Abstract—This paper focuses on development of a wheelchair model and further on proposing a novel control strategy based on fuzzy logic control in order to lift the front wheels and maintain system stability in the upright position. A type-1 TSK FLS is first introduced to handle stability when the wheelchair is in an inverted pendulum-like situation. An extra control loop is designed to stop the motion after the system is stable. Then an interval type-2 TSK FLS is further developed by blurring the boundaries of the membership functions used in type-1 design. The effectiveness of the two type controllers is shown in the simulation results.

Index Terms—wheelchair; balancing; fuzzy logic; TSK, interval type-2;

I. INTRODUCTION

The iBOT is a powered wheelchair developed by Dean Kamen and other engineers at DEKA research in the 1990s [1-2]. It is designed to not only provide mobility for a seated handicapped passenger but also provide several unique advantages over a traditional electric wheelchair. One of the significant advantages is to help disabled people, who are using the wheelchair, reach certain level of height in confined spaces, for example, to put things on shelves or to have conversations with other people at eye-to-eye level.

In order to lift the chair to reach a higher level, the wheelchair has to be in the two-wheeled state, realized by lifting the front wheels (casters) of the wheelchair so that it achieves an upright position. The wheelchair when on two wheels performs as a double inverted pendulum and is thus characterized as a highly nonlinear complex, unstable system. Suitable control is therefore required to lift the chair and stabilize the two-wheeled wheelchair in the upright position.

There are not so many research literature describing about how to do modelling and control of a two-wheel wheelchair. A few examples of related work [3-5] produced by the same research group are given. [3] applies fuzzy logic control scheme to control a two-wheeled wheelchair system modelled as a double inverted pendulum. Three input variables were considered based on Mamdani-type fuzzy rules. [4] is an extension that adjusts the input/output scaling of the fuzzy logic controller using genetic algorithm. [5] introduces another algorithm using an LQR algorithm to regulate a wheelchair system about the upright equilibrium point.

A similar research topic like control of inverted pendulum on two-wheeled robot has been much more studied. [6] gives a survey for balancing and movement of two-wheeled robots either on flat or rough terrain. As described in [6], a variety of controllers that can handle stability of two-wheel robots has been investigated. Among non-linear control methods, backstepping is a common solution. [7] gives a good example where backstepping is used to control tilt angle along with a PD controller for longitudinal position. [8-9] are some examples of using sliding mode controller to simplify the dynamics by applying a large switching control input to force the system state to a desired hyper surface. Fuzzy control is another approach in which the theory is simple and the design is based on humans experience. [10] uses two fuzzy control loops, one for balancing and one for controlling position or speed. [11] adds one more loop for a yaw rotation control. Some other control techniques are optimal control [12-14] (including LQR, $H_{\infty}$, and $H_2$), neural network [15], and adaptive control [16]. A combination of these techniques is also quite popular [17].

Since one of the advantages of fuzzy logic control is an ability to dealing with uncertainties in situations where it is difficult to decide in an ambiguous manner, it is reasonable to apply fuzzy logic control in our problem in which precise knowledge of the dynamic models is lacking and uncertainties are presented. In this paper, we make use of fuzzy logic systems (FLS) to control stability of the wheelchair as in the upright position. Two types of fuzzy controllers are designed based on Takagi Sugeno and Kang (TSK). Type-1 TSK FLS handles uncertainties about the meaning of words (or uncertainties about the antecedents or consequents in rules) by using precise membership functions. On the other hand, type-2 TSK FLS handles such the uncertainties by blurring the boundaries of type-1 membership functions into a footprint of uncertainty (FOU). The effectiveness of the two controllers is compared in the simulation results.
II. NOMENCLATURE

The notations addressed in this paper are listed as follows:

- $m_1$: Equivalent mass of the cart (rear wheels)
- $m_2$: Equivalent mass of the pendulum (front wheels and moving platform)
- $l_1$: Radius of the rear wheel
- $l_2$: Distance from the center of the rear wheel to the end of front wheel
- $J_1$: Equivalent moment of inertia of the cart
- $J_2$: Moment of inertia of the pendulum
- $g$: Gravitational acceleration constant
- $z$: Displacement of the cart
- $\theta$: Angle of the pendulum
- $u$: Total force applied to the rear wheels

III. IBOT-LIKE WHEELCHAIR

A. Hardware design

Fig. 1 shows a standing platform of an iBOT-like wheelchair. The front wheels and rear wheels, connected by transmission chains, rotate simultaneously in the same direction. The wheelchair has three degrees of freedom, e.g., forward/backward, pitch, and yaw, controlled by five DC motors. The first two are used to drive the left rear wheel and the right rear wheel to move the wheelchair forward/backward, or rotate it clockwise/counterclockwise direction. The next two motors are for lifting the left and right arms to perform a wheelie, and the last one is for moving the center of gravity (COG) of the wheelchair to generate momentum to initiate the lifting movement.

