Frontier-based performance analysis models for supply chain management: State of the art and research directions

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ABSTRACT

Effective supply chain management relies on information integration and implementation of best practice techniques across the chain. Supply chains are examples of complex multi-stage systems with temporal and causal interrelations, operating multi-input and multi-output production and services under utilization of fixed and variable resources. Acknowledging the lack of system's view, the need to identify system-wide and individual effects as well as incorporating a coherent set of performance metrics, the recent literature reports on an increasing, but yet limited, number of applications of frontier analysis models (e.g. DEA) for the performance assessment of supply chains or networks. The relevant models in this respect are multi-stage models with various assumptions on the intermediate outputs and inputs, enabling the derivation of metrics for technical and cost efficiencies for the system as well as the autonomous links. This paper reviews the state of the art in network DEA modeling, in particular two-stage models, along with a critical review of the advanced applications that are reported in terms of the consistency of the underlying assumptions and the results derived. Consolidating current work in this range using the unified notations and comparison of the properties of the presented models, the paper is closed with recommendations for future research in terms of both theory and application.

1. Introduction

Supply chain management (SCM) was introduced as a common scientific and managerial term in 1982 (cf. Oliver & Webber, 1992) to describe a hierarchical control system for material, information and financial flows in a potentially multidirectional network of autonomous decision making entities. Although there is a lack of universally accepted definition (Otto & Kotzab, 1999), a well-used and typical definition of a supply chain is ‘a network of organizations that are involved, through upstream and downstream linkages in the different processes and activities that produce value in the form of products and services in the hand of the ultimate consumer’ Christopher (1998, p. 15). The management activity is consequently the coordination of this network, or ‘chain’, of independent processes as to achieve the overall goal in terms of value creation. Three elements are important in our context: the multi-level character of the network, the interdependency and the competitive objective. First, the underlying system is constituted of multiple layers, both horizontally (sequential processing) and vertically (control layers, levels of integration into firms, business units, joint ventures, information sharing, etc.). This implies that the systematic analysis of a supply chain must take into account the level of processing as well as the locus of control in order to understand the organization and its performance. Second, the ‘links’ in the chain form sequential processing stages that are interdependent with respect to potentially three types of flows: material flows in progressive processing, information flows specifying types and quantity of processes to be performed, and financial flows to reimburse or incentivize the units to devote time and resources to the joint activity. Third, a supply chain is not an arbitrary processing plan but it involves multiple independent organizations (conventionally at least three) cooperating under commercial conditions and subject to actual or potential future competition, both collectively in terms of the final output and individually for each processing stage. Taken together, the three observations underline that performance evaluation is of highest importance to assure continuity, competitiveness and, ultimately, survival of the network, but that this evaluation must take into account the specificities of the network character and the decision-making autonomy of the evaluated units.

A wide range of metrics for supply chain performance have been proposed (cf. Melnyk, Stewart, & Swink, 2004; Neely, Gregory, & Platts, 1995) using an equally diverse portfolio of methodologies (cf. Estampe, Lamouri, Paris, & Brahim-Djelloul, 2013). Whereas most SCM literature has been devoted to the elaboration and evaluation of absolute metrics, usually linked to the dimensions cost (profit), time (rates) and flexibility (change of rate), there has also been a growing awareness of the need to perform external benchmarking (Beamon, 1999), the lack of integration of metrics

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The systemic, strategic coordination of the traditional business functions and the tactics across these business functions within a particular company and across businesses within the supply chain, for the purposes of improving the long-term performance of the individual companies and the supply chain as a whole.

Supply chain management takes an integrated system’s view on the design, monitoring and control of the chain. This approach serves to arbitrate the potential conflicts of individual agents in the chain in order to coordinate the flow of products and services to best serve the ultimate customer. We refer to this framework as “centralized”, in that it represents the objective of a hypothetical benevolent supply chain coordinator with authority to implement any necessary decision throughout the chain.

