Dynamic supply chain coordination games with repeated bargaining

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ARTICLE INFO

Article history:
Received 9 May 2014
Received in revised form 29 September 2014
Accepted 14 November 2014
Available online 26 November 2014

Keywords:
Relationship-specific assets
Contract design
Asymmetric information
Rent capture
Bayesian belief

ABSTRACT

Coordination in a supply chains may require investment in relationship-specific assets (RSA) including information systems and human resources from all or a subset of the partners. These investments are typically partially non-verifiable, possibly based on internal resources or opportunity costs. A supplier offers a single-price single-period contract to a downstream manufacturer who accepts or turns to a non-strategic outside option. Both parties invest in relationship-specific assets (RSA) accordingly. Using a game theoretic framework of repeated single-period bargaining under asymmetric information and outside options, we show how a supplier may behave opportunistically. We show how this rent extraction threat is mitigated when the manufacturer mis-informs the supplier or hides information from her. As a result of both behaviors, our model explains how supply chain coordination and efficiency are impaired. On a normative basis, we provide the manufacturer with new justifications for both dual sourcing and distorting information. Numerical examples illustrate the results.

1. Introduction

A supply chain is a network of connected and interdependent organizations mutually and cooperatively working together to control, manage and improve the flow of materials and information from suppliers to end users (Aitken, 1998). There is consensus that supply chain optimization involves emphasis on intra-functional and inter-organizational collaboration, leading to coordination of processes, orders and information in areas such as customer service, production planning, logistics, and capacity utilization. Coordination, in particular, has been a research focus (CSC, 2009). RSA has received intense attention in inter-firm relationship research, and has become an important subject in both marketing channel (Kang, Mahoney, & Tan, 2009) and supply chain management. Asset specificity is mainly researched within the framework of transaction cost economics (Williamson, 1985) and relational exchange theory in which it signals the desire to invest in an enduring relationship (Anderson & Weitz, 1992). Most of that research considers that the RSA are the object of an agreement or even a contract between two successive partners in a supply chain with the purpose of increasing the performance of that supply chain and protecting the investor from ex-post opportunistic behavior. This paper does not consider the contracts which can be set up to coordinate trading between the seller and the buyer. We cover the preliminary evaluation that both must conduct in order to maximize their projected interaction, even before a coordinating contract for their operations can be considered. We are interested here in the ‘selfish’ investment by the downstream partner which is relationship-specific and primarily enhances the investor’s performance.

It is observed frequently in buyer–supplier relationships of very different types of goods and services that RSA are deployed by buyers in absence of agreement or even of knowledge of suppliers. Kneemeyer, Corsi, and Murphy (2003) has surveyed the outsourcing practice of logistic services and shown that it involves investments in specific assets and non-retrievable commitments of resources on the part of outsourcing companies. Sucky (2007) point to the trend towards outsourcing logistic activities as support to the argument that large firms are focusing their activities on their perceived core competencies. A classical case is that of steel-makers or electricity-generating utilities using sulphurous or other low-value coals. For coke-making, coal blends are required to have specified ranges of values for volatile matter, ash and sulfur content (Adieleke & Onumanyi, 2007). The blend, which lowers the purchasing cost, is processed in special equipment which scrubs the sulfur content. The RSA consist in this specialized equipment. Ball and Loncar (1991) modeled the demand elasticity for Australian coal to price fluctuations in the medium term taking into account the quality of the coal, overall economic activity and the price of oil (a substitute for coal in thermal energy generation). The conclusion was
that demand was relatively inelastic to price increases in the short but not in the medium or long run which is consistent with the time required by customers to redeploy their RSA to adapt their cleaning processes to accommodate coal from other suppliers.

Supply chain coordination is a process built on gradually fostered trust in combination with adequately designed, financed, deployed and monitored collaboration and information-sharing instruments. Partners invest time, capital and human resources into adjusting their operations to their supply chain partners’ corresponding processes eg, by changing and coordinating product and packaging dimensions, information technology (IT) communication protocols, bar codes or radio identification codes, product catalogues, product development platforms, production planning (schedule, detail and systems). However, Kampstra, Ashayeri, and Gattorna (2006) notice that progress towards deeper collaboration and coordination with upstream suppliers and downstream customers is slow and frequently disappointing in practice. The authors cite, among other reasons for failure, the lack of trust, fear of external competition, missing infrastructure and financial barriers for the sharing of resources and gains. In particular the infrastructure standard investments that Kampstra et al. (2006) identify as lagging or missing are relationship-specific eg, the adjustment to a given customer’s IT standards has little or no value outside of that supply chain.

We identify three essential characteristics of these investments that influence the ability of the supply chain to achieve coordination. First, the specificity of the investment gives rational reason to fear hold-up (i.e. post-contractual opportunistic action) from the supplier. A recent example is the situation in which Ryanair has found itself when Boeing, sole supplier of planes and technical assistance to the company, decided to increase the price of long-term service contracts (O’Doherty, 2009). In June 2011, partly to escape from this situation, Ryanair signed a cooperation agreement with the Chinese aircraft manufacturer, Comac (Odell, 2011). We are interested in the investment incentives: what matters is the potential exposure to a hold-up.

Second, investments may not be verifiable by the supply chain partner. Moreover, even if the investment is verifiable and well defined, the potential cost sharing among several supply chain affiliations make “open book” procedures ineffective in allocating costs and returns.1

Third, the coordination investments are empirically subject to continuous and repeated financial negotiations within the supply chain, often over several product or contract generations (Cf. Kampstra et al., 2006). Hence the truthful revelation of RSA costs may not occur.

In this paper, we address the question of why supply chains fail to coordinate relationship-specific investments by a stylized dynamic dyadic model of a supply chain. Whereas the literature has suggested a range of remedies to the hold-up problem, such as in Hart and Moore (1990) and references below, most work address the problem from one or two of these perspectives. Our contribution, based upon a game theoretic framework, is both positive and normative. From a positive viewpoint, our model explains the delays in supply chain coordination and its elusive-ness as well as distortions in rent creation and attribution. The normative contribution takes two different viewpoints. From a decision-making stance, two contributions are presented: (a) the Bayesian updating mechanism proposed for the coordinator may be used under more general settings to inform sequential bidding procedures; (b) dual-sourcing is provided with additional justification on opportunistic behavior grounds to protect against holdup within an ongoing relationship.2 From a behavioral stance, our contribution provides a theoretical argument justifying the manufacturer’s secrecy or communicating biased information about his RSA costs, independently from his bargaining power.

