Static and Dynamic Order Scheduling for Recreational Rental Vehicles at Tourism Holdings Limited

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Abstract

Tourism Holdings Limited (THL) operates a fleet of around 4000 recreational rental vehicles of many types, based at a dozen locations in Australia and New Zealand. The allocation of vehicles to bookings is performed centrally. If demand for a particular vehicle type at a location exceeds supply, THL may substitute vehicles of similar types or relocate vehicles where they are needed. The static problem, faced each day, is to find a schedule for vehicles which minimises the tangible and intangible costs of such substitutions and relocations. The dynamic problem is to decide – even as the customer waits on the phone – whether a vehicle will be available to cover a potential booking, and then incorporate that booking into the schedule.

CSIRO have researched, developed and supplied software modules called VASS and D-VASS, to solve both the static and dynamic aspects of schedule creation and maintenance at THL. The system is embedded in THL’s operations. This paper describes the situation at THL, and the systems implemented by CSIRO.
1 Scheduling at THL

Tourism Holdings Limited (THL) is a New Zealand-based company with a turnover in excess of NZ$100 million, which operates fleet of more than 4000 recreational vehicles (e.g. motorhomes and campervans) across New Zealand and Australia. The company has twenty locations in the two countries, all of which are operated centrally.

The solutions to two, linked, questions drive the day-to-day activities at THL. The first is question of which vehicle to assign to each accepted booking. The second is the question of whether to accept a given booking. The reputation, success and even existence of the company relies on being able to find good answers to these questions.

Scientists from the Commonwealth Scientific and Industrial Research Organisation (CSIRO), Australia, have been working with THL over the past five years to implement systems that can help answer these questions, to assist scheduling staff extract maximum efficiency from their vehicle fleet. This paper describes the business issues facing THL, and the systems developed by CSIRO to address those issues.

In Operations Research terms, the problem faced by THL is a scheduling problem. THL need to construct a schedule which assigns a suitable vehicle to each accepted booking. We call this the static problem. The question then arises, “Can this booking be placed into the schedule, and if so, where?”. This is the dynamic schedule maintenance problem. In the remainder of this section, we give an overview of these two scheduling problems.

The vehicle/booking scheduling problem can be stated as follows.

The characteristics of a booking are its start and end date, start and end location, and the booked product. A product is related to a vehicle type, but is more general. A product may specify a four-berth motor-home, but none of the extra distinguishing features of the vehicle type. Products also differ with respect to the periods of hire and insurance conditions.

The main characteristics of a vehicle are its available dates, and its vehicle type. The vehicles include standard four-door saloon cars, four-wheel drive vehicles (SUVs), and campervans with two, four or six berths. Taking into account these and other distinguishing features, such as make and model, there are about 50 distinct vehicle types.
In order to allow the largest set of bookings to be accepted, a large pool of vehicles is considered. The concepts of substitution and relocation are used to maximise the number of vehicles considered.

**Substitution.** The product specified in a booking maps to one or more suitable vehicle types. Some of these vehicles types are a natural match for the product, while others involve a substitution. For example it is permissible, but not desirable to use a six-berth vehicle for a product specifying a four-berth vehicle. A substitution cost table \( c_{pt} \) specifies the cost of using a vehicle of a type \( t \) on a booking for product \( p \). An infinite cost is used to indicate that the vehicle type may never be used for that product. Most feasible substitutions involve a vehicle type of higher quality than would usually be used for a product.

**Relocation.** Relocation allows a vehicle to be moved from one location to another – without a booking – in order to meet demand. A relocation table specifies, for each vehicle type, the cost of relocating a vehicle from one location to another. Relocations of different durations (and cost) are possible. Two drivers might take the vehicle non-stop at high cost. More commonly a single driver performs the relocation over a longer duration. Another alternative is to offer the relocation to backpackers at nominal cost. An allowance is made for an even longer trip, but this is the cheapest of all as the driver does not have to return to the original location.

Other costs in the scheduling problem include

- **Turnaround cost:** When a vehicle returns it must be cleaned and readied for the next booking. An allowance of one day is usually made for this process. However, the process can be sped up at a cost.

- **Usage cost:** Due to higher maintenance costs, some vehicles are preferred over others – for example newer vehicles are used in preference to older ones. This preference is reflected in a per-day usage cost, for each day the vehicle is used on a booking.

