A box model of the seasonal exchange and mixing in Regions of Restricted Exchange: Application to two contrasting Scottish inlets

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ABSTRACT
We present a time-dependent box model of exchange and mixing processes in Regions of Restricted Exchange (RREs). The model is applied to two contrasting Scottish inlets, but can potentially be applied to a wide range of stratified inshore water bodies. The model represents the vertical structure of the inlet using up to three horizontally uniform layers, representing surface, intermediate and deep basin water, respectively, and calculates the daily volume, thickness, salinity and temperature of each layer over an annual cycle. The model is forced by observed time series of daily-mean wind stress, river discharge, surface heat flux, and depth-profiles of the external coastal temperature and salinity. Advection and diffusive fluxes between the layers, and between the inlet and adjacent coastal ocean, are calculated from parameterisations of known physical processes in inshore stratified waters, utilising published analytical solutions and empirically-based formulae. The goal is for the model to be generally applicable to RREs to support improved resource management. As such, the number of free parameters within the model has been kept to a minimum.

The model predictions of layer temperature and salinity are quantitatively compared to observations from year-long sampling programmes in two contrasting Scottish inlets, Loch Creran and Loch Etive. The model showed good agreement with the observed data, reproducing the broad scale seasonal cycle and other shorter-term fluctuations. RMS errors for temperature were less than 1 °C, and for salinity typically of the order 1. The model results suggest that Loch Creran behaved as a well-mixed box, with weak stratification maintained by strong vertical diffusion, and volume exchange between the inlet and coastal waters dominated by tidal forcing. In contrast, Loch Etive exhibited classical fjordic three-layer hydrography and dynamics, with strong stratification, weak vertical diffusion and a density-driven circulation comparable in strength to tidal exchange. The effects of a deep water renewal event on water properties were successfully reproduced by the model. The potential for wider application of the model to resource management issues and to other types of RRE is discussed.

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1. Introduction

Regions of Restricted Exchange (RREs) are elongated stretches of coastal water penetrating inland at mid-high latitudes in both northern and southern hemispheres (Tett et al., 2003). Examples of RREs are the inlets of North America, the fjords of Scandinavia and Chile, the rias of Spain, the voes and sea-lochs of Scotland, and the loughs of Ireland. RREs are typically bounded on three sides by land, such as the fjords of Scandinavia and North America, with water exchange occurring only through the fourth, often narrow, boundary. Around the world, RREs have been subject to increasing pressure from coastal development and coastal industry. In particular, the growth of finfish aquaculture in Europe, North America, Scandinavia and Chile over the past four decades has been concentrated in RREs, and has raised considerable concerns over potential impacts on the surrounding environment (Black, 2001). The concerns include benthic and water quality degradation due to carbon and inorganic nutrient emissions, increased risk of eutrophication in coastal waters, and the potentially toxic effects on native flora and fauna of chemicals used by the aquaculture industry. The potential risk to water quality in RREs is mitigated by the strength of mixing and exchange, both internally within the RRE and externally between the RRE and the adjacent coastal ocean. However, the external exchange in RREs is, by definition, restricted, which, while providing the sheltered waters favourable for aquaculture infrastructure, may exacerbate water quality...
problems. Much interest is focussed, therefore, by coastal managers on understanding and quantifying the mechanisms of water exchange in these environments.

The processes that dominate exchange between RREs and the adjacent coastal ocean are the tides, a density-driven circulation caused by both freshwater discharges into the RRE and fluctuations in the coastal density profile (e.g. Arneborg, 2004), and deep water renewal in fjordic estuaries (Farmer and Freeland, 1983). Within an RRE, entrainment and mixing processes, caused by wind stirring, velocity shear and internal wave activity (e.g. Stigebrandt, 1976, 1977; Farmer and Freeland, 1983; Simpson and Rippeth, 1993), redistribute water properties between surface and deep water layers. The net effect of these processes is to produce a water column structure in RREs that can be effectively represented by two or three horizontally- and vertically-uniform layers, separated by primary and secondary pycnoclines. The upper two layers are in open connection with the coastal ocean, whereas the third, bottom, layer may be isolated from the open ocean by the presence of a sill. This simplified water structure facilitates the use of relatively simple models to quantify the dynamics of exchange in RREs. The goal of this paper is to develop a simple box-type model of RRE exchange and mixing processes that can be widely applied to varying types of RRE to inform resource management and to quantify the dominant mechanisms of exchange in different systems.

Simple box-type models can provide useful insights into the dynamics of coastal channels and inlets. Pawłowicz et al. (2007) used a mixing-box approach to determine residence times in the Strait of Georgia. The interannual variability of volume fluxes and water properties between the Strait of Georgia and Juan de Fuca Strait was investigated with a box model by Li et al. (1999) and Babson et al. (2006) studied the interannual variability of residence times of Puget Sound basins. Simpson and Rippeth (1993) used a one-dimensional (1D) model to quantify the contributions to the mixing regime in the Clyde Sea, a Scottish inlet, by tides, convection, wind stirring and internal waves. They found that the latter two processes dominated the seasonal mixing cycle. Finally, Stigebrandt (1985) developed a 1-D model of the mixed layer depth for the Baltic Sea, which particularly focussed on the balance between surface layer mixing and buoyancy inputs. The physical processes of horizontal and vertical exchange in Scandinavian fjords have been extensively investigated by Stigebrandt et al. over the past three decades (e.g. Stigebrandt, 1980, 1981, 1985, 1999), culminating in the development of the FjordEnv model (Stigebrandt, 2001), a water quality model for inshore waters. For well-mixed and partially-mixed estuaries, box models using exchange rates derived from observed salinity distributions (e.g. Officer, 1980) have been widely used, but such models are not appropriate for the present application due to the relative scarcity of hydrographic observations upon which the models fundamentally depend.

In this paper we present a model, ACEXR, of physical exchange processes in RREs with applications to two contrasting Scottish inlets. Although we focus here on Scottish inlets, we believe the model is generally suited and easily adaptable to other RREs. The physical dynamics of Scottish inlets are not dissimilar to Scandinavian fjords, and the mathematical framework of the FjordEnv model has provided a proven basis on which to build and develop the ACEXR model. The ACEXR model also utilises research describing physical exchange and mixing processes in Scottish (Simpson and Rippeth, 1993; Rippeth and Simpson, 1996; Inall and Rippeth, 2002; Inall et al., 2005) and Scandinavian (Luimungan, 2000; Arneborg, 2004) inlets. We present simulations of the seasonal cycle of water temperature, salinity and exchange in two contrasting Scottish inlets: Loch Creran and Loch Etrive. The performance of the model is evaluated against available data, and the model is then used to describe the dynamical balance of the two systems.

2. Description of the model

A schematic illustration of the model design is presented in Fig. 1. RREs such as inlets and fjords can be considered to consist of three horizontally-uniform layers, representing a surface brackish layer, an intermediate tidally-exchanged layer, and a bottom layer isolated from the adjacent coastal ocean by the presence of a sill. Other RREs, such as rias and voes, do not necessarily contain sills and the isolated bottom water is not present; in these cases, the bottom layer can be excluded from the model and the water body treated as a two-layer system. The layers interact through exchange of mass, volume and heat via a number of physical processes, indicated by the fluxes Q in Fig. 1. Daily variability in the calculated transports, together with surface forcing (Fig. 1), controls the evolution of the thickness, temperature and salinity of each layer over the annual cycle.

Note that tidal exchange operates through the intermediate layer. On each rising tide, seawater from outside the inlet entrance floods into the inlet, sinking beneath the less dense surface water into the intermediate layer below. The outflowing surface layer is considered to be "arrested" by the incoming tidal current and is therefore stationary during flood tides (often leading to the development of "V-shaped" fronts above inlet sills e.g. Booth, 1987). Therefore, in ACEXR, tidal exchange is modelled as an exchange between the intermediate layer and the external coastal ocean (Fig. 1).

