M] \) and \( \text{Re} \) to design
the real part, the synthesis is written as a modulated filter
down:

\[ u(t) = \text{Re} \sum_p w^{-p} (g(q^{-1})[M]U_p(k)). \]

Each complex signal \( U_p(k) \) is oversampled because of the
interpolating low-pass FIR filter \( g(q^{-1}) \), and is translated by
\(+pF_s/2M\) in the frequency domain by multiplication
with \( w^{-p} \). The synthesis of the control \( u(t) \) constitutes
the inverse operation of the analysis. Thus, it can be
interpreted as an inverse discrete short time Fourier
transform.

2.3. Models in the time–frequency domain

Consider \( N \) selected subbands of index \( p \), where each
includes one sinusoid component to reject. The subbands
are selected such that they are not adjacent to each other. For a given subband indexed as \( p \), the model of system (1)-(4), where the \( U_p(k) \) generated by the filter bank contributes to the input \( u(t) \) and where \( Y_p(k) \) is obtained from \( y(t) \), is described by the equations (7)-(10):

\[
Y_p(k) = q^{-d}T_p(q^{-1})U_p(k) + D_p(k) + B_p(k), \quad (7)
\]

\[
B_p(k) = S_p(q^{-1})V_p(k), \quad (8)
\]

\[
D_p(k)D_p(q^{-1}) = 0, \quad (9)
\]

\[
D_p(q^{-1}) = 1 - Q_p q^{-1}. \quad (10)
\]

The downsampling and upsampling of \( M \) in (5) and (6) lead to the use of buffers of \( M \) samples, both for the acquisition of \( y(t) \) and for the reconstruction of \( u(t) \). This block processing saves computation time, but introduces a delay \( d \) of the downsampled clock \( k \). To reduce the number of parameters needed to identify the transfer function \( T_p(q^{-1}) \), it is proposed to replace it by \( T(q^{-1})H_p(q^{-1}) \). The first term, \( T(q^{-1}) \), results from the decimation and the low-pass filtering. With an ideal FIR low-pass filter \( g(q^{-1}) \) of size \( MM_p \) both at the analysis and the synthesis, and with a decimation factor \( M \), the term \( T(q^{-1}) \) converges towards a FIR of size \( 2M_g \). Moreover, in practice, this term can include not only signal processing operations, but also the complete chain of signal acquisition and reconstruction. The second term \( H_p(q^{-1}) \) describes the system behavior in the subband \( p \). For an ideal low-pass filter \( g(q^{-1}) \), the frequency response of the model for the subband \( p \), \( \hat{H}_p(e^{j\omega M}) \), must be equal to the frequency response of the whole model \( \hat{h}(e^{j\omega}) \) in the subband \( p \). Eq. (11) resumes this equality in the frequency domain,

\[
\hat{H}_p(e^{j\omega M}) = \hat{h}(e^{j\omega + \pi/M_p}) + e^{j\xi/M_p}. \quad (11)
\]

For \( M = 1 \), the subband model \( H_p \) is the discrete time model; its impulse response follows from \( h(q^{-1}) \). This is a time-behavior model. For \( M \rightarrow \infty \), the subband model converges towards the frequency model \( \hat{h}(e^{j\omega}) \rightarrow \hat{H}(e^{j\omega/M}) \); the subband model \( H_p \) can come down to a complex coefficient equal to the frequency response of \( h \) for \( \omega = \pi/M \). This is a tonal-behavior model. Between these two extremes, the subband model could be approximated by a FIR filter with \( N_H \) complex coefficients. We could call it a “time–frequency model”. For \( M \rightarrow 1 \) the FIR model needs many coefficients \( N_H \rightarrow \infty \). For \( M \rightarrow \infty \) the FIR model needs few coefficients \( N_H \rightarrow 1 \). Consequently, in open loop, the transient response to a step input \( U_p(k) \) can be estimated to last \( (d + 2M_g + N_H)MT_e \) seconds. This clearly shows the drawback between time and frequency: by increasing \( M \) to converge towards the frequency model, the transient time is also increased because \( MM_g \rightarrow \infty \) and \( N_H \rightarrow 1 \).