We first set up a full information centralized benchmark; common information is not only the reference point for efficiency estimations, but may also exist in vertically integrated organizations (a production division and a distribution organization). We then investigate the case where information about RSA cost is private to the manufacturer (he); the supplier (she) only has some prior belief about the cost.

In the following section, we give some elements of related literature on the subject. We present in Section 3.3 the full information case and Section 3.4 covers the case where the supplier is unaware of the investment costs that the manufacturer faces. A numerical instance positions the different tradeoffs in Section 4. We conclude in Section 5.

2. Literature review

The holdup problem under incomplete contracting and asymmetric information has attracted considerable academic attention in economics, marketing and supply chain management (Cachon & Netessine, 2004, chap. 2). The properties of hold-up, asymmetry of information, renegotiation, incompleteness of contracts, switching costs (Klemperer, 1987) and lock-ins have all been investigated (Garcia Mariñohos, 2001). In González (2004), the agent faces a hold-up situation while making a cost-reducing specific investment unobservable by the principal. To escape the hold-up, the agent randomizes the investment whereas the principal offers screening contracts. The models explored as presented in the marketing literature (Farrell & Klemperer, 2007, chap. 31) are often restricted to full-information, two-period settings with endogenous downstream prices for various market organizations. Within supply chain management, several models explore the influence of a supplier’s offers on the buyer’s decision (Sucky, 2004; Sucky & June, 2006; Esmaeili, Aryanezhad, & Zeephongseksul, 2009); others explore how the supplier can tailor his offers to obtain information private to the buyer (Li, Ritchken, & Wang, 2009). To address the issue of channel inefficiency, a typical approach is to design incentive contracts to provide the downstream partner with flexibility to adapt to volatile demand (Cvs & Gilbert, 2002; Barnes-Schuster, Bassok, & Anupindi, 2002; Wang & Liu, 2007; Zhao, Wang, Cheng, Yang, & Huang, 2010; Zhao, Ma, Xie, & Cheng, 2013). With this incentive, the whole supply chain gains without compromising any of the supply chain members’ profits. In the case where the downstream partner or partners is (are) endowed with operating costs and the possibility to invest, Cho and Gerchak (2005), Plambeck and Taylor (2007) provide several coordination mechanisms for the decentralized chain.

Bargaining Theory is a branch of Game Theory that deals with the bargaining situations between two parties (Wu, 2004, chap. 3). In particular, if the bargaining game is single shot, one may characterize its Nash equilibria. Note that in the above literature, renegotiation does not take place within the model.

Segal and Whinston (2002) provide a survey of mechanism design with renegotiation in settings like the current one, i.e., with hold-up risk and asymmetric information on “selfish investments”. In Tirole (1986), the seller obtains an information rent whereas in our model the buyer does not disclose the investment cost to the seller so as to mitigate the hold-up risk in future periods. Hou

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1 A manufacturer invests in a vertical silo to store a liquid compound which one supplier makes. However, once the silo is there, how is its cost spread among several suppliers who provide equivalent products which can be stored in it?

2 We wish to distinguish here the case of ongoing transactions between a buyer and supplier who know each other from the case where the dual-sourcing is decided because the buyer incurs a high risk of receiving a “bad offer” or bad service from a unique supplier.
Two players, a manufacturer (he) and a supplier (she) may engage in mutually beneficial trade over a finite horizon \( i \) periods, \( i \in \{1, \ldots, n\} \). In any period, both can trade with the other or with a non-strategic outside option. In supply chains represented using game theory several suppliers (or retailers) will interact with a downstream (upstream) player either as cooperating or non-cooperating players (Benz, Jäger, & van Rooij, 2005; Gibbons, 1992; Leng & Parlar, 2005; Leng & Zhu, 2009). In our setting, the non-strategic player does not have the liberty to intervene in the game played by the other two.

If the manufacturer trades with the supplier, he invests \( A \) (strictly positive, irreversible), the cost of some RSA which maximizes his benefit of trading with the supplier. He invests \( a \) (non-negative, irreversible) if choosing to trade with his non-strategic outside option, cost of the corresponding RSA. Symmetrically, the supplier invests \( B \), the cost of some RSA, if trading with the manufacturer or \( b \) if trading with an outside option.\(^4\) The effect of the respective investments is lasting, such that trade is enabled in any successive period following the corresponding investment. Note that this setting means that the successive periods are interrelated and not exchangeable. We assume that the value of trade is always higher than the cost. The manufacturer minimizes his expected cost \( C \) of trading either with the supplier or the outside option.

Typically, one can assume that the outside option for both parties is to turn for their needs to some marketplace or previous trading partner for which the RSA might be considered to be either null because sunk or insignificant (meaning that \( a = 0 \) and, or \( b = 0 \)).\(^5\) In this case, the whole problem revolves around the values of \( A \) and \( B \) for supplier and manufacturer.

The supplier's objective is to maximize her partial profit function \( \Pi_i \).

In time (see Fig. 1), the supplier, as Stackelberg leader, offers a contract. If the manufacturer accepts, both invest in the required specific assets \( A \) and \( B \), unless not done in any prior period. Service is performed and payout takes place.\(^3\) If the manufacturer rejects the offer, both turn to outside options: incurring the corresponding enabling investments \( a \) and \( b \), respectively, unless already sunk (this option is not represented in Fig. 1).

At each period \( i \), the supplier is offering a single-period take-it-or-leave-it price contract \( U^i \) with \( i \in \{1, \ldots, n\} \). The strictly positive contract which the manufacturer may sign with some third party is labeled \( u \) and, without loss of generality, shall be considered to be time invariant. The supplier can also sign a strictly positive time-invariant contract \( v \) with a third party as outside option. We call \( \delta_v \in \{0, 1\} \) the manufacturer's participation decision variable in period \( i \) (notation in Table 1).