Performance measurement is intrinsically anchored in SCM both as a predictive and normative paradigm. It is predictive in the sense that performance management provides productivity estimates useful for the prediction of processing yields and times while planning material and information flows in order to meet stochastic demand, product and process changes. It is also a normative paradigm in the sense that supply chain management interfaces with both operations management and sourcing, providing targets for improvement as well as potentially credible threats of substitution or volume reductions in case of poor [relative] performance. A seminal paper in performance measurement design is Neely et al. (1995), defining the scope of performance assessment as the quantification of effectiveness and efficiency of action. The paper also offers an overview over a wide range of techniques and metrics used as well as their limitations and areas for future research. Conventionally, the operations management literature limited the attention to performance measurement to the mere definition of absolute (e.g. cost per unit) and partial productivity (e.g. labor hours per unit produced) metrics (cf. Melnyk et al., 2004 for a critique of this approach or Lambert & Pohlen, 2001 for an example) without paying attention to their systemic or economic integration, or even to their value as predictors of future profitability or survival in the market place. Neely et al. (1995) provide greater nuance to the analysis of supply chain performance by distinguishing the type of measurement, metric and method based on an analysis of organizational level, integration, organizational support, managerial application and hierarchical level. The authors empirically document that firms frequently neglect non-financial data, use internal cost data of varying quality, deploy methods with no or poor connection to organizational strategy and globally are dissatisfied with their performance assessment system. Shepherd and Gunter (2006) review 362 scientific papers on supply chain performance measurement and conclude that the findings of Neely et al. (1995) in many aspects still are valid. Alternative qualitative approaches exist using tools such as balanced scorecards (cf. Bhagwat & Sharma, 2007). However, the information made available from such models is limited in terms of e.g. decomposing productive and cost efficiency. Nevertheless, the need to identify performance in supply chain management can be of strategic as well as operational value, cf. Gunasekaran, Patel, and McGaughey (2004) and Olugu and Wong (2009), leading us to require consistency in the evaluation methodology between the two levels.