3. Adaptive feedback controller design and analysis

The rejection of the \( N \) sinusoidal components of the disturbance \( d(t) \) from the output \( y(t) \) is equivalent to the rejection of the periodic component \( D_p(k) \) from \( Y_p(k) \) for each subband indexed \( p \). For that purpose, \( N \) independent adaptive feedback controllers are designed; each of them rejecting a \( D_p(k) \) component from \( Y_p(k) \). For each subband, the corrector is implemented as an internal model controller according to Morari and Zafiriou (1989):

\[
\hat{D}_p(k) = Y_p(k) - q^{-d}T(q^{-1})H(\hat{\theta}_p(q^{-1}))U_p(k) \quad (12)
\]

\[
U_p(k) = -R_p \hat{D}_p(k). \quad (13)
\]

For every time \( k \) and every subband \( p \), Eq. (12) estimates the periodic disturbance \( \hat{D}_p(k) \), which is equivalent to observe one unstable pole of the disturbance model (9), (10). A perfect rejection is obtained when the complex number \( R_p \) compensates exactly the gain and the phase shift of the time–frequency transfer function at the frequency of the sinusoidal control signal: \( R_p = (\hat{Q}_p^{-d}T(\hat{Q}_p^{-1})H(\hat{\theta}_p, \hat{\tilde{Q}}_p))^{-1} \). If the estimates \( \hat{\theta}_p \) and \( \hat{\tilde{Q}}_p \) are exact, the corrector realizes a deadbeat control in \( (d + 2M_g + N_H)MT_e \) seconds. This clearly shows the trade-off arising in the filter bank approach for the control: by increasing \( M \) to converge towards the tonal-behavior controller, the settling time is also increased.

Application of this regulator requires knowledge of the system characterized by \( \theta_p \) and the frequency of \( D_p(k) \) characterized by \( Q_p \). The time variation of the physical parameters justifies the implementation of on-line identification of \( \theta_p \) and \( Q_p \). This is done by using two complex recursive least-squares estimators with forgetting factor: CRLS-\( \lambda_1 \) and CRLS-\( \lambda_2 \). Forgetting factor \( 0 < \lambda < 1 \) is mainly used to track time-varying parameters. This factor must be chosen as a trade-off between tracking capability and noise sensitivity. Based on the theory of recursive identification of Ljung and Soderstrom (1983), the CRLS-\( \lambda_1 \) is used to identify the FIR model parameters as follows:

\[
\varphi_p(k) = D(\hat{Q}_p(k-1), q^{-1}(Y_p(k) - \hat{\theta}_p(k-1)\psi_p(k))), \quad (14)
\]

\[
\hat{\theta}_p(k) = \hat{\theta}_p(k-1) + P_{\lambda,1}(k)\psi_p(k)\varphi_p(k), \quad (15)
\]

\[
P_{\lambda,1}(k)^{-1} = \hat{\lambda}_1 P_{\lambda,1}(k-1)^{-1} + \psi_p(k)^{\dagger}\varphi_p(k)^{\dagger} \quad (16)
\]

where

\[
\varphi_p(k) = q^{-d}T(q^{-1})[U_p(k) \ldots U_p(k - N_H + 1)],
\]

\[
\psi_p(k) = D(\hat{Q}_p(k-1), q^{-1})\varphi_p(k)
\]

and

\[
\hat{\theta}_p(k) = [\hat{\theta}_{p,1}(k), \ldots, \hat{\theta}_{p,N_H-1}(k)]^{\dagger}.
\]
The estimator CRLS-\(\lambda_2\) is used to estimate \(Q_p\) using the disturbance model \(D_p(k)\) defined by Eq. (10). More details on the CRLS-\(\lambda_2\) are given in Micheau et al. (1995).