The investments made by the manufacturer are unobservable by the supplier. We first present in Section 3.2 the coordinated benchmark case where decisions and information are centralized. We then study two information scenarios. In Section 3.3, the supplier is informed of the cost to the manufacturer of the investment he realizes. In Section 3.4, the supplier does not possess this information and so relies on her beliefs.

### 3.1. Updating process for the supplier

We assume that the supplier is a Bayesian rationalist who builds her assumptions from her experience at the start of the first period. Using these priors, she calculates her most profitable estimate of the parameters involved and makes the corresponding offer to the manufacturer. Unless the manufacturer agrees to the first contract offered, in the following period the supplier must update her belief about these unknowns in a dynamic updating process (Selten, 1975). This belief is characterized by a continuous cumulative distribution function (cdf) of the probability with increasing failure rates (IFR) as defined in Barlow and Proschan (1965, chap. 1) and a range of possible values for the given unknown. The supplier updates her beliefs using the manufacturer's refusal in what is called a “Bayesian updating with cutoff” (Hart & Tirole, 1986).

We assume that the supplier, as Stackelberg leader, makes all the offers and the manufacturer simply accepts or declines, i.e., we give all the bargaining power to the buyer. This assumption is consistent with conventional literature. Second, we want to abstract from the issues of renegotiation of a contract by an informed party.\(^4\) The possibility of both resolving their needs through another counterparty or taking recourse in a spot market which entails introducing a second random variable to model price and availability has been dealt with in Brusset (2009a).\(^5\) We have chosen not to include this feature in the present model so as to reduce its complexity.

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\(^3\) Without loss of generality, the third parties for the supplier and the manufacturer are assumed to be non-strategic players.
By analogy to Hart and Tirole (1988) in its sales model under non-commitment, the supplier’s strategy is a sequence of contract price offers \(U_1, \ldots, U_n\). The manufacturer’s strategy consists in accepting or rejecting the supplier’s offer, resulting in a series of decision \(d_1, \ldots, d_i\). The corresponding strategies and beliefs, both for the supplier and manufacturer, form a Perfect Bayesian equilibrium (PBE): strategies are sequentially optimal given beliefs (perfection) and beliefs are derived from equilibrium strategies and observed actions using Bayes’ rule whenever possible (Bayesian updating).

3.2. Coordinated benchmark

Consider two firms under common ownership. The per-period benefit of service is positive. The problem is a cost minimization one. Denote the joint cost function when switching to a third party trade for \(k \geq 0\) periods by \(V(k)\).

\[
V(k) = (A + B) \min\{1, n - k\} + (a + b) \min\{1, k\} + k(u - v). \tag{1}
\]

The integrated firm has two options: (i) performing the services internally by the supplier, or (ii) relying on external service provision from period 1. Naturally, the option of changing regime has no value to the integrated firm as the external prices are stationary.

\[
\begin{align*}
\text{Trade together} & \quad V(0) = A + B = V' \\
\text{Trade separately} & \quad V(n) = a + b + n(u - v) \tag{2}
\end{align*}
\]

By inspection, the shifting policy dominates if \(a + b + n(u - v) < A + B\).

We limit our interest to the “mutual beneficial trade” case, defined by the following two conditions:

\[
\begin{align*}
\begin{cases}
    nU + A \leq nu + a, \\
    nU - B \geq nv - b,
\end{cases}
\end{align*}
\]

which lead to the following inequalities

![Fig. 1. Timeline of events when manufacturer and supplier first agree on a contract and to a new relationship. If the manufacturer does not agree to the contract offered, trade with third parties occur (not represented).](image-url)
\[ u + \frac{8x}{\pi} \geq v + \frac{8y}{\pi} \quad \text{Condition 1.} \\
\] \[ u \geq v, \quad \text{Condition 2.} \] \[ \text{Condition 2.} \]

The first condition provides a basis for evaluation of any switching policy, say for \( k \) periods. The coordination loss then equals:

\[ V(k) - V' = a + b + k(u - v). \]

If the second condition is violated, there is no marginal room for beneficial trade. Using Condition 2 from (3), we can then state that the marginal cost of delay is positive and that there are two sources of coordination losses: over-investment in relationship assets \( a + b \) and inferior trade conditions \( u - v \).

### 3.3. Full information

In the full information case, the supplier presents offers using full information about the RSA costs \( A, a \) and \( u \). We use the durable-good deterministic branching process as defined in (Hart & Tirole, 1988): the supplier’s message space at each date is empty and the manufacturer’s message at date \( i \) is a choice between 0 (rejects) and 1 (accepts): the manufacturer decides sequentially on whether to accept or not. The total transfer from manufacturer to supplier depends only upon the accept-reject path.

The game is composed of three states depending on the manufacturer’s investment(s): (S) exclusive investment \( A \) for the supplier, (ST) investments for both the supplier and the third party, (T) investment \( a \) only in third party trade. Table 2 presents the two-period case according to the decisions in the first and second period by the manufacturer. The manufacturer’s optimal contract for each state are presented in Section A.1 of the Appendix.

We present the result of the common information game as a Proposition.

**Proposition 1** (Full information). Given mutual interaction potential for manufacturer and supplier given in (3), the following optimal contract is a (weak) Nash Equilibrium (NE)

\[
\begin{align*}
U^1 & = u - A, \\
U^i & = u, \quad 2 \leq i < n, \\
U^n & = u + a.
\end{align*}
\]

Because this equilibrium is weak, the supplier (but not the manufacturer) is exposed to considerable risk for costly mis-coordination by the manufacturer. This can be reworded in the following remark.

**Remark 1.** The NE in Proposition 1 is neither trembling hand perfect nor a Pareto equilibrium.

The proofs are provided in Section A.2 of the Appendix.

The remark has practical implications: when the manufacturer changes to his outside option, the cost is the same to him but not the profit to the supplier. We note (although not developed here in supplier-driven bargaining) that this condition naturally would be exploited by the manufacturer in an open bargaining process.

