ABSTRACT

The frequency-response masking (FRM) approach was introduced as a means of generating narrow transition band linear-phase FIR filters with a low arithmetic complexity. This paper proposes an approach for synthesizing modulated maximally decimated FIR filter banks utilizing the FRM technique. For this purpose, a new class of FRM filters is introduced. Filters belonging to this class are used for synthesizing nonlinear-phase analysis and synthesis filters. Each of the analysis and synthesis filter banks is realized with the aid of two filters, a cosine modulation block, and a sine modulation block. The overall filter bank is a near perfect reconstruction (NPR) filter bank. The distortion function has a linear phase response but a small magnitude distortion. Further, the filter bank has small aliasing errors. The magnitude distortion and aliasing errors can be made arbitrarily small by properly designing a prototype filter. Compared with conventional cosine modulated filter banks, the proposed ones lower significantly the overall arithmetic complexity at the expense of a slightly increased overall filter bank delay in applications requiring narrow transition bands. Examples are included illustrating the benefits provided by the proposed filter banks.

1. INTRODUCTION

Maximally decimated filter banks find application in numerous areas [1]–[3]. Over the past two decades, a vast number of papers on the theory and design of such filter banks have been published. The attention has to a large extent been paid to the problem of designing perfect reconstruction (PR) filter banks. In a PR filter bank, the output sequence of the overall system is simply a shifted version of the input sequence. However, filter banks are most often used in applications where small errors (emanating from quantizations etc.) are inevitable and allowed. Imposing PR on a filter bank is then an unnecessarily severe restriction which may lead to a higher arithmetic complexity than is actually required to meet the specification at hand. To reduce the complexity one should therefore use near-PR filter banks. For example, it is demonstrated in [4], [5] that the complexity can be significantly reduced by using near-PR instead of PR filter banks.

In this paper, we introduce a class of modulated FIR filter banks utilizing the FRM approach. Modulated filter banks are known to be very efficient since each of the analysis and synthesis parts can be implemented with the aid of one filter and a discrete cosine (or sine) transform [2]. However, when the transition bands of the filters are narrow, the overall complexity may still be high because the order of an FIR filter is inversely proportional to the transition band width [6]. To alleviate this problem, one can use the frequency-response masking (FRM) approach which was introduced as a means of generating narrow transition band linear-phase FIR filters with a low arithmetic complexity [7]–[10]. However, to make the FRM technique suitable for the proposed filter banks, we introduce in this paper a modified class of FRM filters. This modified class has been considered recently in [11], [12], but not in the context of M-channel filter banks. The main difference between these modified FRM filters and the traditional FRM filters is that the new ones have a nonlinear-phase response whereas the traditional ones have an exact linear-phase response. The proposed FRM filters are used as prototype filters for the analysis and synthesis filters in the proposed modulated filter banks. In these banks, each of the analysis and synthesis filter banks is realized with the aid of two filters, a cosine modulation block, and a sine modulation block. The main reason for using the modified FRM filters in the proposed modulation scheme is that the corresponding filter bank structure requires a lower arithmetic complexity than when conventional FRM filters are used. Specifically, one can eliminate the need for an additional cosine modulation block [13].

The proposed overall filter bank is a near-PR filter bank. The distortion function has a linear phase response but a small magnitude distortion. Further, small aliasing errors are present. The magnitude distortion and aliasing errors can however be made arbitrarily small by properly designing the prototype filter. Compared with conventional cosine modulated filter banks, the proposed ones lower significantly the overall arithmetic complexity at the expense of a slightly increased overall delay in applications requiring narrow transition bands. Efficient structures are introduced for implementing the proposed filter banks and procedures are described for optimizing the filter banks in the minimax sense. Examples are included illustrating the benefits provided by the proposed filter banks.

**Figure 1:** M-channel maximally decimated filter bank.
banks.

It should be noted that cosine modulated FIR filter banks utilizing conventional FRM filters recently were considered in [14]. A main advantage of using the proposed modulated filter banks is that the additional upsamplers and downsamplers that are present in the filter banks in [14] then are avoided.