A final consideration are two special events - a maintenance booking, and a disposal. Vehicles have a schedule of routine maintenance. As a maintenance becomes due, a “booking” is placed in the system which effectively reserves the vehicle for the maintenance activity over the given dates.
The vehicle fleet is constantly being replaced. New vehicles are added as older vehicles are decommissioned. The process of removing a vehicle from the fleet is called a disposal.

Both maintenance and disposal events can be treated like normal bookings, in that they have a start and end location, and a start and end date (the end date in the case of a disposal is infinite). Unlike normal bookings, however, maintenance and disposals name a single vehicle, rather than a vehicle type. This complicates modelling as we will see later. Some special VIP bookings also allow nominated vehicles to be specified.

A solution to the (static) scheduling problem assigns a vehicle to each booking, maintenance and disposal, legal with regard to the company’s business rules, so as to minimise the sum of substitution, relocation and other costs. The standard scheduling constraints apply, such as the requirement that bookings assigned to the same vehicle do no overlap. The model employed is described in more detail in Section 5.

The dynamic scheduling problem poses the question “can a vehicle be provided to satisfy a booking for product X over period P from location A to location B?” Such a question must be answered in the course of an operator’s dialog with a customer, that is, in near real time. If the booking is accepted, it must be inserted into the schedule.

Most aspects of the THL’s operation are seen in typical rental-car scenarios, and most features of the recreational hire can be seen in a typical car-rental operation. However, in THL’s experience, the recreational vehicle segment is distinct because of the relative frequency of these characteristics. For example, because of the nature of holiday travel, many bookings are made well in advance, and are for long durations. Unlike many franchise-based car rental companies, THL’s locations operate in concert to maximise company profit, allowing more options in vehicle re-location. And finally, a large percentage of bookings that start and end at different locations. While these characteristics can be seen in car rental firms, they are not the “typical case” seen in the recreational segment.

Relocation is an example of the flexibility available to THL which would not usually be available to a rental-car company. This flexibility is particularly important in view of the large distances between major Australian cities and the availability of reasonably-priced one-way flights, which make one-way vehicle hire a very attractive proposition. Seasonal
patterns can also occur at this larger scale. For example, the southern regions of Australia are popular with travellers in summer months, when the monsoon season makes the tropical north unpleasant. Imbalances of this kind lead to “migratory” patterns of demand, which must be balanced by relocations in the opposite direction. Relocations obviously involve effort that is in an immediate sense unproductive, and they can incur major costs for THL.

THL addresses issue of long lead times by using an “active horizon” of 200 days: bookings beyond the active horizon are automatically accepted, and are dealt with by the scheduling system when they enter the horizon. The schedules for the New Zealand and Australian operations currently comprise several thousand bookings each.

These distinct characteristics require a different emphasis in the approach to solution.

CSIRO have developed a software system for scheduling and vehicle assignment called VASS (Vehicle Assignment and Scheduling System) that offers a solution to the static scheduling problem. The VASS system is operated each night. Its task is to build a schedule that assigns a vehicle to each booking, maintenance and disposal.

For the dynamic scheduling problem we have developed a software system called Dynamic VASS (D-VASS). D-VASS provides support for THL’s front-end reservations system. Its main purpose is to answer availability queries made by THL’s reservation staff, and to incorporate new bookings into the schedule. It also adjusts the schedule in response to operational contingencies such as vehicle breakdowns and late returns.

In handling an availability query, D-VASS first ascertains whether there is a vehicle available to precisely satisfy the request. If not, D-VASS may be used to suggest available alternatives - for example on different dates or with similar products to those requested. When a customer proceeds from an availability query to an actual booking, D-VASS is invoked again to update the schedule by inserting the new booking. At all times, D-VASS maintains a feasible current schedule.

Because D-VASS is a dynamic system, it is important that updates to the schedule should be accomplished with minimal disruption to other activities (e.g. booking enquiries). For this reason updates are carried out as quickly as possible, using a heuristic solution improvement method. Because a long sequence of sub-optimal updates may result in a substantially
inferior current schedule, a background improvement process is invoked whenever there are no other activities current.

This introduction has given a broad description of scheduling at THL. We discuss some of the trade-offs that influence scheduling decisions in Section 3, and in Section 4 we indicate how the system is used in practice. We present our model of the scheduling problem in Section 5, (a mathematical outline is given in Appendix A). VASS, the static scheduler, is described in Section 6, and D-VASS, the dynamic scheduler, in Section 7. In Section 8, we discuss system performance. In Sections 9 and 10 we discuss the benefits that VASS and D-VASS have provided to THL, and conclude by indicating some prospects for further development. But first, we will describe how the project began.