To perform a two-wheel drive, the wheelchair is initially set to its four-wheel function. Then the movable platform, containing packs of batteries and motors, slides rearward causing the COG pushes toward the rear wheels. When the COG is over the rear wheels, the arms (including the front wheels) start lifting easily and at the same time the moving platform keeps sliding forward little by little until reaching the point where it can balance. Once the wheelchair is riding on the back wheels and the moving platform stops sliding, a balancing process can just begin. Fig. 2 presents the four transition phases of changing the wheelchair's COG.

B. Modelling of system dynamics

The non-linear dynamic model of the wheelchair is analyzed based on derivation via the Euler-Lagrange equation. With an assumption that there is no slip between the wheels and the ground. From inspection of Fig. 3, one constructs the differential equations as follows.

\[ \ddot{\theta} = \alpha - \beta \]  

where

\[ \alpha = \frac{\left( m_2 l_2 g \sin \theta \right) \left( m_1 + m_2 + \frac{J_1}{l_1^2} \right)}{\left( \left( m_2 l_2^2 + J_2 \right) \left( m_1 + m_2 + \frac{J_1}{l_1^2} \right) \right) - \left( m_2 l_2 \cos \theta \right)^2} \]

\[ \beta = \frac{\left( m_2 l_2 \cos \theta \right) u + \left( m_2 \frac{2}{l_2} \cos \theta \sin \theta \dot{\theta}^2 \right)}{\left( \left( m_2 l_2^2 + J_2 \right) \left( m_1 + m_2 + \frac{J_1}{l_1^2} \right) \right) - \left( m_2 l_2 \cos \theta \right)^2} \]

and

\[ \ddot{z} = \chi - \delta \]  

where

\[ \chi = \frac{\left( m_2 l_2^2 + J_2 \right) u + \left( \left( m_2 l_2^2 + J_2 \right) \left( m_2 l_2 \sin \theta \right) \dot{\theta}^2 \right)}{\left( \left( m_2 l_2^2 + J_2 \right) \left( m_1 + m_2 + \frac{J_1}{l_1^2} \right) \right) - \left( m_2 l_2 \cos \theta \right)^2} \]

\[ \delta = \frac{\left( m_2 \frac{2}{l_2} g \sin \theta \cos \theta \right)}{\left( \left( m_2 l_2^2 + J_2 \right) \left( m_1 + m_2 + \frac{J_1}{l_1^2} \right) \right) - \left( m_2 l_2 \cos \theta \right)^2} \]
Since the objective is to keep the pendulum upright, it seems reasonable to assume that $\theta(t)$ and $\dot{\theta}(t)$ will remain close to zero. The non-linear model is thus linearized, in order to simplify the analysis and design of controllers, using these approximations: $\sin \theta \approx \theta$, $\cos \theta \approx 1$, $\theta \dot{\theta} \approx 0$, $\theta \ddot{\theta} \approx 0$. (1), and (2), reduce to

$$\dot{\theta} = \tilde{a} - \tilde{\beta}$$

where

$$\tilde{a} = \frac{(m_1 + m_2 + \frac{J_1}{2})(m_2 l_2 g \theta)}{((m_2 l_2^2 + J_2)(m_1 + m_2 + \frac{J_1}{2})) - (m_2 l_2)^2}$$

$$\tilde{\beta} = \frac{(m_2 l_2 u)}{((m_2 l_2^2 + J_2)(m_1 + m_2 + \frac{J_1}{2})) - (m_2 l_2)^2}$$

and

$$\tilde{\delta} = \tilde{\chi} - \tilde{\delta}$$

Choosing the states $x_1 = \theta$, $x_2 = \dot{\theta}$, $x_3 = z$ and $x_4 = \dot{z}$, we obtain the following state model

$$\dot{x} = Ax + bu$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ K_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ K_3 & 0 & 0 & 0 \end{bmatrix} ; b = \begin{bmatrix} 0 \\ K_2 \\ 0 \\ K_4 \end{bmatrix}$$

and the parameter are

$$K_1 = \frac{(m_1 + m_2 + \frac{J_1}{2})(m_2 l_2 g \theta)}{((m_2 l_2^2 + J_2)(m_1 + m_2 + \frac{J_1}{2})) - (m_2 l_2)^2}$$

$$K_2 = \frac{-(m_2 l_2)}{((m_2 l_2^2 + J_2)(m_1 + m_2 + \frac{J_1}{2})) - (m_2 l_2)^2}$$

$$K_3 = \frac{-(m_2 l_2^2 g)}{((m_2 l_2^2 + J_2)(m_1 + m_2 + \frac{J_1}{2})) - (m_2 l_2)^2}$$

$$K_4 = \frac{(m_2 l_2^2) + J_2}{((m_2 l_2^2 + J_2)(m_1 + m_2 + \frac{J_1}{2})) - (m_2 l_2)^2}$$

IV. FUZZY LOGIC CONTROLLER DESIGN

This section discusses the design of FLS used to control stability and mobility of the wheelchair. Fig. 4 demonstrates the block diagram for controlling its pitch angle and for stopping the motion while maintaining stability on two-wheel.