2. FRM APPROACH

This section reviews the conventional FRM approach for generating lowpass linear-phase filters. The proposed modified FRM filters are introduced in the subsequent section.

In the frequency-response masking (FRM) approach, the transfer function of the overall filter is expressed as [7]–[10]

\[ H(z) = G(z^L)F_0(z) + G_c(z^L)F_1(z) \]  

(1)

where \( G(z) \) and \( G_c(z) \) are referred to as the model filter and complementary model filter, respectively. The filters \( F_0(z) \) and \( F_1(z) \) are referred to as the masking filters which extract one or several passbands of the periodic model filter \( G(z^L) \) and periodic complementary model filter \( G_c(z^L) \). The structure is illustrated in Fig. 2. For a lowpass filter, typical magnitude responses for the model, masking, and overall filters are as shown in Fig. 3. The transition band of \( H(z) \) can be selected to be one of the transition bands provided by either \( G(z^L) \) or \( G_c(z^L) \). We refer to these two different cases as Case 1 and Case 2, respectively. Further, we let \( \omega_T, \delta_T, \delta_s \), and \( \delta_c \) denote the passband edge, stopband edge, passband ripple, and stopband ripple, respectively, for the overall filter \( H(z) \). For the model and masking filters \( G(z) \), \( G_c(z) \), \( F_0(z) \), and \( F_1(z) \), additional superscripts \( (G) \), \( (G_c) \), \( (F_0) \), and \( (F_1) \), respectively, are included in the corresponding ripples and edges.

The FRM approach was first introduced in [8] as a means to reduce the arithmetic complexity of linear-phase FIR filters with narrow transition bands. In this case, \( G(z) \) and \( G_c(z) \) are even-order linear-phase filters of equal delays and form a complementary filter pair, whereas both \( F_0(z) \) and \( F_1(z) \) are either even- or odd-order linear-phase filters of equal delays. These filters can be used directly to generate the analysis and synthesis filters in the proposed modulated filter bank scheme to be considered in the following section, but the result is that each of the analysis and synthesis filter bank then requires three modulation blocks. Therefore, we introduce in the next section modified FRM filters that enables us to use only two modulation blocks. These modified FRM FIR filters have been considered recently in [11], [12], but not in the context of \( M \)-channel filter banks.

3. PROPOSED FILTER BANKS

This section gives transfer functions, properties, and realizations of the proposed filter banks.

3.1 Prototype Filter Transfer Functions

For the proposed modulated filter banks, the transfer functions of the analysis and synthesis filters are generated from the prototype filter transfer functions \( P_a(z) \) and \( P_s(z) \), respectively. These transfer functions are given by

\[ P_a(z) = G(z^L)F_0(z) + G_c(z^L)F_1(z) \]  

(2)

\[ P_s(z) = G(z^L)F_0(z) - G_c(z^L)F_1(z) \]  

(3)

Here, \( L \) and the subfilters \( G(z) \), \( G_c(z) \), \( F_0(z) \), and \( F_1(z) \) are selected to satisfy the following criteria:

1) \( L \) is an integer related to \( M \) as

\[ L = \begin{cases} 4M + 1, & \text{Case 1} \\ 4M - 1, & \text{Case 2} \end{cases} \]  

(4)

2) The model filters \( G(z) \) and \( G_c(z) \) are of odd order \( N_G \) and linear-phase FIR filters with symmetrical and anti-symmetrical impulse responses, respectively, with \( G_c(z) \) being related to \( G(z) \) as

\[ G_c(z) = G(-z) \]  

(5)

3) The masking filters \( F_0(z) \) and \( F_1(z) \) are of even order \( N_F \) and linear-phase lowpass filters with symmetrical impulse responses.
With the above selections, the following properties can readily be shown (proofs are given in the appendix):

4) The magnitude responses of $P_a(z)$ and $P_s(z)$ are equal, i.e.,

$$|P_a(e^{j\omega T})| = |P_s(e^{j\omega T})|$$

(6)

5) The filter $P_a(z)P_s(z)$ has a linear phase response.