2.1. Model formulation

The framework for the model lies in the conservation of mass and salt, coupled with parameterisations of the dynamical terms. Notation is given in Table 1. Conservation of volume, $V_j$, in layers $j = 1, 2, 3$ is expressed as:

\[
\frac{dV_j}{dt} = Q_{I, j} - Q_{O, j} + Q_{S} - Q_{E, j} + Q_{T, j} + Q_{G, j}
\]
\[
\begin{align*}
\frac{dV_1}{dt} & = Q_h + Q_{E12} \\
\frac{dV_2}{dt} & = -Q_h - Q_{E12} + Q_{E23} \\
\frac{dV_3}{dt} & = -Q_{E23}
\end{align*}
\]

where \(Q_h\) is the vertical volume flux (m\(^3\) s\(^{-1}\)) between layers 1 and 2 due to wind stirring, and \(Q_{Eij}\) is the vertical flux due to entrainment across the density interfaces between layers \(i\) and \(j\) due to tidal velocity shear. Both the wind-stirring and entrainment terms \((Q_h, Q_{Eij})\) can be positive (upward) or negative (downward).

Transport of a passive tracer \(C_i\) (for example temperature \(T_i\), or salinity \(S_j\)) in layer \(j\) \((j = 1, 2, 3)\) is given by:

\[
\begin{align*}
\frac{d(VC_1)}{dt} & = -Q_{E12}C_1 + Q_{E12}C_3 + Q_{E23}C_2 + Q_{E23} + C_1V_1G
\end{align*}
\]

\[
\frac{d(VC_2)}{dt} = (1 - \delta)Q_{E12}(C_2 - C_1) + \delta Q_{E23}(C_3 - C_2) + (1 - \delta)Q_t(C_2 - C_3) - Q_{E12}C_1 + Q_{E12}C_3 + Q_{E23}(C_3 - C_2) + Q_{E23}C_1 - Q_{E12}C_1
\]

\[
\frac{d(VC_3)}{dt} = \delta(Q_{E12} + Q_{E12})(C_3 - C_1) - Q_{E23}(C_3 - C_2) - Q_{E23}C_1
\]

where \(Q_{Eij}\) is the vertical flux (m\(^3\) s\(^{-1}\)) due to turbulent mixing (eddy diffusion) between layers \(i\) and \(j\), \(Q_t\) is the volume flux due to tidal exchange, \(Q_t\) is the freshwater flux and \(\Gamma\) is the flux of \(C\) through the surface boundary. For salinity, \(\Gamma = 0\) (evaporation and precipitation are neglected), whereas for temperature, \(\Gamma = Q_{SHF}/(C_p\rho_1 - H_t)\), where \(Q_{SHF}\) is the surface heat flux and \(C_p\) is the specific heat capacity of seawater. The tracer flux terms for \(Q_{h}\) and \(Q_{Eij}\) flows, which may be positive or negative, use the upstream value \(C_i\), \(i = 1, 2, 3\). All the other volume flux \((Q)\) terms are always positive.

2.1.1. Gravitational circulation

The strength of the density-driven estuarine circulation is here related to the horizontal density gradient between the surface water inside and outside the estuary. This approach was adopted previously by Simpson and Rippeth (1993) and Li et al. (1999). The volume transport is given by:

\[
Q_e = C_e(\rho_0 - \rho_{ef})/\rho_{ef}
\]

where \(\rho_0\) is the external density and \(C_e\) is a free constant (m\(^3\) s\(^{-1}\)), which is tuned to provide a best fit to observations.

2.1.2. Tidal exchange

The volume flux due to tidal exchange is given by:

\[
Q_t = 2\epsilon a_0 a_0 / T_P
\]

where \(\epsilon\) is the tidal efficiency, \(a_0\) is the horizontal surface area of the water body (m\(^2\)), and \(T_P\) is the dominant (e.g. M\(_2\)) tidal period (s). The efficiency of tidal exchange \((0 \leq \epsilon \leq 1)\) is derived from the estuary exchange method of Wells and van Heijst (2003) i.e.

\[
\epsilon = 1 - (4Y_S/\pi U_5 T_P)^{1/2}
\]

\[
\epsilon = 1 - Y_S/U_5 T_P < 0.13
\]

where \(Y_S\) is the width of the entrance sill (m) and \(U_5\) the current speed over the entrance sill (m s\(^{-1}\)), which for the current case studies is known (Edwards and Sharples, 1986) but could be derived from \(Q_t\) and the known cross-sectional area of the entrance sill or channel. Wells and van Heijst (2003) derived (6) to account for the increased efficiency of estuary exchange when the entrance to the estuary is constricted and produces a jet-like flow on the flood tide. A jet-like flood tide injects external water into the estuary basin beyond the range from which water is drawn for ejection on the subsequent ebb tide, thus increasing the efficiency of the exchange process. If no entrance sill exists in the modelled system, \(\epsilon\) defaults to unity.

2.1.3. Tidal entrainment

Entrainment is the process by which ambient fluid is transferred across a density interface by, and incorporated into, a flowing layer of turbulent fluid. Entrainment is common in environmental flows (Turner, 1986). Here, we consider entrainment from the surface and deep layers into the tidally energetic intermediate layer. As noted earlier, the surfacing fluid layer is considered to be “arrested” by the incoming tidal current and is therefore stationary during flood tides. The lower, deep water layer is quiescent. The turbulent intermediate layer therefore entrains fluid from above and below.
Table 1

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>V_i</td>
<td>Volume of layer (i = 1, 2, 3)</td>
<td>m^3</td>
</tr>
<tr>
<td>H_i</td>
<td>Thickness of layer i</td>
<td>m</td>
</tr>
<tr>
<td>S_i</td>
<td>Salinity of layer i</td>
<td>m</td>
</tr>
<tr>
<td>T_i</td>
<td>Temperature of layer i</td>
<td>°C</td>
</tr>
<tr>
<td>r_i</td>
<td>Density of layer i</td>
<td>kg m^-3</td>
</tr>
<tr>
<td>Q_u</td>
<td>Vertical volume flux between layers 1 and 2, due</td>
<td>m^3 s^-1</td>
</tr>
<tr>
<td></td>
<td>to wind-driven entrainment</td>
<td></td>
</tr>
<tr>
<td>Q_0</td>
<td>Volume flux due to tidal and density driven</td>
<td>m^3 s^-1</td>
</tr>
<tr>
<td></td>
<td>entrainment between layers i and j</td>
<td></td>
</tr>
<tr>
<td>Q_i</td>
<td>Volume flux due to vertical mixing (eddy diffusion)</td>
<td>m^3 s^-1</td>
</tr>
<tr>
<td></td>
<td>between layers i and j</td>
<td></td>
</tr>
<tr>
<td>Q_0</td>
<td>Volume flux due to tidal exchange</td>
<td>m^3 s^-1</td>
</tr>
<tr>
<td>Q_e</td>
<td>Volume flux due to the gravitational estuarine</td>
<td>m^3 s^-1</td>
</tr>
<tr>
<td></td>
<td>circulation</td>
<td></td>
</tr>
<tr>
<td>Q_r</td>
<td>Volume flux of freshwater discharged into the loch</td>
<td>m^3 s^-1</td>
</tr>
<tr>
<td>Q_SHF</td>
<td>Net heat flux through the water surface</td>
<td>W m^-2</td>
</tr>
<tr>
<td>K_0</td>
<td>Vertical eddy diffusion</td>
<td>m^3 s^-1</td>
</tr>
<tr>
<td>A_i</td>
<td>Horizontal interfacial area between layers i and</td>
<td>m^2</td>
</tr>
<tr>
<td></td>
<td>j</td>
<td></td>
</tr>
<tr>
<td>A_e</td>
<td>Horizontal surface area of the loch basin</td>
<td>m^2</td>
</tr>
<tr>
<td>X_i</td>
<td>Vertical cross-sectional area of layer i</td>
<td>m^2</td>
</tr>
<tr>
<td>Y_e</td>
<td>Width of the entrance sill</td>
<td>m</td>
</tr>
<tr>
<td>U_i</td>
<td>Current speed over the entrance sill</td>
<td>m s^-1</td>
</tr>
<tr>
<td>W_t</td>
<td>Wind speed</td>
<td>m s^-1</td>
</tr>
<tr>
<td>T_r</td>
<td>Dominant (e.g., M_2) tidal period</td>
<td>s</td>
</tr>
<tr>
<td>a_0</td>
<td>Amplitude of the surface barotropic tide</td>
<td>m</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration due to gravity</td>
<td>m s^-2</td>
</tr>
<tr>
<td>α</td>
<td>Thermal expansion coefficient</td>
<td></td>
</tr>
<tr>
<td>β</td>
<td>Saline contraction coefficient</td>
<td></td>
</tr>
<tr>
<td>δ</td>
<td>Switch indicating whether deep water renewal is</td>
<td></td>
</tr>
<tr>
<td></td>
<td>occurring (δ = 1) or not (δ = 0)</td>
<td></td>
</tr>
<tr>
<td>C_p</td>
<td>The specific heat capacity of seawater, C_p = 4200</td>
<td>J kg^-1 K^-1</td>
</tr>
<tr>
<td>B</td>
<td>Surface buoyancy flux</td>
<td>m^3 s^-1</td>
</tr>
<tr>
<td>D_p</td>
<td>Drag coefficient</td>
<td>0.0025</td>
</tr>
<tr>
<td>D_s</td>
<td>Surface drag coefficient</td>
<td>0.0015</td>
</tr>
<tr>
<td>C_e</td>
<td>Constant in the gravitational estuarine circulation</td>
<td>m^3 s^-1</td>
</tr>
<tr>
<td>C_k</td>
<td>Constant in the wind-stirred entrainment equation</td>
<td></td>
</tr>
<tr>
<td>ε</td>
<td>Efficiency of tidal exchange</td>
<td></td>
</tr>
<tr>
<td>λ, μ, γ</td>
<td>Constants in the vertical diffusivity equations</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(Equations (15) and (16))</td>
<td></td>
</tr>
<tr>
<td>ρ_a</td>
<td>Density of air, ρ_a = 1.3</td>
<td>kg m^-3</td>
</tr>
<tr>
<td>ρ_ref</td>
<td>The density of seawater at 10 °C and salinity of</td>
<td></td>
</tr>
<tr>
<td></td>
<td>35, ρ_ref = 1027</td>
<td>kg m^-3</td>
</tr>
<tr>
<td>ρ_i</td>
<td>External water density</td>
<td>kg m^-3</td>
</tr>
<tr>
<td>U_w</td>
<td>Friction velocity</td>
<td>m s^-1</td>
</tr>
<tr>
<td>U_wj</td>
<td>Entrainment velocity due to wind stirring and</td>
<td>m s^-1</td>
</tr>
<tr>
<td></td>
<td>buoyancy inputs</td>
<td></td>
</tr>
<tr>
<td></td>
<td>U_wj</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Entrainment velocity between layer i and j due</td>
<td>m s^-1</td>
</tr>
<tr>
<td></td>
<td>to tidal current</td>
<td></td>
</tr>
</tbody>
</table>