### 3.4. Asymmetric information and renegotiation

We assume that the RSA costs \( A \) and \( a \) are now the manufacturer’s private information. To make offers, the supplier relies on a priori information on the distribution for \( A \) supported by \( [A_0, A] \), following a cumulative density function \( F_A \), and for \( a \) as supported by \( [a, a_0] \) and following a cumulative density distribution \( F_a \). We assume without loss of generality that \( 0 \leq A < A \) and \( 0 \leq a < a_0 \) and that these distributions have increasing failure rates (IFR). A property of the IFR density functions is that the set \( \{x | f_A(x) > 0\} \) is connected. Further, \( \forall x \in [A, A_0], F_A(x) < 1 \) but can be arbitrarily close to 1. The distributions which enjoy this property are numerous and include such familiar ones as the normal and uniform distributions (Barlow & Proschan, 1975; Barlow, 2003).

#### 3.4.1. Updating mechanism

The supplier updates her belief about \( A \) and \( a \), using the information conveyed by the rejection of the offers in previous periods. Following a rejection, the supplier makes an estimate of the distribution: mean, range upper and lower limit and standard deviation. The distribution is truncated at a lower limit resulting from her a priori information about investment specific costs \( A \) and \( a \) and mutual interaction distribution conveyed by the rejection of the offers in previous periods. Following a rejection, the supplier makes an estimate of the distribution supported by the supplier’s private information. To make offers, the supplier relies on the distribution for \( A \) and \( a \). We assume without loss of generality that 0 \( \leq A < A \) and that these distributions have increasing failure rates (IFR).

The quantities \( A^i \) and \( L^i \) are evaluated using (25), (27) and the correspondingly seeded distribution of belief. By bootstrapping, knowing \( A^0 \) and \( L^0 \), she then calculates all previous values of \( A^0 \) and \( L^0 \) for \( 1 \leq k \leq n-1 \) up to the initial estimate \( A^1 \) for \( A \) using the procedure described in Section A.3.4. This estimate forms the basis for the offer to the manufacturer in the first period. The manufacturer accepts or rejects. The supplier can now use this information to update her belief by increasing \( A^0 \) to \( A^1 \), modify the estimated distribution of \( A \) before repeating the bootstrapping exercise in the second period.

We spell out a new proposition applicable to the present scenario (proof in Section A.3 in the Appendix).

**Proposition 2** (Asymmetric information). Given asymmetric information about investment specific costs \( A \) and \( a \) and mutual interaction potential defined in Condition 2 of (3), the contracts offered are

\[
\begin{align*}
U^1 & = u - A^1, \\
U^i & = u, \quad 1 \leq i < n, \\
U^n & = u + a.
\end{align*}
\]

If \( A^i < A \), the supplier offers in subsequent periods \( i \) contracts such that

\[
\begin{align*}
U^i & = u - A^i, \quad i \leq n, \\
U^i & = u, \quad i > n, \text{iff } \delta_{m} = 1,
\end{align*}
\]

with

\[
\begin{align*}
A^i + L^i & = n - i + 1, \\
A^i & = \frac{\delta_{m} A^i}{1 - \delta_{m}}(n - i + 1) - v - L^i, \\
L^i & = F_A((n - i + 1) - A^i) + \frac{F_A(A)(n - i + 1) - v - L^i}{(n - i + 1)}, 1 \leq i \leq n - 1, \\
L^n & = F_A((n - i + 1) - A^i) + \frac{F_A(A)((n - i + 1) - v - A^i)}{(n - i + 1)}, \text{(IC)}
\end{align*}
\]

where \( F_A(.) \) represents 1 - \( F_A(.) \).
The rent which accrues to the supply chain composed of the supplier and the manufacturer over the n periods is affected as per the following third proposition (proof in Section A.4 in the Appendix).

Proposition 3. Given asymmetric information about investment specific costs A and a and mutual interaction potential over the n periods, there is a risk that interaction between the supplier and manufacturer will not start in the first period, reducing the supply chain rent. There is also a risk of over-investment equal to a + b by both supplier and manufacturer in the event that the manufacturer decides to fall back on his outside option in any period.

This proposition leads to the following two corollaries.

Corollary 1. Under asymmetric information, the supply chain joint cost function is increasing in the number of periods during which the supplier under-estimates A. In the case of asymmetric information, two cases present themselves: either trade takes place in the first period and we have

$$\delta^0_m = 1,$$

and so

$$\begin{align*}
C^n &= nu + A - A' + a' \\
P^n &= nu - A' + a' - B,
\end{align*}$$

with

$$V(0) = A + B,$$

which means that the joint cost function yields the same cost as in the full information case. But, if joint trade starts in period \( k = j + 1 \), where \( 1 < j \leq n - 1 \) then

$$\delta^j_m = 0, \delta^n_m = 1,$$

$$\begin{align*}
C^n &= nu + A - A' + a \\
P^n &= jv + (n - j)u - A' - B
\end{align*}$$

$$V(j) = A + B + a + b + j(u - v).$$

The joint cost to the supply chain suffers a sudden jump when \( \delta^n_m = 0 \) and then increases further for each period in which the imprecision of the information makes the supplier under-estimate the cost A. Note that the joint supply chain cost is not affected by the exact value of the estimate \( A' \).

Corollary 2. The manufacturer’s rent from the information about A and a which the supplier has to estimate is proportional to the precision of the information available to the latter. This information rent is the difference between the full information and asymmetric information scenarios and can be expressed as

$$R = A' - A.$$

This rent increases with the supplier’s over-estimation of A. It disappears when the supplier under-estimates A because the manufacturer turns to his outside option: there is no interaction with the supplier.

Under opportunistic behavior on the manufacturer’s part, this rent is not null and increases with his ability to influence the a priori beliefs of the supplier. This type of behavior is noticeable in negotiations between buyers and suppliers in very different business to business settings. In the least, the true costs will not be revealed so that, when renegotiation occurs, renewed opportunistic behavior can yield information rents to the downstream partner. It is the authors’ experience that even after working together many years, this type of information withholding behavior over coordinating investments will still take place. This type of behavior has not been addressed in the modeling literature and does not sit easily with the truthfulness and collaboration paradigms.