3.1. Adaptive feedback controller

3.2. Discussion about the estimator convergence

Estimator convergence can be subdivided into two phases: convergence at the beginning (convergence far from the convergence point) and convergence close to the convergence point (asymptotic convergence). In the present case, with a starting point such that the closed loop is stable, there is no problem with the convergence at the beginning because the deterministic transient dynamics provides an appropriate excitation. This is the self-tuning property. However, in steady state, the measured noise provides the persistent excitation, due to the closed loop. In this case, the use of an adaptive feedback corrector can raise a major problem: the correlation between measured noise and the command can force the estimator to converge towards the inverse of the corrector. Usually, to ensure algorithm stability, the estimator is frozen, or a random extra-signal is superimposed to the control. This part investigates analytically the convergence behavior in the case of stochastic noise.

For the proposed adaptive controller, the simulation and the experimentation show that the main problem is the convergence of the CRLS-\(\lambda_2\). Therefore, we limit the convergence discussion to this estimator and we consider that \(\hat{Q} = Q\) anywhere. The objective is to show estimator convergence far from the beginning; in this case, the disturbance is perfectly rejected from the system output. To simplify the expressions, the index \(p\) is omitted.

With \(\hat{Q} = Q\) and \(T(q^{-1})H(\theta, q^{-1}) = \Gamma(q^{-1})\), the a priori error is

\[
\hat{e}(k) = -\psi(k)[\hat{\theta} - \theta] + D(Q, q^{-1})S(q^{-1})V(k),
\]

where \(V(k)\) is a complex white noise and the vector \(\psi(k)\) is given in terms of this white noise. In closed loop and in steady state, all estimator signals are functions of the measured noise \(V(k)\). This includes the vector used by the estimator

\[
\psi(k) = \Delta(\hat{\theta}, q^{-1})[V(k) \ldots V(k - S + 1)],
\]

where

\[
\Delta(\hat{\theta}, q^{-1}) = \frac{R(Q, \hat{\theta})q^{-d}T(q^{-1})D(Q, q^{-1})S(q^{-1})}{1 - R(Q, \hat{\theta})q^{-d}T(q^{-1})H(\theta, q^{-1}) - H(\theta, q^{-1})}.
\]

For the identification of \(H(\theta, k)\), the main problem is the correlation between the vector \(\psi(k)\) and \(V(k)\); this correlation can introduce a divergence on the estimated \(\hat{\theta}\). One way to avoid this drawback is to force a decorrelation between these two terms:

\[
E[\psi(k)^*D(Q, q^{-1})S(q^{-1})V(k)] = 0.
\]

With the persistent excitation condition, \(E[\psi(k)^*\psi(k)^*] > 0\), if the measurement noise is not correlated with the vector \(\psi(k)\), the estimated parameter \(\hat{\theta}\) converges to \(\theta\).

The decorrelation condition (19) is obtained when

\[
d + v^0(T) + v^0(D) > d^0(T) + d^0(S)
\]

where \(v^0(T)\) is the lowest order of the polynomial \(T(q^{-1})\) and \(d^0(D)\) is the highest order of the polynomial \(D(q^{-1})\). According to Eq. (10), \(v^0(D)\) is equal to zero and \(d^0(D)\) is equal to one. The analysis of the stochastic noise implies that \(v^0(S) = 1\). In fact, the first terms of \(T\) could be neglected such that \(v^0(T) = 3\). Therefore, condition (20) is \(d + 3 > d^0(S)\), and it clearly depends on the noise autocorrelation, via \(S(q^{-1})\), and on the system delay \(d = 3\). For a window \(g\) of length \(N_g = M_g M\), the analysis of the stochastic noise implies that \(d^0(S) = M_g - 1\) only when \(M \rightarrow \infty\). To conclude, if a window of length \(N_g \leq 6M\) is used, with a good frequency selectivity, the estimator can converge for \(M\) sufficiently large.