during flood tides, depending on the strength of the velocity shear and the stratification across the interfaces. During ebb tides, stratification increases due to tidal straining, and entrainment is assumed to be inhibited. We have tested several entrainment algorithms developed from laboratory studies by Turner (1986) and Strang and Fernando (2001), but the results presented here used the algorithm presented by Princevac et al. (2005) for natural flows. The entrainment velocity normal to the horizontal interface, \( w_{ij} \), is calculated as:

\[
w_{ij}(x, t) = 0.05U_r^2\text{Re}_r^{0.75}
\]

where \( \text{Re}_r = \Delta B t / U_r^2 \) is a bulk Richardson number, a balance between the buoyancy jump \( \Delta \rho = (g/\rho_{\text{ref}})\Delta \rho \) and velocity shear \( \Delta U \) across the interface. Because, in the current conception of the model, the surface layer is assumed to be stationary during flood tide, \( \Delta U \) is identical to \( U_2 \). The intermediate layer velocity is calculated as:

\[
U_2 = (Q_G + Q_T \sin \omega t) / X_2
\]

where \( \omega = 2\pi / T_h \) and \( X_2 \) is the cross-sectional area of layer 2. When \( U_2 < 0 \) (i.e. during ebb tide), \( w_{ij} \) is set to zero. From the entrainment velocity, the vertical entrainment flux, \( Q_{\text{Eij}} \), is calculated as:

\[
Q_{\text{Eij}} = A_0 w_{ij}
\]

2.1.4. Wind-driven entrainment

The balance between wind stirring and a positive vertical buoyancy flux drives an entrainment velocity, \( w_{\text{es}} \), at the base of the surface layer. Following Stigebrandt (1985), who described a model of the seasonal mixed layer for a large fjordic system, the Baltic, based on the Kato–Phillips entrainment velocity, we specify:

\[
w_{\text{es}} = \frac{2C_G U_r^2}{g} \left( \frac{\Delta \rho}{\rho_{\text{ref}}} H_1 \right) - \frac{B}{\rho_{\text{ref}}} > 0
\]

where \( B \) is the buoyancy input (see below), \( C_G \) is a constant free, and \( U_r \) is the friction velocity (m s^-1) given by:

\[
u_r^2 = \frac{\rho_a C_D S W^2}{\rho_1}
\]

where \( W \) is the wind speed (m s^-1), \( C_D \) a surface drag coefficient and \( \rho_a \) the density of air. The buoyancy flux through the sea surface is given by:

\[
B = g \left( \frac{\alpha}{\rho_1 C_F} Q_{\text{SHF}} + \beta \frac{Q_F}{A_0} \right)
\]

2.1.5. Vertical eddy diffusion

A further vertical flux of scalar water properties between layers (there is no net volume flux) occurs as a result of vertical turbulent mixing driven by vertical velocity shear. We assume that \( K_{\text{SS}} \) takes the general form

\[
K_{\text{SS}} = K_{\text{ZO}} (1 + \gamma \text{Rig})^{\lambda}
\]

where \( K_{\text{ZO}} \) is the diffusivity of non-stratified flow, \( \text{Rig} \) is the bulk Richardson Number as defined above, and \( \gamma \) and \( \lambda \) are constants. Values of \( K_{\text{SS}} \) must be calculated for mixing across both interfaces. The form of (15) is analogous to well-known open ocean parameterisations of vertical mixing (e.g. Munk and Anderson, 1948; Lozovatsky et al., 2006), which used a gradient Richardson
Number rather than the bulk Richardson Number used here. Babson et al. (2006) derived a vertical mixing formulation with the same form as (15), with $\lambda = -1$, for their layered box model of Puget Sound, and we use (15) as an initial approximation of the vertical diffusive flux. Following Babson et al. (2006), the value of $K_{Z2}$ is given by

$$K_{Z2} = \mu C_D U_2 H_2$$

(16)

where $C_D$ is a typical drag coefficient with a value of 0.0025 (Babson et al., 2006) and $\mu$ is a free parameter for calibration.

The shear-induced vertical mixing between model layers 2 and 3 given by (15) is supplemented by an additional term, $K_{Zb}$, representing the mixing caused by breaking internal waves in fjord basins (Stigebrandt and Aure, 1989; Stigebrandt, 2001; Inall and Rippeth, 2002; Cottier et al., 2004; Inall et al., 2004). The calculation of $K_{Zb}$ for silled fjords has been described in detail by Stigebrandt (1999) and we do not repeat the mathematics here. In brief, each sill in the RRE system is assumed to generate a linear internal tide in the stratified water column, and the energy flux is calculated for each one. A small fraction of the total energy flux, generated at all the sills in the system, is expended on vertical mixing; we follow Stigebrandt (1999, 2001) and assume a mixing efficiency (flux Richardson Number) of 5% for ‘wave basins’ and 1% for ‘jet-type’ basins. The vertical diffusive fluxes are therefore given by

$$K_{Z12} = K_{Zs}$$
$$K_{Z23} = K_{Zs} + K_{Zb}$$

(17)

and the volume transport due to vertical eddy diffusion is:

$$Q_{ij} = A_{ij} K_{Zij} / \overline{H_{ij}}$$

Here the overbar indicates the mean of $H_i$ and $H_j$.

2.1.6. Deep water renewal

A particular feature of the model is the capability to simulate the effects of deep water overturn or renewal events, which occur when the external water density, $\rho_0$, becomes greater than the bottom density, $\rho_s$. Renewal is a critical process for the exchange of deep water in fjord basins. In ‘normal’ conditions, when $\rho_0 < \rho_s$, gravitational and tidal exchange are confined to the surface and intermediate layers above sill depth. The intruding waters into layer 2 entrain water from layer 3, so that the thicknesses of layer 2 increases and that of layer 3 decreases. The water properties of layer 3 are modified only by this entrainment process and vertical eddy diffusion. When water denser than the basin water (i.e. $\rho_0 > \rho_s$) crosses the entrance sill, a deep-water renewal event is triggered (Gade and Edwards, 1980), and the intruding water flows down the slope inside the sill, typically as a gravity current, entering the bottom layer. The ambient basin water is lifted up in the water column, from where it can be exchanged and exit the inlet basin. The renewing water fills up the deep basin until the density of the external water is no longer sufficiently dense to replenish the plain.