4. Numerical illustration

A manufacturer and a supplier interact over \( n = 20 \) periods. Let

$$A = 50, \quad u = 100, \quad a = 60, \quad \nu = 80, \quad b = 55.$$  \hspace{1cm} (10)

4.1. Scenario of full information

In the case of common information the offers are straightforward and, according to Proposition 1, yield the following multi-period contract

$$U^i = 100 - 50, \quad \forall i, 2 \leq i < 20,$$

$$U^{10} = 100 + 60.$$  \hspace{1cm} (11)

The manufacturer pays a total of 2060 over 20 periods (including \( A = 50 \), exactly the same cost than his outside option (2060). The supplier earns a larger profit than with her outside option (1545). The chain has no coordination loss.

4.2. Scenario of asymmetric information

We use the updating procedure described in Section 3.4.1 adapted to the present numerical illustration. Initially, we have to determine the last period \( L \) and \( A^L \) as well as \( a^L \). To do so, the seed is set at 10 and the distribution function \( F_{A|X} \) follows a truncated normal distribution with lower bound \( A^L = 10 \) (the seed), \( \mu_{n-j} = \text{seed} + 3\sqrt{\text{seed}} + 100 \) and \( \sigma_{n-j} = \sqrt{\text{seed}} + 100 \). If the ensuing estimation of A leads to a refusal by the manufacturer, in following periods, these a priori beliefs are updated in the following way:

Table 3

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* When a renown specialty minerals manufacturer invests in complementary information systems so that it can work seamlessly with a software as a service web-based logistics service provider, he will disguise the exact cost or extent of the work done to the supplier. This allows the manufacturer to be in a more advantageous position when negotiating the necessary adjustment effort that the supplier has to make to his own information system.
A period increases. profit decreases as the overestimation of the cost to the manufacturer. Note however that the supplier’s strategy induces a temporal protection against opportunistic behavior. As can be seen, the manufacturer only accepts in the first period when the seed is higher than 50. In the first four rows, the manufacturer rejects the first period offer and invests a, thus depriving the supplier of the ability to extract α in the last period.

In Fig. 2, we can observe how the supply chain rent constituted by the difference between the supplier’s profit and the manufacturer’s cost evolves according to the seed. The impact of Proposition 3 can clearly be seen when A^20 ≤ 50. As this seed increases the overall rent increases quickly before reaching a plateau. Note that the first contract offered when A^20 ≥ 100 is a payment by the supplier to the manufacturer!

When the estimate of a is constant whereas the seed for A changes, we see (dashed lines in Fig. 2) that the supplier is able to increase substantially her return even though the cost is almost the same for the manufacturer. Note however that the supplier’s best course is to hold up the manufacturer by the cost of RSA with a third party when engaging with the manufacturer in a multi-period relationship. This hold-up is only partly mitigated when the manufacturer withholds the RSA cost information. We show how, to protect himself, the manufacturer invests in two different relationship-specific assets, thus impairing supply chain efficiency. This provides another motivation for dual-sourcing as a form of protection against opportunistic behavior.

The numerical illustration shows how the manufacturer’s cost is reduced when the supplier over-estimates his RSA costs both for their relationship and the relationship with the outside option. This effect can be related to common anecdotic observations of customers over-declaring the effort deployed to establish the relationship with their suppliers.\(^7\) The manufacturer has an incentive to signal a high RSA cost. The supplier downplays such a message to maximize her profit. Both behaviors, observed in practice, have not been documented in literature. On the other hand, the supply chain efficiency is increased if she obtains precise information about the manufacturer’s RSA costs.

\[ \begin{align*}
\Delta^{i+1} &= A^i \\
\mu^{i+1} &= \mu^i + A^i \\
\sigma^{i+1} &= \sigma^i + 2.2\sqrt{A^i}.
\end{align*} \] (12)

The distribution chosen here is the truncated normal distribution, bounded from below by the lower limit for the range of A.

To illustrate the sensitivity with respect to the initial belief, we have replicated the whole exercise for a range of initial seeds (results in Table 3).

5. Conclusion

In this paper, we have presented a model of a manufacturer and a supplier when they have yet to invest in the RSA required to work with each other and the manufacturer has private information about the cost of such investment. We characterize the NE when the supplier’s Bayesian belief about the manufacturer’s investment costs follow distributions which exhibit Increasing Failure Rates. Under full information, the ensuing NE are weak and not trembling hand perfect. Under asymmetric information, the NE is trembling hand perfect and the manufacturer’s best course is to agree with the supplier’s offers whenever it is less onerous.

We show that the supplier initially has to sweeten her offer in order to later hold up the manufacturer by the cost of RSA with a third party when engaging with the manufacturer in a multi-period relationship. This hold-up is only partly mitigated when the manufacturer withholds the RSA cost information. We show how, to protect himself, the manufacturer invests in two different relationship-specific assets, thus impairing supply chain efficiency. This provides another motivation for dual-sourcing as a form of protection against opportunistic behavior.

The supplier’s and manufacturer’s strategies induce a temporal link among periods: she is motivated to renew the relationship so as to hold up the manufacturer in the last period. The manufacturer is also motivated to stay with the incumbent by the cost of investing in another relationship.

The numerical illustration shows how the manufacturer’s cost is reduced when the supplier over-estimates his RSA costs both for their relationship and the relationship with the outside option. This effect can be related to common anecdotic observations of customers over-declaring the effort deployed to establish the relationship with their suppliers.\(^7\) The manufacturer has an incentive to signal a high RSA cost. The supplier downplays such a message to maximize her profit. Both behaviors, observed in practice, have not been documented in literature. On the other hand, the supply chain efficiency is increased if she obtains precise information about the manufacturer’s RSA costs.