Applications of frontier methods, particularly DEA, to complex multi-stage systems, are relatively rare. An early application to US Army recruitment in Charnes et al. (1986) uses a two-stage approach with intermediate outputs forming the basis of later network models. Färe and Grosskopf (1996a) present a network DEA formulation to examine the internal structure of an entity. Talluri, Baker, and Sarkis (1999) propose a framework based on DEA and multi-criteria decision models for value chain network design, primarily aiming at the identification of an optimal supplier-manufacturer dyad. Löthgren and Tambour (1999) use the network DEA model introduced by Färe and Grosskopf (1996a) to estimate efficiency and productivity for a set of Swedish pharmacies. Hoopes, Triantis, and Partangelo (2000) develop a goal-programming DEA formulation modeling serial manufacturing processes and apply it to data on circuit board manufacturing. Ross and Droge (2002) propose an integrated benchmarking approach for measuring temporal efficiency using some extensions to DEA methodology and then applying their approach into real data including 102 distribution centers in the petroleum business. Talluri and Baker (2002) propose an interesting three-phase approach for designing an effective supply chain using a DEA framework. Phase I evaluates potential suppliers, manufacturers, and distributors in determining their efficiencies using a combination of DEA models and a pair-wise efficiency game. Phase II contains an integer programming model, which optimally selects candidates for supply chain design using a combination of the efficiencies obtained in phase I, demand and capacity requirements.
and location constraints. Phase III includes the identification of optimal routing decisions for all entities in the network by solving a minimum-cost transshipment model. Sexton and Lewis (2003) evaluate managers' management efficiency of 30 Major League Baseball teams in 1999 using a two-stage model. Their model allows identification of efficiency per stage, allowing managers to target inefficient stages of the production process. Lewis and Sexton (2004) view a baseball team as a network and extend Sexton and Lewis (2003) to consider efficiency at each node of a network. Troutt, Ambrose, and Chan (2001) that takes productivity optimization of a serial processes into account for optimal throughput planning. Ross and Droge (2002) is an another early DEA application to a large-scale supply chain distribution network. In particular they investigate how DEA results may inform about differences in managerial and scale efficiency by identified clusters in a larger dataset. Narasimhan, Talluri, and Das (2004) consider a two-stage framework, namely flexibility competency and execution competency, for discussing the relationship between manufacturing flexibilities and manufacturing performance for a set of firms. Their model uses the reduced DEA model under constant returns to scale (CRS) proposed by Andersen and Petersen (1993) to measure the efficiency of each stage independently. Sheth, Triantis, and Teodorović (2007) evaluate the overall performance of an agency’s bus routes by using network DEA (Färe & Grosskopf, 2000) and goal programming (Athanassopoulos, 1995) taking into account preference indicators from suppliers, consumers, and general society. Yu and Lin (2008) use a multi-activity network DEA model for estimating passengers and freight technical efficiency, service effectiveness and technical effectiveness for 20 global railway firms. Yu (2008a) uses a multi-activity DEA model for measuring the efficiency of both highway and urban bus services in the presence of environmental variables. In addition, they use a shared output-model proposed by Yu and Fan (2006). Yu (2008b) presents a network DEA approach consisting of the two stages, the production and consumption stages, to evaluate the technical efficiency, the service and technical effectiveness of a selected sample of 40 global railways. Yu and Fan (2009) propose a specific two-stage process model (first stage parallel, second stage unique) for DEA process evaluation, demonstrated empirically on a set of national bus transit providers. Vaz, Camanho, and Guimarães (2010) propose a method to measure Portuguese retail stores performance based on the network DEA (Färe, Grabowski, Grosskopf, & Kraft, 1997), taking into account the interdependencies of the departments composing the store. Tables 1 and 2 succinctly show applications of performance assessment to supply chain and other multi-level systems, respectively, in the frontier analysis context.

3. Data envelopment analysis

The data envelopment analysis (DEA) approach to efficiency measurement is a deterministic method that does not require the definition of a functional relationship between inputs and outputs. In economic terms, DEA utilizes a non-parametric mathematical programming approach to estimate best practice production frontiers (envelope). The basic frontier production model introduced by Debreu (1951) and Farrell (1957) was later developed and named data envelopment analysis by Charnes, Cooper, and Rhodes (1978) as a data-driven method for evaluating the relative efficiency of a set of entities, called decision making units (DMUs), with multi-inputs and multi-outputs. DEA has a rapid and continuous growth in different areas since 1978: Emrouznejad, Barnett, and Gabriel (2008) reported more than 4000 DEA research studies published in journals or book chapters. A taxonomy and a general model framework for DEA can be found in Cook and Seiford (2009).

Let us introduce the technology set $T$ or the production possibility set (PPS):

$$T = \{(x, y) \in \mathbb{R}_+^m \times \mathbb{R}_+^n | x \text{ can produce } y \}.$$

The background of the DEA is production theory, and the idea is that the DMUs have a common underlying technology $T$. In reality, we usually cannot specify the technology set but DEA deals with the

![Table 1: Supply chain applications using frontier analysis.](http://dx.doi.org/10.1016/j.cie.2013.02.014)

<table>
<thead>
<tr>
<th>Reference</th>
<th>Method</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Talluri et al. (1999)</td>
<td>DEA and multi-criteria decision models</td>
<td>Supply chain</td>
</tr>
<tr>
<td>Löhgren and Tambour (1999)</td>
<td>Network DEA (Färe &amp; Grosskopf, 1996a)</td>
<td>Swedish pharmacies</td>
</tr>
<tr>
<td>Hoopes et al. (2000)</td>
<td>Goal-programming DEA</td>
<td>Circuit board manufacturing</td>
</tr>
<tr>
<td>Ross and Droge (2002)</td>
<td>DEA models (CRS and VRS)</td>
<td>Petroleum distribution facilities</td>
</tr>
<tr>
<td>Vaz et al. (2010)</td>
<td>Network DEA (Färe et al., 1997)</td>
<td>Retail stores</td>
</tr>
</tbody>
</table>