It is obviously seen that the FLS is designed to handle the discrete time model obtained by discretizing (5) using a sampling rate of 1 KHz. The inputs to the upper FLS are the angle error $E_\theta(z)$ and the angular velocity $\theta(z)$, the force $U_\theta(z)$ is, meanwhile, defined as the output. The objective of the upper FLS is to generate some force that the wheelchair can incline to a desired angle $\theta_d(z)$. In this case, $\theta_d = 0$. For the lower FLS, the one-time step difference of the distance and the velocity is inputted to the FLS. The output is generated to stop the motion after reaching steady state. The sum of $U_\theta(z)$ and $U_z(z)$ is then applied to the wheelchair (rear wheels).

![Fig. 4. Block diagram of pitch and wheel direction control based on FL scheme.](image)

Since the purpose of this paper is to investigate the performance of type-1 TSK FLS and interval type-2 TSK FLS and compare them, the next sub-sections will give some brief overviews of type-1 TSK FLS and interval type-2 TSK FLS.

To simplify the problem, let us use the same type TSK FLS for the upper and lower FLS in the block diagram in Fig. 4.

A. Zero-order type-1 TSK FLS

Here we consider two zero-order type-1 TSK models with a rule base of 25 rules, each having 2 antecedents; the rules are expressed in Table I. The first model as highlighted is designed to get the desired angle, which is zero deg. in our work, and the second model is for stopping the motion of the wheelchair after reaching steady state.
TABLE I
TYPE-1 FUZZY RULES FOR CONTROLLING THE ANGLE AND DISPLACEMENT.

<table>
<thead>
<tr>
<th>Antecedents</th>
<th>NB</th>
<th>NS</th>
<th>ZO</th>
<th>PS</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>PM</td>
<td>PB</td>
<td>PB</td>
<td>PB</td>
<td>PBB</td>
</tr>
<tr>
<td>NS</td>
<td>PS</td>
<td>PM</td>
<td>PB</td>
<td>PB</td>
<td>PBB</td>
</tr>
<tr>
<td>ZO</td>
<td>NM</td>
<td>NS</td>
<td>ZO</td>
<td>PS</td>
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</tr>
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<td>PS</td>
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<tr>
<td>PB</td>
<td>NBB</td>
<td>NB</td>
<td>NM</td>
<td>NS</td>
<td>NBB</td>
</tr>
</tbody>
</table>

The antecedents shown in Fig. 5 are type-1 fuzzy sets; meanwhile, the consequents presented in Fig. 6 are crisp numbers. The output of type-1 TSK FLS is obtained as

\[ y_{TSK}(x) = \frac{\sum_{i=1}^{M} f^i(x) y^i(x)}{\sum_{i=1}^{M} f^i(x)} \]  \hspace{1cm} (6)

where \( y^i(x) \) is the output of the \( i^{th} \) rule and \( f^i(x) \) is rule firing level, defined as

\[ f^i(x) = \min \left[ \mu_{F_l^1}(x_1), \mu_{F_l^2}(x_2) \right] \]  \hspace{1cm} (7)

The parameter \( \mu_{F_l^k}(x_k) \) denotes the membership function of the antecedent \( k \) in the rule \( l \).

B. Interval Type-2 TSK FLS

Next, we consider a zero-order interval type-2 TSK model with the same rules and number of antecedents as in type-1 model. The only difference is the primary memberships of the inputs containing the uniformly shaded FOUs in the fuzzy sets as depicted in Fig. 7. Note that the secondary membership functions are interval sets.

Let \( \mu_{\tilde{F}_l^k}(x_k) \) and \( \nu_{\tilde{F}_l^k}(x_k) \) denote the lower and upper membership functions for \( \mu_{F_l^k}(x_k) \) where \( k = 1, 2 \) (number of antecedents) and \( l = 1, 2, 3, \ldots, 25 \) (number of rules). For an interval type-2 TSK FLS, the result of input and antecedents operations is an interval type-1 set, the firing interval, which is a set of \( [f^i_l, \tilde{f}^i_l] \) determined by using a definition of minimum t-norm as

Fig. 5. Primary membership functions of the antecedents.