In the model, the deep-water renewal process is accomplished by setting the switch $\delta$ in Equation (2) to unity when $\rho_0 > \rho_s$. Otherwise, when $\rho_0 < \rho_s$, $\delta = 0$. When $\delta = 1$, the density-driven and tidal exchanges are directed into layer 3 rather than layer 2, as reflected in the transport of external water properties (Equation (2)). Density currents are well known to entrain fluid from the ambient environment (Turner, 1986); in the model, the entrainment term $Q_{23}$ is made negative, to reflect the downward entrainment from layer 2 into layer 3 during renewal events. The simulated renewal event continues until either $\rho_0 < \rho_s$, or the vertical location of the interface between layers 2 and 3 is elevated to sill depth, at which point $\delta$ is reset to zero. The approach described here clearly does not simulate the dynamics of deep water renewal events; our intention is only to attempt to reproduce the effects of the renewal process on water properties in this simple model.

2.2. Boundary conditions

The model is forced by daily or hourly wind stress at the surface, total daily freshwater discharge into the surface layer and, if surface heat flux is to be included in the simulation, daily values of local sea level pressure, air temperature, dew-point or wet-bulb temperature, relative humidity, cloud cover and solar irradiation are required. These are used to derive the net daily heat flux, $Q_{\text{swin}}$, into the surface layer of the modelled inlet.

For the open boundary condition, daily profiles of coastal ocean temperature and salinity, external to the modelled basin, are required. In U.K. coastal waters, data of this frequency are few and far between. More common, though by no means widespread, are monitoring stations which may be sampled weekly or fortnightly. The model has been written to accept available low-frequency data and interpolate these data linearly into daily resolution. Linear interpolation represents the simplest method of interpolating sparse data, and makes the fewest assumptions about the nature of the data. However, it could be argued that data with a predictable seasonal cycle, for example coastal temperature, could be interpolated using a sinusoidal curve, and that option may be added to the model in future.

2.3. Solution procedure and implementation

Equations (1)–(18) form a closed set of equations that can be solved iteratively from an initial set of conditions to calculate $V_i$, $S_i$, $T_i$ and $\rho_i$. From the hypsography of the model domain, the layer thickness, $H_i$, can be derived from $V_i$. The parameters $C_{ih}$ in (4), $C_{ij}$ in (10), $\lambda$, $\gamma$ in (15) and $\mu$ in (16) are free parameters that can be used to calibrate the model. They control the strength of, respectively, the estuarine circulation, wind-driven entrainment and vertical mixing.

Initial values of $H_i$, $S_i$ and $T_i$ must be specified. From these, $V_i$ and $\rho_i$ are derived. The model uses an Euler forward scheme in time, with a time step of $\Delta t$ presently set to 0.1 days. A 5–day spin-up period is typically invoked. The tidal entrainment terms are calculated using a sub-time-stepping procedure to ensure stability. Values of the forcing data $Q_{it}$ and $Q_{st}$ are derived, or supplied, at a daily resolution. Values of $Q_{ij}$, $Q_{hi}$, $Q_{ij}$, $Q_{hi}$ and $Q_{ij}$ are calculated every time step. At each time step, the values of all the transport variables are first calculated using the most recent values of the layer properties. The layer volume and thickness, $V_i$ and $H_i$ are then updated, followed by $T_i$, $S_i$ and $\rho_i$. At the end of each simulated day, values of the layer properties and the daily-mean values of the transport terms are stored.

The model was coded in the Mathworks MATLAB® scripting language, and can be operated under Windows, Mac and-linux operating systems. The model is written in modular form, allowing easy modification and development of individual components of the code. A one-year simulation takes about two minutes.

2.4. Evaluation of model performance

The ability of the model to reproduce the observations is tested quantitatively. First, the model results are linearly regressed against the corresponding observed data and the coefficient of the
regression, \( r^2 \), is determined. Second, we calculate the root mean square error between model and data, given by

\[
E_{\text{RMS}} = \left[ \frac{1}{N} \left( \sum_{j=1}^{N} (p_j - o_j)^2 \right) \right]^{1/2}
\]

(19)

where \( p_j \) and \( o_j \) are predicted and observed values respectively and \( N \) is the number of data points in the calculation. Model skill is estimated using an index of agreement, \( d_2 \) (Willmott et al., 1985):

\[
d_2 = 1 - \frac{\sum_{j=1}^{N} (p_j - o_j)^2}{\left( \sum_{j=1}^{N} (|p_j - \bar{o}| + |o_j - \bar{o}|) \right)^2}
\]

(20)

where \( \bar{o} \) is the arithmetic mean of the observed data. A perfect agreement between model and data would give \( d_2 = 1 \), with decreasing values indicating declining performance.

2.5. Ensemble simulations

The model was run repeatedly, with values of \( C_C \), \( C_E \), \( \lambda \), \( \gamma \) and \( \mu \) chosen at random from prescribed ranges. For both modelled systems, Loch Creran and Loch Etive, an ensemble of 5000 members was performed. The ensemble approach allowed us to identify the parameter set that best reproduced the observations, and to investigate the sensitivity of the model to the empirical constants. The index of agreement, \( d_2 \), was calculated for temperature and salinity in each layer for every ensemble member. From these six values, an overall (average) skill for the ensemble member was derived, enabling the best parameter set to be identified for each inlet system modelled.

In addition to the ensemble approach, some simulations testing the sensitivity of the model to the boundary conditions were performed. The density of these fjordic RREs is dominated by salinity, which in turn is controlled by the open ocean boundary condition and the riverine freshwater input. Given the scarcity, in particular, of adequate coastal ocean data, we tested the response of the model to the open boundary condition by repeating the two simulations with constant coastal temperature and salinity values. The values chosen were the mean values of the data for 1978 and 2000 for the Creran and Etive simulations respectively. Thus a vertical gradient was maintained, but remained constant throughout the simulations. Similarly, the simulations were repeated with constant freshwater inputs (again, mean values for 1978 and 2000 were used), and finally, simulations were performed with both the open boundary conditions and river flows constant.

3. Observations

The model has been applied to two contrasting inlets in western Scotland: Loch Crean and Loch Etive (Fig. 2). These sites were chosen largely on the basis of data availability, which are required both to force the model and evaluate the results. Loch Creran has been the subject of a sporadic observation programme over four decades. We have chosen to model 1978, when one of the most comprehensive annual datasets was collected (Tyler, 1983). Loch Creran is a small inlet, about 13 km long, with a maximum water depth of 42 m (Table 2). It contains three sills, of which the entrance sill is the shallowest with a maximum depth of 7 m. Loch Creran has a relatively low freshwater input, with a relatively small watershed (164 km\(^2\)) compared to its surface area (15.1 km\(^2\)).

Freshwater discharge data into Loch Creran during 1978 were derived from gauged flow on the River Creran by Tyler (1983), accounting for the increased catchment area of the loch relative to the river and local variations in climatological rainfall between individual river catchments. Wind speed data were obtained from the Tiree meteorological station and atmospheric

Fig. 2. Locations of observation stations C2–C6 in Loch Creran and RE1–RE8 in Loch Etive on the west coast of Scotland. The sill in Loch Creran lies seaward of C2; the Bonawe sill in Loch Etive lies between RE5 and RE6. The locations of the monitoring station LY1, which provided boundary data for the Creran simulations, and the weather stations at Dunstaffnage and Tiree are also marked.
variables required for heat flux calculations were recorded at Dunstaffnage. The coastal temperature and salinity profiles came from station LY1 in the Firth of Lorn, which was sampled once or twice each month during 1978. The forcing data for Loch Creran from 1978 are shown in Fig. 3.

Model results were compared with field observations of water temperature and salinity. Full depth profiles of temperature and salinity at six fixed stations in Loch Creran (C1–C6, Fig. 2) were recorded at regular intervals throughout 1978 (Table 3). Here, we used data from 4 m (the shallowest depth consistently available), 10 m and 40 m depths, as representative of layers 1, 2 and 3 respectively, to compare against the model results. We did not use depth-averaged data since the thickness of each model layer was not known a priori. The arithmetic-mean of the data from all stations inside the loch (C2–C6) was calculated to give a basin-average value. Depth-integrated data could not be used because of the variable thickness of the model-predicted layers. Boundary forcing data consisted of profiles of temperature and salinity obtained at station LY1 (Fig. 2) during the same surveys of stations C2–C6.