\(^7\) When the author wished to sell remote sensing equipment to a water distribution utility, much was made of the cost the telecommunication link to relay the information back from the field.
Appendix A

A.1. Contracts offered under full information scenario

We present the evaluations for the offers made by the supplier according to the state the manufacturer finds himself in under full information (from Section 3.3).

STATE S:
Manufacturer has made a single investment, A in period one.
Under strict rationality, in period n the supplier’s dominant offer must thus be: \( U^n = u + a \). In the last period \( n \), the manufacturer’s cost becomes \( C^n = \min(U^n, u + a) \). In period \( n \), the supplier can hold the manufacturer up by asking for \( u + a \) without the latter defecting. Can the supplier repeat this hold up in the previous period \( n - 1 \)? She could, but if she did the manufacturer now would defect and turn to the other supplier by investing \( a \) and paying \( u \). If the manufacturer does not defect in period \( n - 1 \), he faces the same hold up in the last period. This strategy is clearly not optimal to him since he would have a cost of \( 2u + 2a \). Hence the manufacturers cost for the 2 periods \( n \) and \( n - 1 \) is: \( C^{n-1} + C^n = \min(U^{n-1} + u + a, 2u + a) \), bounding \( U^{n-1} \leq u \) for mutual trade.

It follows that in any period \( j \), \( 1 < j < n \), the manufacturer has to solve \( C^j = \min(U^j + (n-j)u + a, (n-j-1)u + a) \). So, to ensure incentive compatibility, the supplier is limited to offering \( U^j \leq u \), \( \forall j \). To ensure \( U^j \leq u \), \( \forall j \).

So: to ensure incentive compatibility, the supplier is limited to offering \( U^j \leq u \), \( \forall j \). The overall minimized cost function becomes \( C = nu + a \). The supplier’s maximal profit is \( \Pi_s = -nu + a - A - B \).

The supplier’s optimal strategy under Condition 2 in (3) consist of the profit maximizing contracts as an \( n \)-sized vector \( \Delta^s = (u - A; u; \ldots; u) \). Let us call the manufacturer’s strategy when he is in state S the \( n \)-sized vector comprised of the decisions \( \Delta^s \) in response to the offers as

\[
\nu^s_i = (\Delta^s_1, \Delta^s_2, \ldots, \Delta^s_n), \quad \forall i, 1 \leq i \leq n, \Delta^s_m = 1. \tag{13}
\]

The strategy sets \( R^S \) and \( R^T \) for each player are reduced to just one vector in each.

STATE ST:
The manufacturer has invested \( a \) and \( A \), the supplier has invested \( B \) and \( b \). In the last period \( n \), the supplier’s cost function is \( C^n = \min(U^n, u) \) for acceptance. By extension, \( U^j \leq u \), \( \forall j \). The threat strategy for the manufacturer is to accept the introductory offer \( u - A \) in period 1 and then reject the renewal of the contract in period 2, resulting in \( C^2 = u + a \), whether or not the supplier offers \( u \). The total cost for this strategy over \( n \) periods is \( C = nu + a \). In the best case where the manufacturer switches back to the supplier from period 3 to \( n \), the supplier’s corresponding profit becomes

\[
\Pi_s = (n-1)u + v - A - B - b \tag{14}
\]

If in period \( k \), \( 1 < k \leq n \), the manufacturer invests \( a \), the profit function becomes

\[
\begin{align*}
\Pi^u_k \geq & (k-1)u - A - B - b + v + (n-k)u, \\
\Pi^P_k \leq & (k-1)u - A - B - b + v + (n-k)u,
\end{align*}
\tag{15}
\]

and \( \Pi^u \leq \Pi^P \).

The strategies of manufacturer and supplier are now described by the following sets of vectors

\[
R^S_m = \\{ \nu^S_i, \nu^S_j = (u - A); u; \ldots; u \} \tag{16}
\]

\[
R^T_m = \\{ \nu^T_i, \nu^T_j = (\Delta^S_1, \Delta^S_2, \ldots, \Delta^S_m) \}, \quad \exists i, 1 < i < n, \Delta^S_m = 0 \tag{17}
\]

A.2. Full information: proof of Proposition 1

Proof 1. When comparing the strategies among the three states, we see that in the first state, the supplier exploits the incumbent’s advantage of the relationship specific investment with the manufacturer’s outside option only in the very last game. If she were to try doing so before, the manufacturer would simply defect. If the supplier’s participation constraint \( n u + a - A - B \geq n v - b \), (18)
is satisfied, the profit extracted in state 1 is larger than those in either other states. This justifies that she offers in all periods \( U^j = u \), \( \forall i \).

Given that the supplier is the Stackelberg leader and the manufacturer is reduced to accepting or rejecting the offers that both work in a full information environment, the NE strategy is the one presented in the case of state 1: the manufacturer agrees to the contract offered in the first period and, under sequential rationality (Watson, 2002), subsequently accepts all the offers made by the supplier without deviating by working with his outside option.

We will now investigate the weakness of this NE. The cartesian set \( R_s \times R_m \) represents all the available strategies of both players. This set is larger than the union of the three sets defined when describing the three states in which the manufacturer finds herself. However, all the strategies which do not belong to the sets \( R^S_m, R^{ST}_m \), and \( R^T_m \) are evidently not profit maximizing ones or do not satisfy the participation constraints as binding constraints and shall be discarded.