![Table 2: Other multi-level applications using frontier analysis.](http://dx.doi.org/10.1016/j.cie.2013.02.014)

<table>
<thead>
<tr>
<th>Reference</th>
<th>Method</th>
<th>Application</th>
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<tbody>
<tr>
<td>Charnes et al. (1986)</td>
<td>Two-stage approach</td>
<td>US Army recruitment</td>
</tr>
<tr>
<td>Löthgren and Tambour (1999)</td>
<td>Network DEA (Färe &amp; Grosskopf, 1996a)</td>
<td>Swedish pharmacies</td>
</tr>
<tr>
<td>Sexton and Lewis (2003)</td>
<td>Two-stage approach</td>
<td>Baseball teams</td>
</tr>
<tr>
<td>Yu (2008a)</td>
<td>Multi-activity DEA model</td>
<td>Highway and urban bus services</td>
</tr>
<tr>
<td>Yu (2008b)</td>
<td>Two-stage approach</td>
<td>Global railways</td>
</tr>
<tr>
<td>Yu and Fan (2009)</td>
<td>Two-stage approach</td>
<td>Bus transit providers</td>
</tr>
</tbody>
</table>
problem by estimating the PPS, $T^*$, from observed data on actual production activities according to the minimal extrapolation principle.

The mathematical programs can be obtained when we combine the idea of minimal extrapolation with Farrell’s idea of measuring efficiency as a proportional improvement opportunity.

Assume that there are $n$ DMUs to be evaluated where every DMU, $j = 1, 2, \ldots, n$, produces $s$ outputs, $Y^j = (y^j_1, \ldots, y^j_s) \in \mathbb{R}^s$, using $m$ inputs, $X^j = (x^j_1, \ldots, x^j_m) \in \mathbb{R}^m$. The $s \times n$ matrix of output measures is denoted by $Y$, and the $m \times n$ matrix of input measures is denoted by $X$.

The input-oriented technical efficiency of a specific DMU is denoted by $E(X, Y, \gamma)$ where $\gamma$ represents the returns to scale. Concretely, $E(X, Y, \gamma)$ is calculated by using the following mathematical model

$$\begin{align*}
\min & \quad E(X, Y, \gamma) = \theta^*, \\
\text{s.t.} & \quad X^j \leq \theta^* X^*, \\
& \quad Y^j \geq Y^*, \\
& \quad \lambda \in \Omega(\gamma),
\end{align*}$$

(1)

where $\Omega(\gamma)$ specifies the shape of the frontier through the constraints put upon the multipliers used to create the best-practice frontier. In short, we can define six following classical DEA models as follows (cf. Bogetoft & Otto, 2010 for CRS-FDH, see Agrell & Tind (2001) for ERH):

- The constant returns to scale (CRS) model with $\Omega(\text{crs}) = \{ \lambda \in \mathbb{R}^m_+ | \lambda \geq 0 \}$.
- The decreasing returns to scale (DRS) model with $\Omega(\text{drs}) = \{ \lambda \in \mathbb{R}^m_+ | \lambda \leq 0, \lambda \leq 1 \}$.
- The increasing returns to scale (IRS) model with $\Omega(\text{irs}) = \{ \lambda \in \mathbb{R}^m_+ | \lambda \geq 0, \lambda \geq 1 \}$.
- The varying returns to scale (VRS) model with $\Omega(\text{vrs}) = \{ \lambda \in \mathbb{R}^m_+ | \lambda \geq 1 \}$.
- The free disposability hull (FDH) model with $\Omega(\text{fdh}) = \{ \lambda \in \mathbb{R}^m_+ | \lambda = 1, \forall h : \lambda_h = \{0, 1\} \}$.
- The free replicability hull (FRH) model with $\Omega(\text{frh}) = \{ \lambda \in \mathbb{R}^m_+ | \forall h : \lambda_v h = Z_+ \}$.
- The elementary replicability hull (ERH) model with $\Omega(\text{erh}) = \{ \lambda \in \mathbb{R}^m_+ | \forall h : \lambda_v h = 0, \lambda_2 \in Z_+ \}$.