2 Project History

CSIRO’s involvement in the VASS project goes back to 1999. A vehicle rental company called Britz:Australia had developed in-house vehicle scheduling software which used a simple heuristic to assign bookings to vehicles in an incremental fashion. However as Britz was expanding, the problem size was reaching the limits of what could be handled in this way. After searching the market for a suitable software product to replace their existing system, Britz:Australia concluded that there was no existing commercial solution that would meet their needs. At this point CSIRO was commissioned to undertake a pilot study to demonstrate a capability to provide a suitable scheduling system. The pilot study involved the construction of a prototype static scheduling system, a series of tests in which the prototype was applied to several trial datasets, and a comparison of the results with those obtained using Britz:Australia’s in-house system.

The success of the pilot study led to the commissioning of VASS in June 1999 for operations spanning the whole of Australia. A significant test of the robustness and scale-ability of VASS came shortly afterwards, when THL acquired Britz:Australia, thereby almost doubling the size of the fleet that had to be scheduled. No significant problems were encountered in dealing with this increase in problem size, and VASS has been in continuous use since then.
THL were sufficiently pleased with the performance of VASS that they subsequently commissioned the implementation of D-VASS as part of a complete re-write of their computerised reservations, booking and vehicle management systems. D-VASS has been in use in New Zealand since September 2002, and in Australia since November 2002. Every new booking is passed to D-VASS to be incorporated into the schedule. D-VASS also aids the company in planning for unusual conditions and in responding to contingent situations such as late returns, breakdowns, and so on. VASS and D-VASS work in tandem, D-VASS running continuously during the day to maintain the schedule, while VASS is invoked at night to return the schedule to a near-optimal state.

3 Scheduling trade-offs

We have given a broad-brush picture of scheduling at THL in Section 1. There are, of course, many subtle details that contribute to that picture. At its heart, the scheduling task involves the management of a series of multifaceted trade-offs. The choice between a relocation as against a substitution to satisfy a particular booking can be seen as a trade-off of customer satisfaction versus cost minimisation. But the decision must also take into account the probable demand for the substituted vehicle, which usually is of higher value than the requested vehicle: when the initial scheduling decision is made there may be no known competing demand for the substituted vehicle, but the opportunity cost of the decision is not negligible.

Another important question follows a decision to plan a relocation: how much time should be allowed for the trip? The longer period allowed for a relocation, the lower the cost, so there is an incentive to decide early to relocate. On the other hand, other scheduling changes subsequent to the booking may eliminate the need for relocation altogether. There is therefore a counter-incentive, to push relocation decisions as far as possible into the future.

D-VASS handles these conflicts by using relocation in preference to substitution when a booking is first made, thus matching the booking as far as possible with the product specified by the customer. As the date of the booking comes closer, the system devotes increasing attention to the possibility of using a substitution instead of a relocation, in a trade-off
involving potential customer dissatisfaction (due to a substitution) versus dollar cost (for a relocation).

Another factor is the possibility of a delay in the realisation of a booking, which can occur as a result of a late return or breakdown, for example. A delay of this kind, of up to two days, is allowed at a high notional cost. THL does not plan for delays – if there is no vehicle available when a booking request is made, the booking is not allowed to proceed.

A final trade-off is associated with *turnaround* – the process of cleaning and preparing a vehicle after it is returned. A standard time-interval is allowed for a turnaround (depending on vehicle type), but it can be rushed or done overnight if required. A notional cost is assigned to any turnaround interval less than the standard interval. The system will consider short turnarounds in order to accommodate bookings when the schedule is tight.

## 4 Scheduling in practice

The scheduling system is seen in practice from three main points of view. The first is that of THL’s reservation staff, who use the system when responding to enquiries and booking requests made by customers. For this purpose D-VASS is accessed by way of a front-end module that handles pricing, payment and other details that are ancillary to the core scheduling function.

The second group of users are scheduling specialists and other staff members responsible for management of THL’s vehicle fleet. Vehicle purchase, maintenance, decommissioning and related actions are recorded in the database and transmitted to D-VASS. Schedulers at THL now focus their attention on less routine tasks such as planning for busy periods and handling exceptional circumstances, such as where a VIP is to be allocated a particular vehicle. Another task for the scheduling staff is to identify extra vehicles that can be hurried through maintenance or commissioning processes in response to breakdowns, late returns, and other contingencies.