### 3.2 Analysis and Synthesis Filter Transfer Functions

The analysis filters $H_{ak}(z)$ and synthesis filters $H_{sk}(z)$ are obtained by modulating the prototype filters $P_a(z)$ and $P_s(z)$ according to

$$H_{ak}(z) = \beta_k P_a(z^{W_{2M}^{(k+0.5)}}) + \beta_k^* P_a(z^{-W_{2M}^{(k+0.5)}})$$

(7)

and

$$H_{sk}(z) = cf(-1)^k[\beta_k P_a(z^{W_{2M}^{(k+0.5)}}) - \beta_k^* P_s(z^{-W_{2M}^{(k+0.5)}})]$$

(8)

respectively, for $k = 0, ..., M-1$ with

$$c = \begin{cases} 
1, & N_G + 1 = 4m \\
-1, & N_G + 1 = 4m + 2
\end{cases}$$

(9)

for some integer $m$. Case 1, and

$$W_M = e^{-j2\pi M}, \quad \beta_k = W_{2M}^{(k+0.5)}N_F/2$$

(10)

For Case 2, (9) is negated. Note that this type of modulation is slightly different from the one that is usually employed in cosine modulated filter banks [2]. The main reason for this is that our prototype filters have a nonlinear phase response whereas the conventional ones have a linear phase response.

### 3.3 Distortion and Aliasing Transfer Functions

For the $M$-channel maximally decimated filter bank in Fig. 1 the $z$-transform of the output signal is given by

$$Y(z) = \sum_{m=0}^{M-1} V_m(z)X(z^{W_M^m})$$

(11)

where

$$V_m(z) = \sum_{k=0}^{M-1} H_{ak}(z^{W_M^m})H_{sk}(z)$$

(12)

Here, $V_0(z)$ is the distortion transfer function whereas the remaining $V_m(z)$’s are the aliasing transfer functions. For a PR (near-PR) filter bank, it is required that the distortion function is (approximately) a delay, and that the aliasing components are (approximate) zero. By inserting the expressions for $H_{ak}(z)$ and $H_{sk}(z)$ as given by (7) and (8) into (11) and (12), we obtain expressions for all $V_m(z)$’s that are useful in the filter design. Details are given in [13].

### 3.4 Filter Bank Properties

This subsection gives three important properties of the proposed filter banks. Proofs are given in the appendix.

1) The magnitude responses of $H_{ak}(z)$ and $H_{sk}(z)$ are equal, i.e.,

$$[H_{ak}(e^{j\omega T})] = [H_{sk}(e^{j\omega T})]$$

(13)

2) The distortion function has a linear phase response with a delay of $LN_F+N_F$ samples.

3) The filter bank is readily designed so that: a) the analysis and synthesis filters are arbitrarily good frequency selective filters, and b) the magnitude distortion and aliasing errors are arbitrarily small.

### 3.5 Filter Bank Structures

We consider first the analysis filter bank structure. We begin by expressing $G(z)$ and $G_c(z)$ in polyphase forms according to

$$G(z) = G_0(z^2) + z^{-1}G_1(z^2)$$

(14)

$$G_c(z) = G_0(z^2) - z^{-1}G_1(z^2) = G(-z)$$

We can now rewrite $P_a(z)$ in (2) as

$$P_a(z) = G_0(z^{2L})A(z) + z^{-L}G_1(z^{2L})B(z)$$

(15)

where

$$A(z) = F_0(z) + F_1(z), \quad B(z) = F_0(z) - F_1(z)$$

(16)