where $Z_+$ is the set of non-negative integers.

The CRS, DRS and VRS models are convex formulations solvable using linear programming (LP) problems while FDH, FRH and ERH are non-convex models. The dual problem for convex models of (1) is:

$$\begin{align*}
\max & \quad \theta^* = u Y^* + u_0, \\
\text{s.t.} & \quad v X^* \leq 1, \\
& \quad u^\top Y - v^\top X + u_0 \leq 0, \\
& \quad u, v \geq 0, \\
& \quad u_0 \in \Phi(\gamma),
\end{align*}$$

(2)

where $\Phi(\text{crs}) = \{0\}$, $\Phi(\text{drs}) = R_-$, $\Phi(\text{irs}) = R_+$ and $\Phi(\text{vrs}) = R$. In model (2), $u$ and $v$ are the weight vectors assigned to the output and input vectors, respectively. For the duality results for non-convex models, see Agrell and Tind (2001). Note that in the dual program of CRS, $u_0 = 0$ since there are no restrictions on $\lambda$ in the primal model, thus, it becomes

$$\begin{align*}
\max & \quad \theta^* = u Y^*, \\
\text{s.t.} & \quad v X^* = 1, \\
& \quad u^\top Y - v^\top X \leq 0, \\
& \quad u, v \geq 0.
\end{align*}$$

(3)

A DMU with $E(X, Y, \gamma) = 1$, is called technically input-efficient with respect to the technology set $T(X, Y)$ and the returns to scale $\gamma$; otherwise, $E(X, Y, \gamma) < 1$ is called inefficient. Problem (1) is referred to as the envelopment or primal problem, and (2)–(3) the multiplier or dual problem.

4. A generic SCM model

The inefficient DMUs are notably interested in the factors that cause the inefficiency, although it is obvious that either reducing inputs or increasing outputs will improve their performance. To answer this question, much effort has been devoted to breaking down the overall efficiency into components so that the sources of inefficiency can be identified. One type of decomposition focuses on the structure of the DEA models. The general multi-level/multi-stage structure for performance evaluation in a complex real-world environment is illustrated in Fig. 1. This model involves the direct inputs and outputs for each stage, the intermediate flows between the two stages, the common inputs among all levels of the system and shared inputs among stages of each level.
each we use (1996b, 2000) expanded this modeling approach, the studies of Whittaker (1995) and Färe and Grosskopf (1996a) for electricity generation plants. Although constructed multi-plant efficiency measures and illustrated their performance evaluation of DMUs with known internal structure. They in the mid-1980s, structure of DMUs is generally ignored, and the performance of a.....