For the supplier, it can easily be seen that, with \( \nu^s_m \) as defined in (13),

\[
\forall \nu^s_m \in R^S_m \text{ and } \nu^s_m \neq \nu^s_1, \Pi_s(\nu^s_m) > \Pi_s(\nu^s_1) \tag{19}
\]

For the manufacturer, evidently \( R^S_m, R^{ST}_m, R^T_m \), so \( \forall \nu^s_m \in R^S_m \),

\[
C(\nu^s_m) = n u + a \Rightarrow \exists \nu^s_m \in R^S_m \text{ } C(\nu^s_m) > C(\nu^s_1). \text{ Hence, the NE is weak.}
\]

If the manufacturer chooses other responses, these strategies can be assimilated to the “Trembling Hand” ones (Selten, 1975). The three states presented above are in fact occurrences of this

<table>
<thead>
<tr>
<th>Outcome</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n u + a - A - B )</td>
<td>State 1</td>
</tr>
<tr>
<td>( (k-1)u - A - B - b + (n-k) \max (u,v) )</td>
<td>State 2</td>
</tr>
<tr>
<td>( (k-1)u - b + (n-k+1)u - A )</td>
<td>State 3</td>
</tr>
<tr>
<td>( n v - b )</td>
<td>No contract</td>
</tr>
</tbody>
</table>
Trembling Hand argument: as can be seen in Table 4, the manufacturer may costlessly play a different strategy which hurts the supplier. Due to the Stackelberg structure, we do not explore the other rent-appropriation possibilities and conclude that the NE is not a trembling-hand perfect equilibrium. □ 

A.3. Asymmetric information: proofs of Proposition 2

We present proofs substantiating the second proposition in Section 3.4 when A and a are the manufacturer’s private information.

The supplier must evaluate optimal offers for each of the manufacturer’s enabling investment(s) states. She then formulates offers as functions of thresholds $A^{i}$, $a^{i}$ and eventually the updated thresholds $A^{i}$ in periods $i$ if $i \in \{2, \ldots, n\}$ so that she maximizes her profit. We first evaluate the strategies open to both players before presenting in Section 3.4 when $n$ the calculations of those thresholds which help to determine the supplier’s bids.

A.3.1. Strategies and manufacturer’s states

STATE S:

This state is attained only if $A^{i} \geq A$ to incite an acceptance by the supplier. So, given the sequential rationality of the manufacturer, he works with the supplier for this and all periods up to period $n - 1$ with probability 1 as long as the reservation utility is met. In the last period, the manufacturer accepts the final hold-up only if $a^{i} \leq a$.

The supplier’s strategy is structurally analogous to the full information case in Section 3.3:

\[
\begin{align*}
U^{i} &= u - A^{i}, \\
U^{i} &= u, \\
U^{i} &= u + a^{i}, \\
\end{align*}
\]

under the adjusted participation constraint for the supplier: $nu - A^{i} + a^{i} \geq nu - b(1/P)$. The manufacturer’s cost and supplier’s profit over the $n$ periods are $C = nu - A^{i} + A + a^{i}$, $\Pi_{i} = nu - A^{i} - B + a^{i}$.

When period $n$ starts, the supplier must now show her offer using $a^{i}$. If $a^{i} > a$, the manufacturer rejects the offer, $\delta_{j}^{n} = 0$. The manufacturer’s cost and supplier’s profit when terminating in state S are $C = nu - A^{i} + A + a$, $\Pi_{i} = (n - 1)u - A^{i} + B + v - b$.

STATE ST:

The double-investment state is attained at the earliest in period 2 if the supplier’s threshold $A^{j} \leq A$ made the manufacturer reject the initial offer and accept it in a subsequent period $j$, $1 \leq k \leq n$. The supplier has incurred $b$ and the manufacturer has incurred investments $A + a$ up to this period.

The dominant strategy for state ST is trivial, there is no opportunity for hold-up in the last period since the manufacturer has a sunk investment with the third party. Thus, the resulting strategy for the supplier for all subsequent periods $k + 1 \leq j \leq n$ is $U^{i} = u$, which follows from Condition 2.

The manufacturer cost and supplier profit expressions are, respectively:

\[
\begin{align*}
C &= nu - A^{i} + A + a \\
\Pi_{i} &= (n - k)u - A^{i} - B + kv - b. \\
\end{align*}
\]

STATE T:

State T results from $A^{i} < A$ in all periods from 1 to $k$. The manufacturer has invested $a$ in a third-party relationship. In period $j$, $k < j < n$ the supplier may offer a new contract based upon $A^{i}$ that, if accepted by the manufacturer, would change the state to

St. However, an updated offer by the supplier is made only if the manufacturer’s incentive compatibility (IC) constraint $A^{i} < A^{i+1}$, $k < j < n$, is satisfied. In the following period, the supplier cannot make an acceptable offer to the manufacturer because doing so would violate her participation constraint.

If the game terminates in state T, the manufacturer and supplier have never worked together: the manufacturer’s overall cost is $C = nu + a$ and supplier’s overall profit is $\Pi_{s} = nu - b$.

Fig. 3 represents the three states and the manufacturer’s decisions leading to them.

A.3.2. Bid determination

We now present the evaluations of the thresholds that enable the supplier to calibrate her offers. The supplier’s partial objective function over the $n$ periods is

\[
\max_{A} \Pi_{t}(A^{t}) = FA_{1}(A^{1}) \left\{ (n - 1)u - A^{1} - B + FA_{2}(a^{1})(u + a) + FA_{3}(a^{1})(v - b) \right\} \\
+ FA_{2}(A^{t})[(v - b + FA_{3}(A^{t}))(n - 1)u - A^{t} + B] \\
+ FA_{3}(A^{t})(v + FA_{3}(A^{t}))(n - 2)u - A^{t} + B \\
+ \cdots FA_{n}(A^{n-1})(j + 1)u - A^{n-1} + B + v \\
+ FA_{n-1}(A^{n-1})(j + 1)u - A^{n-1} + B + v \cdots \}
\]

s.t.

\[
\begin{align*}
A^{(i-j+1)} &\leq A^{(i-j)} & 1 < j < n, \quad \text{(IC)} \\
uu + a - A^{i} - B &\geq nu - b, \quad \text{(PC).}
\end{align*}
\]

Note that $j < n$ because there is no interest for the supplier to make an offer in period $n$ lower than $u$. Justification is presented in Proof 2 in Section A.3.4.
The two above inequalities represent the participation constraint (PC) and incentive compatibility constraint (IC). We solve for $a$ in the next subsection and $A'$ in the following one.

A.3.3. Belief about $a$

We first turn to the belief about $a$ which the supplier has to make in period $n$ (if at all).

Since we have assumed that $f_a$ has an IFR and $f_a(a) \neq 0$, there exists an interior value $a^*$ such that $a^* - F_a(a^*) = v - b - u$.