A third point of view is that of the customer service staff who handle dispatching and arrivals of vehicles. For example, when a customer arrives to commence a rental, the service staff
will query D-VASS to identify the vehicle that has been allocated to the rental, and then notify the system when the vehicle has been dispatched.

5 Scheduling Model

VASS and D-VASS share a common model of the scheduling task. The time resolution is half a day, and the main task is to assign bookings to vehicles, based on the following inputs.

Index Sets The sets of all locations, products, vehicle types and rental types relevant to the problem.

Vehicles Each vehicle has a unique identifier, a vehicle type, and a specification of its availability (i.e. the location and date when it will next be available for rental).

Bookings The work to be scheduled is represented as a set of bookings. Each booking specifies a start date and location, an end date and location, and a product identifier to indicate the range of vehicles that can satisfy the booking. A number representing the value or priority of the booking helps to determine which bookings to leave unscheduled when not all bookings can be feasibly scheduled. For some specialised purposes a booking may be defined as a fixed assignment, requiring a particular vehicle or one of a limited subset of vehicles.

Maintenance Maintenance activities nominate a particular vehicle. They have fixed start and end dates, and a specific location.

Disposal Same information as a maintenance record.

Relocations Defines all legal relocations in terms of the source and destinations cities, time required and cost. A given pair of cities may have several different relocations available.

Substitutions A table listing, for each product, the vehicle types which may feasibly be used. “Natural” vehicle types have a zero cost, and others – nominally a substitution – have a larger cost.

Parameters Additional parameters influencing the composition of a scheduling solution include the time-horizon over which to schedule, an incentive factor to delay relocations as much as possible, the maximum allowable delay, a penalty cost factor for the delays and so on. Normally the values of these parameters do not change between runs of
VASS or D-VASS, but they give THL additional control over the schedules produced by the software.

Further details of the model used are provided in Appendix A.

6 VASS - Static scheduler

VASS uses a two-phase approach based on a network flow formulation of the problem (see Appendix A). The first phase involves the solution of a master problem using a commercial integer linear programming code. In the absence of a requirement that specific vehicles need to be allocated to bookings, the schedule produced by VASS is optimal with respect to the modelled costs. That is, the master problem exploits the fact that all vehicles of a particular type are essentially identical. The requirement for some specific vehicles to be allocated to particular activities (maintenance, disposal) disrupts this structure. The problem is akin to “tail assignment” in aircraft scheduling. A heuristic is used to perform the assignment of specific vehicles to activities, based on the solution of the master problem.

7 D-VASS – Dynamic scheduler

The main task for D-VASS is to insert a new activity (e.g. a booking or a maintenance activity) into the schedule. In the case of an availability query the insertion is temporary, but in other cases the revision is accepted and the current schedule is updated accordingly. In order to maintain acceptable on-line performance, D-VASS’s view of the scheduling problem ignores some of the details addressed by VASS. The most important simplification in this respect is that D-VASS preserves all substitutions already programmed by VASS; that is, it does not try to change the vehicle type assigned to an activity.

D-VASS applies an heuristic procedure to an assignment formulation of the scheduling problem. In summary, the schedule is regarded as a time-ordered sequence of activities. This sequence implies a successor or “assignment” relationship between each pair of activities carried out by a given vehicle, and a graph in which each node is an assignment. The task then
is to find the shortest path connecting these nodes, for each vehicle in the fleet. The method used by D-VASS to incorporate a new activity in the schedule is based on the successive shortest path method for solving the assignment problem described by Engquist [1]. A more complete description of the algorithm is given in [2].

A key advantage of this approach is that it can also be used in a “continuous optimisation” scheme. Improvements in the schedule appear as negative cycles in the graph used to solve the assignment problem. The continuous improvement operator simply looks for such negative cycles, and implements them.

8 Operating Environment and Performance

The scheduling system is configured for a Microsoft Windows™ platform, currently driven by a 1.5 GHz Pentium processor. Both VASS and D-VASS are written in C++ and implemented as Dynamic Link Libraries. VASS uses the CPLEX™ linear programming library to solve the network flow problem.

Access to VASS is via D-VASS, which in turn is invoked by a COM+ server in a piece of middleware written especially for this purpose. The use of COM+ facilitates distributed access to the scheduling system from the front-end clients mentioned in Section 4. For efficiency, schedule information is retained in memory between client calls rather than being re-read from database every time the server is invoked. Implementation of this arrangement has required special measures to protect the integrity of the database and to ensure robust operation in other respects.