The analysis filters $H_{ak}(z)$ can now be written as

$$H_{ak}(z) = G_0(-z^{2L})A_k(z) + s(-1)^kz^{-L}G_1(-z^{2L})B_k(z)$$

(17)

where

$$A_k(z) = \beta_k A(z^{W_{2M}^{(k+0.5)}}) + \beta_k^* A(z^{-W_{2M}^{(k+0.5)}})$$

$$B_k(z) = \beta_k B(z^{W_{2M}^{(k+0.5)}}) - \beta_k^* B(z^{-W_{2M}^{(k+0.5)}})$$

(18)

and

$$s = \begin{cases} 
1, & \text{Case 1} \\
-1, & \text{Case 2}
\end{cases}$$

(19)

Let $a(n)$, $b(n)$, $a_k(n)$, and $b_k(n)$ denote the impulse responses of $A(z)$, $B(z)$, $A_k(z)$ and $B_k(z)$, respectively. We then get from (10) and (18) that $a_k(n)$ and $b_k(n)$ are related to $a(n)$ and $b(n)$ through

$$a_k(n) = 2a(n)\cos\left(\frac{(2k+1)\pi}{2M} \left( n - \frac{N_F}{2} \right) \right)$$

$$b_k(n) = 2jb(n)\sin\left(\frac{(2k+1)\pi}{2M} \left( n - \frac{N_F}{2} \right) \right)$$

(20)

Since $b_k(n)$ is purely imaginary, $H_{ak}(z)$ is obviously the transfer function of a filter with a real impulse response. It can be written as

$$H_{ak}(z) = G_0(-z^{2L})A_k(z) - s(-1)^kz^{-L}G_1(-z^{2L})B_k(z)$$

(21)

where

$$B_{kr}(z) = -jB_k(z)$$

(22)

Through a similar derivation as above, the synthesis filters $H_{sk}(z)$ can be rewritten as

$$H_{sk}(z) = \left(-1\right)^L G_0(-z^{2L})B_{kr}(z) + sz^{-L}G_1(-z^{2L})A_k(z)$$

(23)
The specifications in (24)–(26) are met when $\delta \leq \delta_1$. The adjustable parameters in (27) are the filter coefficients of the model filter $G(z)$ and masking filters $F_0(z)$ and $F_1(z)$, and $\delta$.

The problem in (27) is a non-linear optimization problem and therefore requires a good initial solution. For this purpose, we have derived formulas that enables one to separately optimize $G(z)$, $F_0(z)$, and $F_1(z)$ so that (24)–(26) are “almost satisfied”. These filters serve as good initial filters for further optimization according to (27). Details are given in paper [13].

5. DESIGN EXAMPLE

As a means of demonstrating the proposed design method, a 5-channel cosine modulated filter bank is designed. The specifications of $H_i(z)$ and $V_m(z)$ in (24)–(26) are the following: $\delta_1 = 0.01$, $\delta_i = 0.01$, $\delta_0 = 0.01$, $\delta_1 = 0.005\pi$. The specifications are met with filter orders $N_G = 43$ and $N_{F0} = N_{F1} = 58$. Magnitude responses of the analysis filters, distortion function, and aliasing functions are plotted in Figs. 7–9. The overall filter bank (including the analysis and synthesis parts) requires 91.2 multiplications per input/output sample plus the cost to implement the cosine and sine modulation blocks. With further optimization, the filter orders and complexities can be reduced. Details of this and more examples will be given in future work, [13].

As a comparison, a regular linear-phase FIR filter fulfilling the frequency selectivity only, would need a filter order of about 400. Therefore, at least about 160 multiplications per input/output sample is needed in the filter part of a regular NPR FIR cosine modulated filter bank. As usual when employing the FRM approach, we achieve more savings when the transition band becomes more narrow. The price to pay for the decreased complexity is, as always when using the FRM approach, a longer overall
delay. In this example, the delay is about 75% longer for the proposed filter bank compared with the regular filter bank. This figure is higher than for conventional FRM filters [7]. It is explained by the fact that L in our case is restricted according to (4). Without this restriction, there is more freedom to choose L so that the delay of the minimum-complexity FRM filter is only slightly higher than for a regular FIR filter.