5.1. Network DEA

In the black-box approach of conventional DEA, the internal structure of DMUs is generally ignored, and the performance of a DMU is assumed to be a function of the chosen inputs and outputs. In the mid-1980s, Färe and Primont (1984) started working on performance evaluation of DMUs with known internal structure. They constructed multi-plant efficiency measures and illustrated their models by analyzing utility firms each of whom operated several electricity generation plants. Although Färe (1991), Färe and Whittaker (1995) and Färe and Grosskopf (1996a) further expanded this modeling approach, the studies of Färe and Grosskopf (1996b, 2000) in the literature are known as a pioneering line of research at developing a general multi-stage model with intermediate inputs-outputs commonly called network DEA. Cook, Chai, Doyle, and Green (1998) discussed a general framework for hierarchies in DEA, grouping DMUs and their individual and aggregate performance indexes. Cook, Hababou, and Tuento (2000) presented a non-linear DEA model for measuring the efficiency of two components (i.e., service and sales) in a banking system in the presence of shared resources. Cook and Green (2004) modified the DEA model developed by Cook et al. (2000) in order to specify the core business performance in multi-plant firms. Jahanshalloo, Amirteimoori, and Kordrostami (2004a) determined the progress and regress of each component of a DMU upon the basis of Cook et al. (2000). Jahanshalloo, Amirteimoori, and Kordrostami (2004b) linearized the model proposed in Cook et al. (2000) in the presence of discretionary and non-discretionary shared resources. Yang, Ma, and Koike (2000) proposed a DEA evaluation model for multiple independent parallel subsystems in which the efficiency of the overall process equals to the maximum of the efficiencies of all sub-processes. Castelli, Pesenti, and Ukovich (2001, 2004) and Amirteimoori and Shafiee (2006) discussed some types of the network structure using DEA-like models. Golany, Hackman, and Passy (2006) simultaneously measured the efficiency of the whole system and each sub-system as a special case of the Färe and Grosskopf network framework (2000). Chen (2009) developed a dynamic production network DEA model by introducing an alternative efficiency measure for evaluating the performance of various hierarchical levels in the dynamic environment along with discussing returns to scale properties of production networks. Chen and Yan (2011) recently developed three different network DEA models based on the concept of centralized, decentralized and mixed organization systems along with discussing the relationship between their efficiencies. A later complementary network DEA formulation is the non-radial slacks based approach in Tsutsui and Tone (2008). This approach has applications in Fukuyama and Weber (2009), Fukuyama and Weber (2010) for bad outputs, Akkian (2009) for banking and Yu (2010) to airport operations. An important modeling contribution to the Tsutsui and Tone (2009) model is made in Chang, Tone, and Wei (2011), where the focus is on the ownership-control for the formulation of a full set of efficiency metrics. We summarize the aforementioned DEA network methods in Table 3.

The two-stage process, a special case of Färe and Grosskopf’s multi-stage framework, captures a large number of real evaluation problems. An excellent review of DEA models exploring internal structure in general, including some of our work, is found in Castelli, Pesenti, and Ukovich (2010). In the ensuing paper, we present a literature review of DEA models, applying two-stage processes, game theory and bi-level programming to the supply chain management area.

5.2. Two-stage DEA

Wang, Gopal, and Zionts (1997) were the first, to the best of our knowledge, to apply a two-stage structure for performance assessment. Their model is composed of \( X_1 \), \( Z \) and \( Y_2 \) which are the input vector of stage 1, the intermediate vector and the output vector of stage 2, respectively (see Fig. 2). Wang et al. (1997) ignored the intermediate measures and obtained an overall efficiency with the inputs of the first stage and the outputs of the second stage (see model (4)). Similarly, Seiford and Zhu (1999) proposed a two-stage method to obtain the profitability and marketability of the top 55 US commercial banks, consisting of \( X_1 \), \( Z \) and \( Y_2 \) presented in Fig. 2. Seiford and Zhu (1999) used independent CRS models (4)–(6) to measure the overall efficiency and the efficiencies of stage 1 and stage 2:
outputs, thereby reducing the efficiency of the first stage. Such an action may, however, imply a reduction in the first stage's inputs (intermediate measures) to achieve an efficient status. The intermediate measures can create potential conflicts between the two stages. For example, the second stage may reduce the presence of intermediate measures. Although model (7) includes the intermediate measure \( Z \), it does not consider the relationship between the first and second stages because of the decoupling of the two stages through different multipliers. This does not suggest an ideal integrated supply chain (Liang, Yang, Cook, & Zhu, 2006). Chen and Zhu (2004) suggested the following linear model for the two-stage process based upon a VRS formulation consisting of \( X_1 \), \( Z \) and \( Y_2 \) presented in Fig. 2:

\[
\begin{align*}
\text{Stage 1:} & \quad \max \quad E(X_1, Y_2, Z, \text{crs}) = uY_2, \\
& \text{s.t.} \quad wX_1 = 1, \\
& \quad uY_2 - vX_1 \leq 0, \\
& \quad u, v \geq 0.
\end{align*}
\]

\[
\begin{align*}
\text{Stage 2:} & \quad \max \quad E(X_1, Y_2, Z, \text{crs}) = uY_2, \\
& \text{s.t.} \quad wZ = 1, \\
& \quad uY_2 - wZ \leq 0, \\
& \quad u, w \geq 0.
\end{align*}
\]

The intermediate measures can create potential conflicts between the two stages. For example, the second stage may reduce its inputs (intermediate measures) to achieve an efficient status. Such an action may, however, imply a reduction in the first stage outputs, thereby reducing the efficiency of the first stage. Zhu (2000) applied a method similar to that of Seiford and Zhu (1999) to a set of Fortune Global 500 companies.

When the model consists of \( X_1 \), \( Z \) and \( Y_2 \), a conventional approach is to use the intermediate measure as an output (\( Z + Y_2 \)) for measuring the overall efficiency. Chen and Zhu (2004) demonstrated that such DEA model specification fails to correctly characterize the two-stage process and that it may distort the interpretation of the DEA frontier, i.e., the performance improvement of one stage affects the efficiency status of the other, because of the presence of intermediate measures. Zhu (2003) and Chen and Zhu (2004) have shown that DEA model (4) does not correctly characterize the performance of the two stages, because it only considers the inputs and outputs of the whole process and ignores intermediate measures \( Z \) associated with the two stages. Alternatively, one can consider the following DEA model that is the average efficiency of the two stages:

\[
\begin{align*}
\text{Stage 1:} & \quad \max \quad E(X_1, Y_2, Z, \text{crs}) = \frac{1}{2} \left[ \frac{w_1Z - vX_1}{vY_2} \right], \\
& \quad \text{s.t.} \quad w_1Z - vX_1 \leq 0, \\
& \quad uY_2 - w_2Z \leq 0, \\
& \quad w_1, w_2, u, v \geq 0.
\end{align*}
\]

Although model (7) includes the intermediate measure \( Z \), it does not consider the relationship between the first and second stages because of the decoupling of the two stages through different multipliers. This does not suggest an ideal integrated supply chain (Liang, Yang, Cook, & Zhu, 2006). Chen and Zhu (2004) suggested the following linear model for the two-stage process based upon a VRS formulation consisting of \( X_1 \), \( Z \) and \( Y_2 \) presented in Fig. 2:

\[
\begin{align*}
\min \quad E(X_1, Y_2, Z, \text{crs}) = \gamma_1X_1 - \gamma_2\beta, \\
& \text{s.t.} \quad \begin{cases}
1\lambda = 1, \\
\lambda \geq 0, \\
Z \mu = \beta Y_2, \\
1\mu = 1, \\
\mu \geq 0,
\end{cases}
\end{align*}
\]

where \( \gamma_1 \) and \( \gamma_2 \) are the predetermined weights reflecting the preference over the two stages’ performance and \( Z^* \) as a decision variable represents an intermediate measure for a specific DMU under assessment. According to model (8), if each stage is efficient,
(that is $\alpha^* = \beta^* = 1$) then the two-stage process also is efficient. Note that model (8) not only measures the overall efficiency, but also obtains optimized values for the intermediate measures of a DMU under evaluation. Chen and Zhu (2004) claimed that model (8) can determine the DEA frontier for the two-stage process so as to project inefficient observations onto the efficient frontier. Chen et al. (2006) applied a DEA model to assess the IT impact on firm performance by considering both stages of the scenario studied in Wang et al. (1997) and Chen and Zhu (2004). They decomposed some inputs from the first stage into the second stage. Another application of this approach is Saranga and Moser (2010) to purchasing and supply management performance.