Loch Etive is one of the larger Scottish inlets, with a length of 30 km and a maximum depth of 140 m (Table 2). It has six sills, the entrance sill being the shallowest with a maximum depth of 7 m. This sill, known locally as “The Falls of Lora”, has a strong choking effect on the tide, reducing the external spring tidal range from 3.6 m to 1.8 m inside the inlet. The deepest basin lies landward of the sixth sill, with the sill itself located between RES and RE6 (Fig. 2). The sill depth is 13 m. Loch Etive has a large watershed of 1350 km² compared to its surface area of 29 km², with much of the freshwater discharging via the River Awe close to station RES; the water in the inlet is more brackish (deep water salinity values are typically 27) than is usual in Scottish inlets.

From July 1999–March 2001, the REES experiment in Loch Etive was conducted by the Scottish Association for Marine Science (SAMS unpublished data). We used data from 2000 when the field programme continued throughout the calendar year, with regular (approximately monthly) sampling trips to Loch Etive (Table 3). During each trip, conductivity–temperature–depth (CTD) profiles were made and water samples collected at fixed station locations, named RE1–RE8 (Fig. 2). To force the model, gauged flow data from the River Awe during 2000 were adjusted to account for the whole Etive catchment. Wind speed data came from the Tiree meteorological station, atmospheric variables from Dunstaffnage, and the coastal temperature and salinity profiles came from REES CTD station RE6 (Fig. 2). The model was used to simulate only the upper basin of the loch, upstream of the sill at Bonawe. [Experiments to run the model for the whole of Loch Etive using data from RE8 as boundary forcing were not successful. The model failed to run due to the extreme flow speeds and

Table 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Loch Creran</th>
<th>Loch Etive</th>
</tr>
</thead>
<tbody>
<tr>
<td>L (km)</td>
<td>13.1</td>
<td>30.0</td>
</tr>
<tr>
<td>A₀ (km²)</td>
<td>15.1</td>
<td>16.1</td>
</tr>
<tr>
<td>Hmax (m)</td>
<td>42</td>
<td>140</td>
</tr>
<tr>
<td>Catchment (km²)</td>
<td>164</td>
<td>1350</td>
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<tr>
<td>Y₀ (m)</td>
<td>325</td>
<td>188</td>
</tr>
<tr>
<td>U₀ (m s⁻¹)</td>
<td>0.82</td>
<td>0.66</td>
</tr>
<tr>
<td>T (s)</td>
<td>44,712</td>
<td>44,712</td>
</tr>
<tr>
<td>a₀ (m)</td>
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</tr>
<tr>
<td>Cₜ</td>
<td>0.86</td>
<td>0.59</td>
</tr>
<tr>
<td>Cⱼ</td>
<td>4.0 × 10⁴</td>
<td>5.0 × 10⁴</td>
</tr>
<tr>
<td>μ</td>
<td>2.90</td>
<td>1.94</td>
</tr>
<tr>
<td>γ</td>
<td>0.10</td>
<td>0.28</td>
</tr>
<tr>
<td>λ</td>
<td>-2</td>
<td>-2</td>
</tr>
</tbody>
</table>

Fig. 3. Daily boundary data from 1978 used to force the physical exchange model for a simulation of Loch Creran: riverine freshwater discharge (Q_f, m³ s⁻¹), wind speed (W, m s⁻¹), net surface heat flux (Q_{SHF}, W m⁻²), external coastal temperature (Tₑ, °C), salinity (Sₑ) and density (ρₑ, kg m⁻³) at 4 depths (1, 10, 20, 30 m).
turbulence that occur over the sill at the Falls of Lora; the configuration and application of the model to the whole of Loch Etive remains a challenging problem. The forcing data are shown in Fig. 4.

Data from depths of 3 m (the shallowest data depth consistently available), 15 m and 110 m at RE1–RE5 were averaged to provide representative values for each layer. As with Loch Creran, we did not use depth-averaged data since the thickness of each model layer was not known a priori.

### Table 3

Sampling dates in each of the two simulated years for the two lochs, Creran and Etive. The date, and the Julian day, of each observation are given. See text for observation depths. The means of all available station data at each depth and sample date were used to generate a basin-average for comparison with the model predictions.

<table>
<thead>
<tr>
<th>Loch Creran, 1978</th>
<th>Loch Etive, 2000</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Date</strong></td>
<td><strong>J. day</strong></td>
</tr>
<tr>
<td>16 January 1978</td>
<td>16</td>
</tr>
<tr>
<td>30 January 1978</td>
<td>30</td>
</tr>
<tr>
<td>13 February 1978</td>
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<td>27 February 1978</td>
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<td>3 April 1978</td>
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<td>29 May 1978</td>
<td>149</td>
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<tr>
<td>12 June 1978</td>
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<td>3 July 1978</td>
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<td>17 July 1978</td>
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<td>9 October 1978</td>
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<td>25 October 1978</td>
<td>298</td>
</tr>
<tr>
<td>6 November 1978</td>
<td>310</td>
</tr>
<tr>
<td>18 December 1978</td>
<td>352</td>
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</tbody>
</table>

### 4. Results

#### 4.1. Model calibration, evaluation and sensitivity

Observed water temperatures from Loch Creran in 1978 were dominated by the mid-latitude seasonal cycle (Fig. 5). Surface temperatures ranged from 5.5 °C in February (Days 32–60) to almost 14 °C in summer. In the intermediate and deep layers, the temperature range was slightly less, with marginally warmer winter temperatures and cooler summer ones. However, observations at each depth co-varied strongly, and thermal stratification in the water column was evidently weak throughout the year. Little high-frequency variability in temperature was apparent from the observations, either due to inadequate sampling frequency or an absence of such variability. The observations indicate a smoothly varying temperature cycle over the year. The salinity of the water column was less affected by the seasonal cycle and was driven more by discharge events. The salinity data reveal three major freshening events during 1978, in January (Days 16–30), April (Day 93) and October–November (Days 282–310). These events clearly occurred in response to periods of increased river discharge (Fig. 3). Each event was clearly evident throughout the water column, and the salinity difference between surface and deep water was generally less than 2, again suggesting relatively weak stratification of the water column throughout the year.

The ensemble spread for the Loch Creran simulation was quite confined (Fig. 5), indicating that for this system the model was relatively insensitive to the free parameter values. The predicted temperature in particular, for all three model layers in Loch Creran, was very consistent between ensembles. Water temperature variability was dominated by the external coastal temperature and the seasonal heating cycle. The predicted salinity time series for Loch Creran showed greater ensemble spread, but were still quite tightly confined around the best simulation. As we shall see below, Loch Creran acts as a rather well-mixed box, and the internal dynamics

![Fig. 4. Daily boundary data from 2000 used to force the physical exchange model for a simulation of Loch Etive: riverine freshwater discharge ($Q_F$, m$^3$ s$^-1$), wind speed ($W$, m s$^-1$), net surface heat flux ($Q_{SHF}$, W m$^-2$), coastal temperature ($T_E$, °C), salinity ($S_E$) and density ($\rho$, kg m$^-3$) at 4 depths (1, 10, 20, 30 m) at location RE6.](image-url)
have rather little effect on its water properties. The system could conceivably be modelled accurately by a much simpler, single-box model, and the sensitivity of the model predictions to chosen parameter values is limited.

The model skill assessment suggests that the model predicted temperature better than salinity, with higher $d_2$ values and RMS errors that were a smaller fraction of the observed standard deviation (Table 4). In each layer, temperature was predicted with an RMS error of less than 1 °C. In late spring and summer, predicted temperatures in the surface and intermediate layers were slightly warmer than observed, with RMS errors of 0.63 °C and 0.73 °C respectively. But the model predictions compared well with data through autumn and winter, and overall the seasonal cycle was well reproduced. The deep water temperature was very accurately predicted throughout the year (RMS error = 0.24 °C).