The proof is presented in Brusset (2014).

### A.3.4. Updated belief about the manufacturer’s RSA with the supplier $A^{n-j}$

In period $n-j$ with $1 < j < n$ the supplier updates $A^{n-j}$. Differentiating in $A^{n-j}$:

$$\frac{\partial \Pi_{n-j}^{(n-j)}(A^{n-j})}{\partial A^{n-j}} = f_{\Pi_{n-j}}(A^{n-j})(j + 1)(u - v) - A^{n-j} - B$$

$$- f_{\Pi_{n-j}}(A^{n-j}).$$

(24)

The Second Order Condition (S.O.C.) requires that $f_{\Pi_{n-j}}(A^{n-j})$ $(j + 1)(u - v) - A^{n-j} - B - 2f_{\Pi_{n-j}}(A^{n-j}) < 0$. The proof is similar to the one used above.

So, we can deduct the optimal threshold $A^{n-j}$ in period $n-j$ as solution to

$$A^{n-j} + B + f_{\Pi_{n-j}}(A^{n-j})f_{\Pi_{n-j}}(A^{n-j}) = (j + 1)(u - v).$$

(25)

Using backward induction, we can now evaluate the threshold for $A$ in the previous period starting with period $n-j-1$. The profit function in preceding periods $i$, $1 \leq i \leq n-j-1$, can be written as

$$L^i = f_A(A^i)((n - i + 1)u - A^i - B) + f_A(A^i)(v + L^{i+1}).$$

(26)

In period $n-j$, the last term of the series before the supplier’s Participant Constraint (PC) is violated, $L^{n-j}$ is written as

$$L^{n-j} = f_{\Pi_{n-j}}(A^{n-j})((j + 1)u - A^{n-j} - B) + f_{\Pi_{n-j}}(A^{n-j})(j + 1)v.$$  

(27)

The optimal threshold in each period $i$, $2 \leq i \leq n-j-1$, is the result of evaluating the First Order Condition (F.O.C.) and S.O.C. of the expression $L^i$ differentiated in $A^i$. We obtain

$$A^i + B + f_{\Pi_{n-j}}(A^i)f_{\Pi_{n-j}}(A^i) = (n - i + 1)(u - v) - L^{i+1}.$$  

(28)

Proceeding in a bootstrapping iteration we evaluate all the preceding $L^i$ back to $L^2$ and $A^2$:

$$A^i + B + f_{\Pi_{n-j}}(A^i)f_{\Pi_{n-j}}(A^i) = (n - 2)(u - v) - L^2.$$  

(29)

Hence the threshold $A^i$ is a function of the distribution of the belief about $A$, the difference between the outside options’ prices, and the length of the game.

Remark 2. Witness how each threshold is determined using the supplier’s posterior a priori beliefs about cost $A$. However, the supplier relies upon updating her belief using an “updating with cutoff” procedure. That is, using the information imparted by the manufacturer’s decision to turn down her previous offer, she sets $A^{n-j} = A^{n-j-1}$, and updates the cdf accordingly. Hence, given the way our model is built, this means that the posterior beliefs rely upon updating the current period’s belief. Theoretically our supplier should not be able to resolve this circular reasoning. To get around this contradiction, the supplier must initially set a range and a distribution function for period $n-j$, use a procedure\(^\text{10}\) to bootstrap her a priori belief in period 1, make an offer and use the manufacturer’s decision to update the a priori belief of period $n-j$. In this way, as periods go by, the supplier updates sequentially the a priori belief about $A$ in the last period which satisfy IC and PC in (23) and bootstraps back her range of beliefs $[A^L, A^H]$ and consequent distribution function $F^*$ to the period she finds herself in.

To be incorporated in contracts to be offered to the manufacturer, the thresholds evaluated above must follow the strategy which we described when the manufacturer is in state ST or T, namely that $A^L < A^{n-j}$ for the first periods.

We prove that the supplier’s PC and manufacturer’s IC in Proposition 2 and in (23) the supplier’s objective function (22) are satisfied.

Proof 2. By definition, $L^i$ represents the expected profit to the supplier going forward in periods $i + 1$ to $n$ when she erred in her evaluation of $A$. As $i$ increases, this expected profit decreases since each period’s profit is not negative, even if it involves choosing her outside option. So $L^{i+1} < L^i$. Further, from (28), in each period, the period’s threshold is higher than the previous one’s. Moreover, since the supplier offers a sequence of monotonously increasing bids, we have $A^1 < A^2 < \ldots < A^n$. We reach a point where $A^i$ becomes large compared to $(n-i+1)u$ within $L^i$ in (26). By construction, after that point $A^i$ can no longer increase and in fact decreases as can be seen in the numerical illustration presented in Section 4. So the supplier cannot make an offer more enticing than the previous one to the manufacturer. Instead, she turns to her outside option. \(\Box\)

Note that period $n-j$ becomes the true last period for mutual beneficial trade.

A.4. Proof of Proposition 3

Proof 3. We have seen in the integrated supply chain that if the mutual interaction is delayed by $k$ periods, then the loss in rent to the supply chain is given by the difference between the cost to the manufacturer and the profit to the supplier:

$$\begin{align*}
C &= a + nu + A - A^k + 1; \\
\Pi_1 &= kv - A^j - B - (n - k)u,
\end{align*}$$

(30)

Whereas, if the interaction starts in period 1, the partners’ objective function work out as

$$\begin{align*}
\mathcal{C} &= A - A^1 + nu + a^*; \\
\Pi_1 &= nu - A^j - a^*,
\end{align*}$$

(31)

This leads to the difference between both cases of $\Delta = a + b + k(u - v)$, which is strictly positive because $v \leq u$ according to the PC in (3).

Note that this result is independent of the period in which the manufacturer decides to change supplier. It is also independent of the supplier’s estimates of $a$ and $A$. \(\Box\)

References


\(^{10}\) We present such a procedure in our numerical illustration. Any Bayesian updating with cutoff procedure which satisfies $A^L < A^{n-j}$ and enables the supplier to characterize distribution functions of her belief over the different periods could be used.