6. CONCLUSION

This paper introduces an approach for synthesizing modulated maximally decimated FIR filter banks using the FRM technique. For this purpose, a new class of FRM filters was introduced. Each of the analysis and synthesis filter banks is realized with the aid of two filters, a cosine modulation block, and a sine modulation block. The overall filter bank is a near perfect reconstruction (NPR) filter bank.

Compared with regular cosine modulated filter banks, the proposed ones lower significantly the overall arithmetic complexity at the expense of an increased overall filter bank delay in applications requiring narrow transition bands. This was demonstrated by means of a design example.

APPENDIX

This appendix shows some of the properties of the filter bank focusing on the prototype filters, the analysis filters and the synthesis filters.

We first regard the magnitude response of the prototype filters and the phase response of \( P_a(e^{j\omega T}) \) (properties 4 and 5 in 3.1). The frequency responses of \( G(e^{j\omega T}) \), \( G_c(e^{j\omega T}) \), \( F_0(e^{j\omega T}) \), and \( F_1(e^{j\omega T}) \) can be written as

\[
G(e^{j\omega T}) = e^{-jN_c \omega T/2}G_R(\omega T)
\]

\[
G_c(e^{j\omega T}) = je^{-jN_c \omega T/2}G_{cR}(\omega T)
\]

\[
F_0(e^{j\omega T}) = e^{-jN_r \omega T/2}F_{0R}(\omega T)
\]

\[
F_1(e^{j\omega T}) = e^{-jN_r \omega T/2}F_{1R}(\omega T)
\]

where the subscript \( R \) denotes zero-phase frequency response. We rewrite the magnitude responses of the prototype filters (2), (3).

\[
P_a(e^{j\omega T}) = G(e^{jL \omega T})F_0(e^{j\omega T}) + G_c(e^{jL \omega T})F_1(e^{j\omega T})
\]

\[
= e^{-j(N_cL+N_r) \omega T/2}(G_R(\omega T) + jG_{cR}F_{1R})
\]

\[
P_s(e^{j\omega T}) = G(e^{jL \omega T})F_0(e^{j\omega T}) - G_c(e^{jL \omega T})F_1(e^{j\omega T})
\]

\[
= e^{-j(N_cL+N_r) \omega T/2}(G_R(\omega T) - jG_{cR}F_{1R})
\]

From (29) it follows that the squared magnitude response of the two prototype filters are, leaving out \( \omega T \) and \( L\omega T \) for a shorter presentation,

\[
|P_a(e^{j\omega T})|^2 = G_R^2F_{0R}^2 + G_{cR}^2F_{1R}^2 = |P_s(e^{j\omega T})|^2
\]

The product of the two magnitude responses has linear phase, as can be seen in (31).

\[
P_a(e^{j\omega T})P_s(e^{j\omega T}) = e^{-j(N_cL+N_r) \omega T}
\]

\[
\cdot (G_RF_{0R} + jG_{cR}F_{1R})(G_RF_{0R} - jG_{cR}F_{1R})
\]

\[
= e^{-j(N_cL+N_r) \omega T}(G_R^2F_{0R}^2 + G_{cR}^2F_{1R}^2)
\]

Second we show that the magnitude responses of the analysis filters and the synthesis filters are equal, and that the product of \( H_{ak}(e^{j\omega T}) \) and \( H_{sk}(e^{j\omega T}) \) has the linear phase \( LN_G + NF_r \) (properties 1 and 2 in 3.4). We use the notation \( H(z^k) \) which stands for

\[
H\left( e^{j\frac{2\pi z^k}{2M}} \right)
\]

and rewrite the transfer functions of the analysis filters, (7), and the analysis filters, (8), as

\[
H_{ak}(z) = \beta_k P_k(z) + \beta_{ak} P_{k}(z)
\]

\[
= \beta_k[G^{(k)}(z)LF_{0k}(z) + G_c^{(k)}(z)LF_{1k}(z)]
\]