4. Simulation

Simulation results illustrate the trade-off between the controller dynamics and the estimator convergence as a function of \(M\). The deterministic disturbance comprises \(L = 4\) sinusoidal components of zero phase which are classified according to their amplitude. For the two sinusoidal components which are to be controlled, the amplitudes \(A_0 = A_1 = 1.0\) and the frequencies \(V_0 = 147\) Hz, \(V_1 = 303\) Hz are employed. For the two sinusoidal components which are not to be controlled, the amplitudes \(A_2 = A_3 = 0.1\) and the frequencies \(V_2 = 69\) Hz, \(V_3 = 382\) Hz are employed (Fig. 1). The frequency sampling is \(F_s = 1000\) Hz. To simulate a complex system, we use the transfer function

\[
h(q^{-1}) = \frac{\alpha_4q^{-13} + 1 + \alpha_1 q^{-1} + 0.9q^{-3} + 0.81q^{-5}}{1 - \alpha_4 q^{-27}},
\]

where \(\alpha_3 = 1.3\) and \(\alpha_4 = 0.7\). The stochastic noise \(\delta(t)\) is generated with a white noise \(\nu(t)\), filtered by the filter

\[
s(q^{-1}) = \frac{1}{1 + 0.49q^{-1} + \alpha_4 q^{-27}}.
\]

Only this measurement noise serves to excite the system via the feedback loop. The low-pass filter is designed to obtain filter banks with a good selectivity both in time and frequency, and the maximal length of \(N_g = 6M\). The following simple expression is proposed to generate the low-pass filter which presents at least a 30 dB attenuation outside the band: \(g_{l} = 0.275 - 0.5\cos(2\pi l/6M) + 0.225\cos(4\pi l/6M)\) for \(l = 0, 1, \ldots, 6M - 1\). We must choose the decimation order \(M\) for the STFT, which characterizes the discretization of the time-frequency
plane. According to the above discussion and to the frequency localization of the sinusoidal components, $M$ must be greater than 13. However, the number $M$ must be chosen as a trade-off between the controller dynamics and the estimator convergence. For this purpose, the two cases $M = 16$ and 256 are simulated.

With $M = 16$, the undersampling rate is 62.5 Hz and the two independent controllers use the subbands indexed $p = \{5, 10\}$ (Fig. 2). For each subband, a sufficiently accurate model of the system is obtained with transfer functions $H(\hat{\theta}_p, q^{-1})$ using $N_H = 20$ complex parameters. Fig. 3 shows that the controller needs 300 samples to obtain a significant rejection from the system output. In regulation phase, Fig. 4 shows that the identified parameters are not stabilized. This simulation clearly shows parameters divergence: condition (20) is not respected. With this system and $M = 16$, the controller cannot be used with the adaptive loop.

With $M = 256$, the undersampling rate is 3.9 Hz and the two independent controllers use the subbands indexed $p = \{76, 156\}$. For each subband, a sufficiently accurate model of the system is obtained with transfer functions $H(\hat{\theta}_p, q^{-1})$ using $N_H = 3$ complex parameters. Fig. 5 shows that the controller needs 3000 samples to obtain a notable rejection from the system output. From sampled time $\ell = 2.56 \times 10^5$ to $\ell = 10^6$, system parameters $x_3$ and $x_4$ are linearly set to zero; this modifies the system transfer function and the measurement noise spectrum. The estimator RLS-$\lambda_1$ used a forgetting factor $\lambda_1 = 0.999$. Fig. 6 shows that the on-line identification tracks the variation of system parameters and stabilizes afterward. Condition (20) is respected. With this system and $M = 256$, the controller can be used with the adaptive loop.

5. Conclusions

The adaptive controller using filter banks proposed in this paper presents a link between “time behavior” and “tonal behavior” controllers. Filter banks provide efficient tools to observe and to control only the sinusoidal
component to reject. Filter banks allow the use of simplified models for both the system and the disturbance. In the selected subband, the simplified models used are an IIR with one complex parameter for the disturbance, and a FIR with few complex coefficients for the system. A correct choice of decimation order, or subband number, allows to implement an adaptive controller using the measurement noise to excite and identify the system, in spite of the presence of the feedback loop. The main contribution of this proposed scheme is to save calculation time by undersampling, and to allow modeling of complex structures. A drawback results from the time–frequency duality, since we cannot design a window with good selectivity both in time and in frequency. With low $M$, the controller response is fast but the decorrelation condition (20) cannot be respected; and vice versa, for $M \to \infty$, the decorrelation condition (20) can be respected but the transient is long. The number of subbands must be chosen as a trade-off between controller dynamics and estimator convergence. The main future development will concern its application to MIMO systems.

References


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