The model largely reproduced the three major freshening events observed during 1978. In the surface and intermediate layers, the minimum predicted salinity was less than the minimum observed salinity, but the predicted minima occurred slightly earlier than the observed ones and it is possible that the discrepancy is due to under-sampling; the model salinity at the time of the observed minima matched the observed values closely. In the deep layer, the agreement between model and data during these events was less good, with the predicted salinities being too high. Overall, salinity in Loch Creran for 1978 was reproduced satisfactorily, with RMS errors less than 1, and skill score for the surface and intermediate layers exceeding 0.9 (Table 4). The model successfully reproduced the key features of the observed system: a strong seasonal cycle in temperature, major freshening events in the salinity records, and relatively weak thermal and haline stratification.

In Loch Etive during 2000, observations again revealed a strong seasonal cycle in both temperature and salinity in the surface and intermediate layers, both having lower values during winter and rising over spring and summer (Fig. 6). Over the seasonal cycle, the water temperature and salinity in the surface layer varied over ranges of 10 °C and 20 respectively; the seasonal variability of both parameters was slightly reduced in the intermediate layer. In the bottom layer, no seasonality was evident, with changes in water properties dominated by a deep water renewal event during days

![Fig. 5. Predicted temperature (a, c, e) and salinity (b, d, f) time series for three water layers in Loch Creran for 1978. All 5000 individual ensemble members are shown as thin grey lines. The solid black line indicates the overall 'best fit' simulation. Observed data, from Tyler (1983), are indicated by the open circles.](image)

![Fig. 6. Predicted temperature (a, c, e) and salinity (b, d, f) time series for three water layers in Loch Etive for 2000. All 5000 individual ensemble members are shown as thin grey lines. The solid black line indicates the overall 'best fit' simulation. Observed data are indicated by the open circles.](image)

### Table 4

<table>
<thead>
<tr>
<th></th>
<th>Temperature</th>
<th>Salinity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SD $r^2$ $\varepsilon_{\text{RMS}}$ $d_2$</td>
<td>SD $r^2$ $\varepsilon_{\text{RMS}}$ $d_2$</td>
</tr>
<tr>
<td><strong>Loch Creran 1978</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer 1</td>
<td>2.88 0.96 0.63 0.988 1.25 0.74 0.64 0.919</td>
<td></td>
</tr>
<tr>
<td>Layer 2</td>
<td>2.75 0.96 0.73 0.984 1.05 0.83 0.54 0.932</td>
<td></td>
</tr>
<tr>
<td>Layer 3</td>
<td>2.57 0.99 0.24 0.998 0.80 0.74 0.51 0.844</td>
<td></td>
</tr>
<tr>
<td><strong>Loch Etive 2000</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer 1</td>
<td>3.50 0.88 1.54 0.941 0.79 0.95 0.12 0.983</td>
<td></td>
</tr>
<tr>
<td>Layer 2</td>
<td>2.27 0.91 0.91 0.964 5.04 0.97 1.15 0.984</td>
<td></td>
</tr>
<tr>
<td>Layer 3</td>
<td>1.03 0.76 0.52 0.927 0.48 0.97 0.41 0.888</td>
<td></td>
</tr>
</tbody>
</table>
135–225; this event will be discussed further in Section 4.3. Before and after the renewal event, both temperature and salinity varied only slowly. In contrast to Loch Creran, therefore, Loch Etive exhibits classical fjordic characteristics, with strong vertical stratification and a deep layer isolated from the general circulation and surface heat and freshwater fluxes (Edwards and Edelsten, 1977).

The model captured all main features of the observed data (Fig. 6). The best overall model simulation (i.e. that with the highest skill), had a better skill for salinity than for temperature in the surface and intermediate layers (Table 4). The RMS errors for salinity in these two layers, while quite large in absolute terms, were about 22% of the observed standard deviation. For temperature, the RMS errors were 40% of the observed standard deviation. As for Loch Creran, the model tended to over-predict temperature in these layers during the spring and summer. In the deep layer, the model captured the major changes in temperature and salinity, including the absence of the seasonal cycle, indicating that the exchange between intermediate and bottom layers is parameterised accurately. The major discrepancy between model and data occurred in the over-prediction of salinity following the deep water renewal event. The RMS error for the bottom layer salinity of 0.41 was 85% of the observed standard deviation; thus the model skill for the bottom layer salinity was less than for the surface and intermediate layers.

A minor, but interesting, discrepancy between model and data during this simulation is found in the evolution of bottom water temperature before the renewal event (i.e. before Day 135). During the first 100 days of the simulation, the observed bottom temperatures increased slightly, whereas the modelled temperature decreased slightly (Fig. 6e). This observation suggests that a weak vertical temperature gradient existed in the deep layer, with temperature continuing to increase towards the seabed; the net effect of vertical diffusion was therefore to gradually increase the temperature at the observation depth. In the model, however, the bottom layer is by necessity treated as a homogeneous water mass, and the layer is gradually cooled by upward diffusion of heat to the intermediate layer. In contrast, the gradual reduction of salinity observed in the bottom layer, likewise caused by the upward diffusion of salt, is well reproduced by the model (Fig. 6f).

The simulation of Loch Etive shows more sensitivity to the free parameter set than Loch Creran, particularly in the intermediate layer for both temperature and salinity, and in the surface layer salinity. This is because the wind-stirred entrainment and density-driven circulation terms, which contain free parameters, play a greater role in the dynamics of the system. Sensitivity in the temperature and salinity of the bottom layer is negligible. This is because the water properties are largely modified only through the vertical mixing term $K_{23} = K_{25} + K_{26}$ (Equation (17)). But, although $K_{25}$ contains three free parameters, in the deep water vertical mixing is dominated by the internal wave mixing term, $K_{26}$, which does not contain any, and the exchange between intermediate and deep layers is determined by the internal tide energy flux.

### 4.2. Volume and salt fluxes

The volume transport terms in Loch Creran were dominated by the vertical mixing and tidal exchange terms (Fig. 7). The density-driven circulation $Q_d$ was relatively weak (Fig. 7a). The wind-driven entrainment flux was sporadic, and generally weaker than the tidal exchange (apart from a strong entrainment event during the last few days of the simulation, Fig. 7b). Tidal entrainment was negligible (Fig. 7c). The fortnightly spring–neap cycle was clearly evident in both the tidal exchange and the vertical mixing transport. The simulated density-driven circulation was stronger at the start of the year and during the first two freshwater discharge events, before becoming much weaker from Day 100 onwards.

The salt flux terms for each model layer (Fig. 7d–f) further indicate the predominant balance in Loch Creran between tidal exchange, vertical mixing and the density-driven circulation. Import of salt into the system throughout the year occurs mainly by tidal exchange into Layer 3 (Fig. 7f), and is balanced by an upward diffusive flux into Layer 2. This diffusive salt flux into Layer 2 is in turn balanced largely by diffusive transport into Layer 1, although the model attributes some upward salt flux to the density-driven circulation. In the surface layer, the diffusive input of salt is balanced by an export of salt due to the density-driven circulation. The model indicates that Loch Creran, during 1978 at least, was a weakly stratified, strongly mixed system, with internal salt fluxes dominated by vertical eddy diffusion.

Modelled volume and salt fluxes in Loch Etive 2000 (Fig. 8) exhibited several contrasting features to Loch Creran. The density-driven circulation was of a comparable magnitude to tidal exchange, though more irregular. There was a much greater exchange between Layers 1 and 2 due to wind-driven entrainment (Fig. 8b), and vertical mixing was much weaker (Fig. 8c). In particular, the exchange between Layers 2 and 3 due to mixing was very small; between Layers 1 and 2, mixing increased slightly during the deep
water renewal event, when stratification was weakened, but was otherwise minimal. The strong haline stratification of the water column, combined with the choked tidal range, evidently severely inhibited vertical mixing in Loch Etive.

The modelled salt flux terms reinforce the perception of Loch Etive as a 'classical' fjord. Tidal exchange imported salt into Layer 2 (except during the deep water renewal event), and salt was exported largely through wind-driven entrainment into Layer 1. Salt export from Layer 1 was dominated by the density-driven circulation. So the dynamical balance in the upper water column lay predominantly between advective (tidal and density-driven) exchange and wind-driven entrainment. Except during the renewal event, the bottom layer was largely isolated from salt balance dynamics, with only a weak diffusive flux modifying the bottom water salinity. During the renewal event, salt was imported into the bottom layer by tidal and density-driven exchange, and little salt was exported.