VASS is activated by an automatic process each evening, when booking activity is low. VASS typically requires five to twenty minutes to produce a schedule. D-VASS is available for use on a more or less continuous basis, and includes a “background improvement” procedure that it runs when not attending to incoming calls (see Section 7 above). To avoid excessive delays, each standard (non-VASS) call to D-VASS is subject to a five-second time limit. This provides a comfortable ceiling, even for booking queries, indicating an adequate level of on-line performance.
9 Benefits

The pilot study (see Section 2) showed the potential for significant savings in the quality of schedules created by VASS. Compared with the schedules produced by the system in use at the time, VASS obtained a reduction in substitution costs of over 30% and reduced relocations by 2%, while at the same time decreasing by 20% the number of bookings left unassigned in the schedule.

Apart from cost savings, an important benefit has been the flexibility delivered by the system. THL’s scheduling staff formerly spent many hours each day developing a schedule manually. An internally developed scheduling system improved the situation. The introduction of VASS delivered a more reliable schedule, reducing further the amount of time spent by schedulers developing a schedule. The schedulers have gradually developed a trust in the system and its ability to produce effective solutions, and their efforts are now concentrated on handling “exceptions” to normal operation: late returns, breakdowns, incorrect vehicles dispatched, and so on. As a result, events of these kinds are handled much more effectively than in the past. The system has also enabled THL to keep its business running with much less dependence on a scheduling staff on call seven days per week.

THL’s operations have changed markedly since the commissioning of VASS and D-VASS. The company has changed hands. World events that have led to a down-turn in tourism elsewhere have affected THL’s business. Customer profiles have changed – some market segments have shrunk, others have grown.

All these changes have the potential to affect THL’s bottom line. Take for example the shifts in patterns of demand encountered by THL since September 11 2001. Although changes in fleet composition cannot always exactly match changes in demand, VASS and D-VASS have assisted the company in taking an adaptive approach in this respect. In particular, the scheduling system has facilitated the phasing-out of old types of vehicles and the introduction of new ones, through fine-grained control over substitution policy, and by extracting maximum utility from the fleet.

The form of the parameter tables allows THL’s scheduling staff to respond quickly to unusual events. For example, a “special event” (e.g. a music festival) can cause an unexpected spike
in demand, which can be handled by specialised substitution or relocation rules applied during the period of the event. The end result is better customer service: whenever possible, a customer can obtain the vehicle he or she desires, or a close substitute.

It is known that the new systems have yielded savings in direct operating costs of about 2% per annum, and anecdotal evidence indicates that VASS and D-VASS have had significant additional impacts on THL’s bottom line, for example through what amounts to an increase in fleet capacity. These impacts are difficult to quantify because of the change of company ownership which occurred shortly after the introduction of VASS. There is no doubt however that the new systems enable THL to accept many more bookings, more rapidly, with increased fleet utilisation.

VASS and D-VASS form an excellent platform for the implementation of predictive techniques. More intelligent relocation and substitution decisions can be made by incorporating expected demand into the VASS and D-VASS systems – making scheduling decisions that allow for the presence of vehicles that can be expected to appear at each location.

A related predictive approach is Revenue Management, the science of balancing pricing against expected demand to achieve the best possible result. See [3] for a review of concepts and techniques. For instance, the general flow of bookings for THL in New Zealand is North to South, and many relocations are required to bring vehicles to the North again. A reasonable question is “What is a suitable discount to offer customers to provide an incentive to travel against the flow?” Techniques from Revenue Management may provide the key.

10 Conclusions

CSIRO have produced a software system incorporating two main modules – VASS and D-VASS – that assist a large recreational vehicle rental company in their day-to-day operations. The software is used to handle every availability query, and to incorporate every new booking into the vehicle schedule. It untangles difficult questions that are central to the efficient running of the company:

- What re-locations do I need to do today to meet demand?
• When should I use a substitution rather than a relocation?
• Do I have a vehicle available to cover a requested booking?

In the future, THL and CSIRO may examine ways of incorporating Revenue Management concepts into VASS and D-VASS, as a basis for more informed pricing and discounting decisions. CSIRO and THL would like to develop their relationship further to examine these and other questions.