Fig. 6. The consequents.

Fig. 7. Pictorial representation of a type-2 fuzzy set.
\( f^i(x) = \min \left[ \mu_{F_1}(x), \mu_{F_2}(x) \right] \) (8)

\( \overline{f}^i(x) = \min \left[ \mu_{F_1}(x), \mu_{F_2}(x) \right] \)

The fired output consequent \( \mu_{\overline{B}}(y) \) of rule can be obtained from the fuzzy rules demonstrated in Table I. For type reduction, and interval set determined by its two end points can be expressed as

\[
y_l = \frac{\sum_{i=1}^{M} f^i y^i_l}{\sum_{i=1}^{M} f^i}
\]

\[
y_r = \frac{\sum_{i=1}^{M} f^i y^i_r}{\sum_{i=1}^{M} f^i}
\]

where \( M \) is 25 in this design. We finally defuzzify the interval set using the average of

\[
y(x) = \frac{y_l + y_r}{2}
\] (10)

V. SIMULATION RESULTS

This section presents simulation results using the parameters of the dynamic model given in Table II.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_1 )</td>
<td>3.2 kg</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>36.176 kg</td>
</tr>
<tr>
<td>( l_1 )</td>
<td>0.1450 m</td>
</tr>
<tr>
<td>( l_2 )</td>
<td>0.4025 m</td>
</tr>
<tr>
<td>( J_1 )</td>
<td>0.025024</td>
</tr>
<tr>
<td>( J_2 )</td>
<td>1.73634</td>
</tr>
<tr>
<td>( g )</td>
<td>9.81 m/s^2</td>
</tr>
</tbody>
</table>

With an assumption that the initial angle is at -5 deg, the performance of the two proposed controllers is compared and shown in Fig. 8. The plot in the upper left corner shows how good they can track the desired angle. It is obviously seen that both of them can track the desired angle within a few seconds. However, type-1 seems to have 35% lower %OS within 1.3 times longer settling time. Based on the current information, it is unclear to summarize which one performs better. The plot in the lower left corner gives us more information about the displacement of the wheelchair. Type-2 stops moving around 0.38 m from the original position, type-1, on the other hand, stays steady at 0.43 m from the original position. As a result, type-2 performs better in the sense that it can reach the steady state and stop moving faster. The angular velocity and linear velocity of the wheelchair are demonstrated in the upper right and lower right corners, respectively.

Fig. 8. Comparison of type-1 and type-2 TSK FLS with the initial angle at -5 deg.

Fig. 9 illustrates the tracking error of the angle (upper) and the total force applied to the wheelchair to get the desired response (lower). As seen in Fig. 8, type-1 and type-2 perform not much different for tracking the angle according to the angle error plot but type-2 requires a more significant number of force to push the angle back to zero and reach steady state. Note that the maximum required force for type-1 is about 40N and for type-2 is around -70 N. One might need to consider if it is reasonable to use type-2, requiring more force but less settling time, in the real world applications.

Fig. 9. Angle error (upper) and force applied to the wheelchair (lower) corresponding to Fig. 8.

Since the steady state responses of the angle using type-1 and type-2 are exactly the same, let us consider the comparison of transient responses using both controllers. For different initial angles, Table III summarizes the characteristics of the transient responses comparing between the two controller designs. The performance of type-1 is shown in the shaded
area. Type-2 gives a little slower rise time and a bigger size of %OS with a noticeably better performance in settling time and displacement in all cases. However, more force is required to get the desired response in a faster time.

**TABLE III**

Performance indicators in transient responses of the angle at different initial angles.

![Table](image)

**VI. CONCLUSION**

The iBOT wheelchair allows users to raise themselves to eye level, climb stairs and gives people with disabilities a greater sense of independence and freedom to go where they want to go. The purpose of this paper is to design and construct fuzzy controllers to stabilize an inverted pendulum-like wheelchair to provide a performance like the iBOT. The controllers based on a type-1 TSK FLS and an interval type-2 TSK FLS are designed in order to get the desired angle and stop the motion after reaching steady state. The performances using both controller designs are demonstrated and compared. Type-2 can reach the steady state faster but require higher exerted force to balance the wheelchair. One needs to consider the hardware limitation when implementing the design in the real world.

**ACKNOWLEDGMENT**

This research project is supported The National Teseach Council of Thailand (NRCT) and The King Mongkut's University of Technology Thonburi, Department of Mechanical Engineering for supporting in design and create Wheelchair Based and The Department of Control System and Instrumentation Engineering for supporting Scholarships for Self-Balancing Robot Research for the Master of Engineering(Electrical Engineering) Program Thai Master degree, Department of Electrical Engineering, King Mongkuts University of Technology.

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