The contrasts between the two inlet systems are marked. The entrance flux due to wind stirring, $Q_h$, was much stronger in Loch Etive than in Loch Creran, despite the two water bodies having similar surface areas and experiencing not dissimilar wind stirring strength over the two simulated years. The values of the parameter $C_E$ for the ‘best’ simulations were also of similar magnitude (Table 2). The annual-mean modelled thickness of the surface layer in Loch Etive was only 60% of that in Loch Creran, allowing greater entrainment at the base of the surface layer (Equation (10)). Loch Etive is considerably more stratified all year round, due to its greater watershed and consequent higher freshwater input, and vertical mixing in the water column was strongly damped as a result. Most significantly, the deep water in Loch Etive was isolated from the general circulation and its water properties were modified only through vertical diffusion. Deep water isolation lasts, on average, 16 months (Edwards and Edelsten, 1977). In contrast, the deep water of Loch Creran was in open connection with the adjacent coastal ocean through tidal and density-driven exchange, and the water column was relatively weakly stratified due to strong vertical mixing.

### 4.3. Deep water renewal and exchange

A particular feature of the simulation for Loch Etive in 2000 is that the model successfully reproduced the observed deep water renewal that occurred in a series of events between May–August 2000 (days 115–225). The onset of renewal was marked in a number of model indicators: an increase in bottom layer thickness (Fig. 9a), and simultaneous sharp changes in bottom layer temperature and salinity (Fig. 9b and c). The model indicates that the renewal event began with an intrusion of cold, but less saline water, which was nevertheless of greater density than the ambient basin water (Fig. 9d). Unfortunately, observations are not available from the initial phase of the renewal event, and the predicted initial drop in bottom water temperature and salinity characteristics cannot be verified. The first observations obtained after the onset of the renewal event indicate that the water was indeed colder than the original bottom water, but at the time of the observation the salinity was already greater than the pre-renewal water (Fig. 6).

After this initial incursion, the salinity and temperature of the intruding water both increased throughout the remainder of the renewal event, until on 13 August 2000 (day 225), the salinity of the external water fell sharply and the renewal event ceased. The temperature of the external water continued to warm for another month, but not sufficiently to significantly affect the external water density. The temperature of the intermediate layer closely tracked the external values, whereas the intermediate layer salinity exhibited some dilution relative to the external values.

Prior to the deep water renewal event, water was entrained into the inflowing intermediate layer both from above and below (Fig. 9f). The entrainment from below led to the gradual erosion of the bottom layer, reducing its thickness by about 4 m over the 115 days before the event (Fig. 9a). The corresponding increase in layer 2 thickness was modified by wind-driven entrainment between layers 1 and 2, leading to minor fluctuations in the intermediate layer thickness superimposed on the increasing trend. During the renewal event, water was entrained from the intermediate layer into the intruding flow in the deep layer (i.e. $Q_{23} < 0$), whilst entrainment from the surface layer was set to zero during deep water renewal. The renewal event restored the bottom layer thickness to its original thickness before the entrainment fluxes reverted back to pre-renewal conditions when the event was complete.

A passive tracer, initialised in the bottom layer with a value of unity at the start of the simulation, was quickly flushed from the inlet following the onset of the renewal event (Fig. 9e). On the eve of the renewal event, on day 114, the bottom layer concentration was still greater than 0.98. By day 142, after 27 days of deep water exchange, the concentration has dropped to 1% of its initial value. The surface and intermediate layers exhibited a peak in tracer...
concentration during the renewal event, as tracer was lifted up into
the overlying water before being exported from the inlet. The tracer
time series provide an indication of the rapid replacement of bot-
tom water during renewal events.

4.4. Vertical mixing

The volume and salt fluxes discussed in Section 4.2 reveal a key
difference between the two modelled inlets. While the simulation
of Loch Creran in 1978 featured strong vertical mixing and ex-
change between the layers, diffusive fluxes in Loch Etive were
minimal during 2000. The stratifi-
cation in Loch Etive was consis-
tently stronger than that in Loch Creran (Fig. 10), due to the greater
input of freshwater per unit surface area of the inlet, and resulted in
bulk Richardson numbers that were two or three orders of mag-
nitudes greater (Fig. 10c and d). Vertical mixing coef-
ficients were therefore strongly damped in Loch Etive compared to Loch Creran
(Fig. 10e and f). In both inlets, the mixing between layers 2 and 3
was dominated by the background mixing term, K_{2b}, as the stratifi-
cation was sufficiently strong to largely suppress the shear-driven
component (Fig. 10g and h).

Annual-mean modelled diffusivities during 1978 for Loch Creran
were 11 cm² s⁻¹ and 14 cm² s⁻¹ for the upper and lower interfaces
respectively, whereas for Loch Etive in 2000 the corresponding
values were 0.02 and 0.04 cm² s⁻¹ respectively. The modelled
turbulent mixing was therefore on average roughly five hundred
times greater in Loch Creran than in Loch Etive.

The mean vertical diffusivity between layers 2 and 3 for Loch
Etive during autumn and winter, i.e. in the absence of the deep
water renewal event, can be converted into a vertical mixing rate of
0.35 m³ s⁻¹. This can be recalculated as an exchange rate (E = Q/V)
of 0.05 year⁻¹. By way of comparison, Edwards and Edelsten (1977)
calculated the vertical exchange rate in Loch Etive deep water
during stagnation periods from 1971 to 1973, and obtained values
of 0.2 year⁻¹ for salt and 0.5 year⁻¹ for temperature. Our value is
thus considerably lower than theirs, yet the modelled rate of
change of salinity during the period before and after renewal event
in 2000 compares well with the observations (Fig. 6f). This raises
the question, as yet unresolved, of whether the mixing regime in
Loch Etive has changed since the early 1970s, or whether the dif-
fERENCE in estimated diffusion rates is due to the different
methodologies.
4.5. Exchange rates and flushing times

Daily exchange rates were calculated for the above-sill water column \( E_{1,2} \), below-sill water \( E_3 \) and the whole inlet \( E_W \) as follows:

\[
E_{1,2} = \frac{|Q_G + (1 - \delta)Q_T + Q_T + Q_{K23}|}{(V_1 + V_2)}
\]

\[
E_3 = \frac{|\delta Q_T + Q_T + Q_{K23}|}{V_3}
\]

\[
E_W = \frac{|Q_G + Q_T + Q_T|}{(V_1 + V_2 + V_3)}
\]

(21)

The entrainment term, \( Q_{K23} \), is not included in these expressions since entrainment erodes the layer, rather than exchanging layer properties. The flushing times for each layer, \( T_{fl} \), are given by the reciprocal of the exchange rate e.g. \( T_{fl,2} = E_{1,2}^{-1} \).

The modelled exchange rate of the below-sill water in Loch Creran during 1978 was regular and periodic (Fig. 11a), which may be expected given the dominance of tidal forcing to the exchange process. The rates varied by a factor of about three over the fortnightly spring–neap cycle. Little seasonal variation was apparent. The exchange rate predicted for the bottom water was more rapid than that above sill depth because of the strong tidal forcing and vertical mixing, and the smaller water volume. Exchange of the upper water column also exhibited a strong spring–neap dependency, but was more variable due to the contribution made by the density-driven circulation. On occasions, exchange rates both above- and below-sill depth dropped almost to zero for short periods (Table 5), giving very large maximum above- and below-sill depth dropped almost to zero for short periods without deep water renewal, the upper water column had a median flushing time of 8 days (Table 5), whereas exchange of the deep water was extremely slow. During 2000, the flushing time of the bottom water was less than 10 days for 30% of the year, but for the rest of the year it exceeded 100 days (Fig. 11d). The median flushing time for the surface layers was 7.7 days, which compares remarkably closely to the tidal prism estimate of 7.6 days (Table 5). This agreement is fortuitous, but indicates that for Loch Etive, unlike Loch Creran, the tidal prism method can give a useful first order estimate of the flushing rate of above-sill water. The tidal prism method does not, however, give any indication of the variability of the flushing rate, which for Loch Etive was high. The median flushing time for the whole inlet (11.9 days), was shorter than the tidal prism estimate (13.9 days) because of the deep water renewal event that occurred during the year.