References


A Problem formulation

In this Appendix we indicate how the problem referred to in Section 5 is specified mathematically. For definitions of the terms used, please see the main text.

The following terminology will be used to define a rental fleet and its associated activities.

\[ V \] – Set of vehicles in the rental fleet.
\[ B \] – Set of current rental bookings to be scheduled.
\[ F \] – Set of activities with a fixed vehicle required, such as maintenances and pre-assigned bookings.
\[ P \] – Set of products available.
\[ T \] – Set of vehicle types.
\[ V_t \] – Set of vehicles of type \( t \in T \).
\[ F_v \] – Set of fixed activities for vehicle \( v \in V \).
Included in $F_v$ is a dummy activity representing the current activity for vehicle $v$. This activity implies the location and time the vehicle will next be available.

The following terms define other scheduling elements and conditions.

- $sl_a$ – The location where activity $a$ is to commence.
- $el_a$ – The location where activity $a$ is to terminate.
- $st_a$ – The time when activity $a$ is to commence.
- $et_a$ – The time when activity $a$ is to terminate.
- $v_b$ – Profit obtainable from booking $b$.
- $reloc-t(v, m, n)$ – Time to relocate vehicle $v$ from $m$ to $n$. Time is zero if $m = n$.
- $reloc-c(v, m, n)$ – Cost to relocate vehicle $v$ from $m$ to $n$. Cost is zero if $m = n$.
- $subst-c(v, b)$ – Substitution cost of using vehicle $v$ for booking $b$. This is determined by the product of $b$ and the vehicle type of $v$.
- $turn-t(v)$ – Turnaround time needed to prepare vehicle $v$ for a rental.
- $delay-c(b, t)$ – Cost of delaying booking $b$ by a time $t$. $delay-c(b, 0) = 0$.
- $D_a$ – Maximum delay for activity $a$.
- $d_a$ – Actual delay for activity $a$.

A schedule is defined by the list of activities $A_v \subseteq (B \cup F)$ assigned to each vehicle $v \in V$:

- $A_v$ – List of activities scheduled for vehicle $v$.
- $A$ – Set of all activities assigned to vehicles = $\cup_{v \in V} A_v$.
- $U$ – Unassigned bookings = $B \setminus A$.
- $a^i_v$ – The $i^{th}$ activity scheduled for vehicle $v$.

With the provision for delays, the timing of a rental or other activity performed on a given vehicle $v$ may be delayed beyond its stipulated commencement-time $st_{a^i_v}$ by tardiness in completion of the activity’s predecessor $a^{i-1}_v$. The delay is zero for $i = 1$, while for $i > 1$ it is defined as follows.

$$st'_{a^i_v} = et_{a^{i-1}_v} + turn-t(v) + reloc-t(v, el_{a^{i-1}_v}, sl_{a^i_v})$$  \hspace{1cm} (1)

hence

$$d_a = \max(st'_{a^i_v} - st_a, 0)$$  \hspace{1cm} (2)
Four main constraints apply to the construction of a schedule $A$. In order these are

- each vehicle must perform the fixed activities pre-assigned to it
- each activity can be scheduled on at most one vehicle:
- the activities assigned to each vehicle must be arranged in temporal sequence
- an upper limit is placed on any delay in commencing an activity

$$ F_v \subseteq A_v \quad \forall v \in V $$

$$ A_v \cap A_w = \emptyset \quad \forall v \neq w \in V $$

$$ st_{a_{v-1}} \leq st_{a_{v}} \quad \forall v \in V, \forall i : i > 1, a_v^i \in A_v $$

$$ d_a \leq D_a \quad \forall a \in A $$

The following cost components are defined for a schedule $A$.

Profit foregone from rentals that are not included in the current schedule:

$$ CP = \sum_{b \in U} v_b $$

Relocation costs:

$$ CR = \sum_{v \in V} \sum_{i: i > 1, a_v^i \in A_v} reloc-c(v, el_{a_v^{i-1}}, sl_{a_v^i}) $$

Substitution costs:

$$ CS = \sum_{v \in V} \sum_{a \in A_v \cap B} subst-c(v, a) $$

Costs due to delays beyond requested commencement times.

$$ CD = \sum_{v \in V} \sum_{i: i > 1, a_v^i \in A_v} delay-c(a_v^i, d_a^i) $$

The problem then is to construct the schedule $A$ so as to minimise the weighted sum of $CP$, $CR$, $CS$ and $CD$, subject to conditions 3-6.