\[
+ \beta_{ak}[G^{(k)}(z)LF_{0k}(z) + G_c^{(k)}(z)LF_{1k}(z)]
\]

\[
H_{sk}(z) = cj(-1)^j[\beta_k P_{k}(z) - \beta_{ak} P_{k}(z)]
\]

\[
= cj(-1)^j[\beta_k G^{(k)}(z)LF_{0k}(z) + G_c^{(k)}(z)LF_{1k}(z)]
\]

\[
+ \beta_{ak}[G^{(k)}(z)LF_{0k}(z) - G_c^{(k)}(z)LF_{1k}(z)]
\]

We use the expressions
This gives us a relation between $G(z)$ and $G_c(z)$:

$$G^{(k)}(z) = G_0(-z^2L) \pm j(-1)^k z^{-L}G_1(z^2L)$$

$$G^{(k)}_e(z) = G_0(-z^2L) \pm j(-1)^k z^{-L}G_1(z^2L)$$

$$G^{(k)}_c(z) = G_0(-z^2L) \mp j(-1)^k z^{-L}G_1(z^2L)$$

(36)

Now we rewrite the transfer function of the analysis and synthesis filters as

$$H_{ak}(z) = G^{(k)}(z)\left[\beta_k F_0^{(k)}(z) + \beta_c F_1^{(k)}(z)\right]$$

$$+ G^{(k)}_e(z)\left[\beta_k^e F_0^{(k)}(z) + \beta_c^e F_1^{(k)}(z)\right]$$

$$H_{sk}(z) = (G^{(k)}_e(z)\left[\beta_k^e F_0^{(k)}(z) + \beta_c^e F_1^{(k)}(z)\right] - G^{(k)}_c(z)\left[\beta_k^c F_0^{(k)}(z) + \beta_c^c F_1^{(k)}(z)\right] c j(-1)^k$$

(38)

We use (38) to write their frequency responses as

$$H_{ak}(e^{j\omega T}) = e^{\frac{j\pi(k+0.5)\omega}{N_G}} G^{(k)}(F_{0R}^{(k)} + F_{1R}^{(k)})$$

$$+ j G^{(k)}_e(F_{0R}^{(k)} + F_{1R}^{(k)}) e^{\frac{j\pi(k+0.5)\omega}{N_G}}$$

$$H_{sk}(e^{j\omega T}) = c j(-1)^k e^{\frac{j\pi(k+0.5)\omega}{N_c}}$$

$$[G^{(k)}(F_{0R}^{(k)} + F_{1R}^{(k)}) - j G^{(k)}_e(F_{0R}^{(k)} + F_{1R}^{(k)})]$$

$$e^{2\frac{j\pi(k+0.5)\omega}{N_c}}$$

(39)

(40)

From this, it follows that the magnitude of the frequency responses are equal, as can be seen in (41).

$$H_{ak}(e^{j\omega T}) = |G^{(k)}(F_{0R}^{(k)} + F_{1R}^{(k)}) + j G^{(k)}_e(F_{0R}^{(k)} + F_{1R}^{(k)})|$$

$$H_{sk}(e^{j\omega T}) = |G^{(k)}_e(F_{0R}^{(k)} + F_{1R}^{(k)}) - j G^{(k)}(F_{0R}^{(k)} + F_{1R}^{(k)})|$$

(41)

Finally, it is obvious from (42) that the product of the filters $H_{ak}(e^{j\omega T})$ and $H_{sk}(e^{j\omega T})$ has the linear phase $L N_G + N_F$. 

$$H_{ak}(e^{j\omega T})H_{sk}(e^{j\omega T}) = e^{-j(N_G L + N_c)\omega T}$$

$$\cdot c j(-1)^k [G^{(k)}_R(F_{0R}^{(k)} + F_{1R}^{(k)})^2]$$

$$+ G^{(k)}_e^2(F_{0R}^{(k)} + F_{1R}^{(k)})^2)$$

(42)

REFERENCES