4.6. Sensitivity to boundary forcing

Simulations to assess the sensitivity to forcing data indicated that accurate, time-dependent open boundary data are essential to reproduce the internal evolution of inlet water properties over a seasonal cycle (Fig. 12). We consider mainly salinity here, since salinity dominates the water density and therefore inlet dynamics, and also because temperature is strongly influenced by the surface heat flux and is therefore less sensitive to the open boundary and river forcing. The results for both inlets demonstrate that bottom layer salinity during the simulated years was strongly dependent on the external salinity (Fig. 12). When the open boundary conditions were held constant, the model failed entirely to reproduce the evolution of bottom layer salinity seen in the baseline simulation (Fig. 12e and f). Notably, in the case of Loch Etive, the deep water renewal event was entirely missed, and the bottom water salinity steadily reduced throughout the year. In contrast, when the river forcing was held constant at its annual-mean value, the model still largely reproduced the baseline bottom water salinity time series in both systems.

The surface and intermediate layers in Loch Creran and Loch Etive responded rather differently to the forcing data. In Loch Creran, the modelled salinities in layers 1 and 2 were only weakly influenced by open boundary conditions but responded strongly to the modified river forcing. In Loch Etive, the surface layer was significantly affected by both river and open boundary forcing, but most strongly by the latter; the intermediate layer, however, was only weakly influenced by the river forcing, and strongly reflected the open boundary condition.

The modelled temperature (not shown) revealed similar dependencies on the open and river boundary conditions. The variability in the surface layer was less than for the salinity, since the
surface temperature was also strongly influenced by the surface heat flux. In the intermediate layer, temperature in both systems was strongly influenced by the open boundary and only weakly affected by river forcing. In the deep layer, the open boundary forcing dominated basin conditions. Overall, it is evident that realistic data for both the river inputs and the open boundary are required for the model to produce predicted time series that are comparable with observations (as was demonstrated for the baseline time series).

5. Discussion

Regular observations of temperature and salinity over a seasonal cycle in Loch Creran and Loch Etive, albeit collected over different years, revealed contrasting hydrography in the two systems, with the former being weakly stratified and the latter strongly stratified. The application of the ACExR model to these two fjordic RREs successfully reproduced the distinct hydrographic conditions, with RMS errors for temperature and salinity typically less than 1°C and of the order of 1 respectively. These RMS errors are comparable to other simple models of circulation and mixing in RREs for which error calculations have been published (e.g., Liungman, 2000; Babson et al., 2006). In addition, the model provided quantitative estimates of the dynamic processes that result from, and sustain, those hydrographic conditions, allowing us to characterise the systems. According to the model, Loch Creran behaved as a well-mixed box, with weak stratification maintained by strong vertical diffusion and volume exchange between the inlet and coastal waters dominated by tidal forcing. In contrast, Loch Etive exhibited classical fjordic three-layer hydrography and dynamics, with strong stratification, weak vertical diffusion and a density-driven circulation comparable in strength to tidal exchange.

The focus of this paper has been the application of the ACExR model to two fjordic RREs in Scotland. However, we believe the model can be applied to a wider range of RRE types such as rias, voes and lagoons. The model can operate with two or three layers, with or without a constricted entrance, and has been provisionally applied to Sandsound Voe, a sill-less inlet in the Shetland Islands off North Scotland. The model as currently conceived assumes that only the intermediate layer is tidally exchanged and that the surface layer only experiences tidal forcing through secondary mechanisms (entrainment and mixing). For well-mixed and partially-mixed estuaries, the model may be limited in its applicability without further refinement; certainly it is currently untested in those particular environments. In estuaries where flushing is dominated by strong river input, the traditional freshwater–salinity box models described by Officer (1980) may remain a better tool. However, Officer (1980) also describes the box model concept as a balance between an upstream flux of salt due to tidal exchange and the net circulation and a downstream flux due to river flow, which is not inconsistent with our conceptual model. A further modification required to make the model suitable for other types of RRE, particularly those in lower latitudes, is to incorporate evaporation and precipitation processes, the lack of which may have contributed to the slight over-prediction of surface layer temperatures in both simulations reported here.

In order to facilitate wider application of the model to other systems, the availability and influence of the open boundary data needs to be further considered. A simple sensitivity experiment described here revealed the importance of realistic boundary...
forcing to reproduce the evolution of internal water properties accurately, particularly in the deeper layers. This is not surprising given the connection between the tidally-exchanged (intermediate or bottom) layer and the external water. The use of a climatology of U.K. coastal temperature and salinity, developed by the U.K. Hydrographic Office, to drive the model is being investigated. The disadvantage of using a climatology is that short-term events, such as a freshening of coastal salinity due to high rainfall, which may strongly influence conditions inside coastal inlets, are lost in the averaging process. Whilst weakening direct comparisons between model predictions and observations, it is not clear however that such events, despite modifying internal water properties, have much impact on the flushing and exchange of the inlet, which remains the ultimate objective of the model. The exchange of the surface layers is dependent on both the density-driven circulation and tidal exchange terms (Equation (21)), and only the former is dependent on water properties, as is the vertical mixing term indirectly. The errors introduced into the model simulations of water properties and exchange rates by using a climatological boundary condition need to be assessed further before wider application can be justified. In addition, more case studies are being conducted, and a further option is to use output from the current generation of continental shelf hydrodynamic models (e.g. Wakelein et al., 2009) to drive the box model.

One of the key outcomes of the model is the ability to calculate the mean and daily variability of the flushing time of an RRE. The flushing time (or variations thereof, such as turnover time and residence time) is a key parameter used by coastal managers to try and ensure sustainable development of inshore waters. In both systems studied here, the predicted flushing times for the combined surface and intermediate layers were largely consistent with values estimated using the traditional tidal prism method (Dyer, 1997). The tidal prism method has generally been considered to overestimate exchange rates, due to the inherent assumptions of complete vertical mixing every tidal cycle and fully efficient exchange of water between consecutive ebb and flood tides. Our results indicate that, though simplistic, the prism-based resulting flushing times for Loch Creran and Loch Etive were not unreasonable. The inefficiencies of tidal exchange, ignored by the tidal prism method, are compensated to some extent by the additional exchange prompted by vertical mixing and the density-driven circulation. This provides some confidence in the historical application of the simple ECE model (Gillibrand and Turrell, 1997; Tett et al., 2003; Amundrud et al., 2009) used to advise on aquaculture development in Scotland.

Whereas the tidal prism method of calculating exchange provides a quasi-constant flushing rate (depending whether variability in tidal range is incorporated), the present model illustrates the temporal variability in the net volume exchange of the surface and intermediate layers. That variability ranged over two orders of magnitude, seasonally-varying information that would enable regulators and managers to make more informed decisions about assimilative capacity. For example, surface layer exchange may be weakest in summer, when river flows and wind stress are minimal; regulators may want to set aquaculture production limits based on summer exchange rates to reduce the potential for summer eutrophication. In the two inlets modelled here, the seasonal variation in flushing was rather small, reflecting the dominance of tidally-driven exchange. The coupling of the ACExR model with the LESV ecological box model (Portilla et al., 2009) will allow regulators to investigate and rapidly test the ecosystem response to different production limits (Tett et al., 2011). This approach of using simple models to manage aquaculture development has been adopted in Norway (e.g. Ervik et al., 1997; Stigebrandt et al., 2004). The ACExR–LESV models are written in MATLAB® scripts, making them easily available to academics and regulators with access to MATLAB software, with quick run-times on a desktop computer.

A wide variety of fluid dynamic processes occur in RREs, particularly where stratified tidal flow interacts with topography (Farmer and Freeland, 1983; Inall and Gillibrand, 2010). Representing some of these complex processes through simple analytical expressions inevitably leads to a loss of realism in model simulations and introduces errors into model-data comparisons. If improved understanding of those processes is required, or spatially-resolved maps of the water circulation desired, then the spatially-averaged ACExR model will not suffice; for those purposes, a full three-dimensional hydrodynamic model must be used. Nevertheless, effective resource management in these valuable environments and ecosystems requires appropriate tools that allow regulators and managers to control development at sustainable levels. Previous models used for this purpose, in Scotland for example (Gillibrand and Turrell, 1997), while having served their purpose, are no longer adequate (Amundrud et al., 2009), in particular because they did not capture seasonal and shorter-term variability in the exchange of the system, which has important implications because of the seasonal nature of emissions from aquaculture for example. Although still relatively simple, the ACExR model described here improves the representation of physical processes in these regions compared to previous models intended for regulation and management (e.g. Gillibrand and Turrell, 1997; Ross et al., 1993, 1994). The ACExR, coupled with the LESV model (Portilla et al., 2009; Tett et al., 2011), therefore offers a potentially significant step forward towards improved management tools for Regions of Restricted Exchange.

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