**Remark 1.** If $\gamma_1 = \gamma_2 = 1$ and we provide the condition of CRS by removing $\lambda_1 = 1$ and $\mu_1 = 1$ the optimal $\alpha^*$ in model (8) is always equal to unity and the optimal $\beta^*$ represents the overall efficiency for the entire process (Chen, Liang, & Zhu, 2009).

Chen et al. (2006) developed a shared two-stage DEA with respect to $X_i$, $Z$, and $Y_j$. Assume, therefore, that $X_i$ is split into two parts $x_iX_1$ and $(1 - x)X_2$. The average CRS ratios of stages 1 and 2 in the program (9) are used to measure the overall efficiency with common input and output weights for the two stages.

$$
E = E_1 \times E_2 = \frac{wZ}{vX_1} = \frac{uY_2}{wZ} = \frac{uY_2}{uX_1},
$$

Consequently, the overall efficiency $E$ under the CRS assumption can be obtained through:

$$
\begin{align*}
\max & \quad E(X_1, Y_2, Z, crs)^{th} = uY_2, \\
\text{s.t.} & \quad \nu X_1 = 1, \\
& \quad uY_2 - nx_1 \leq 0, \\
& \quad wZ - nx_1 \leq 0, \\
& \quad uY_2 - wZ \leq 0, \\
& \quad u, v, w \geq 0.
\end{align*}
$$

The constraint set of (12) is the envelope of those of models (4)–(6). Note that the weight associated with $Z$ in the constraints is assumed to be the common. It means that it does not matter whether the intermediate measures play the role of output or input. This assumption permits the conversion of their original non-linear program into a linear programming problem. This assumption also links the two stages. Note also that the constraint $uY_2 - nx_1 \leq 0$ is redundant in model (12) because of the two constraints $wZ - nx_1 \leq 0$ and $uY_2 - wZ \leq 0$. If $u^*$, $v^*$ and $w^*$ are the optimal multipliers of (12), the overall efficiency, the efficiencies of stages 1 and 2 are calculated by $E = uY_2$, $E_1 = wZ/uX_1$, $E_2 = wZ/wZ^*$, respectively. The optimal multipliers of (12) may not be unique; hence, the decomposition of $E = E_1 \times E_2$ would not be unique. Kao and Hwang (2008) proposed the following model so as to find the set of multipliers that produces the largest $E_1$ while maintaining the overall efficiency score at $E$ calculated from (12):

$$
\begin{align*}
\max & \quad E_1(X_1, Y_2, Z, crs)^{th} = wZ^*, \\
\text{s.t.} & \quad \nu X_1 = 1, \\
& \quad uY_2 - E (\nu X_1) \leq 0, \\
& \quad wZ - \nu X_1 \leq 0, \\
& \quad uY_2 - wZ \leq 0, \\
& \quad u, v, w \geq 0.
\end{align*}
$$

Chen et al. (2009) investigated the relationship between the approaches of Chen and Zhu (2004) and Kao and Hwang (2008) for performance evaluation of two-stage processes. Note that Kao and Hwang’s (2008) model was developed under the CRS technology in the multiplier DEA model (see model (12), while Chen and Zhu (2004)’s model was developed under the VRS technology in the envelopment DEA model (see model (8)).

**Remark 2.** If there is only one intermediate input, then the non-linear DEA model (9) becomes a linear program (Chen et al., 2006).

Contrary to previous studies (e.g. Seiford & Zhu (1999)), which treated the whole process and the two sub-processes as independent, Kao and Hwang (2008) considered a series of relationships between the whole process and the two sub-processes in measuring the efficiencies when a production process is composed of $X_1$, $Z$ and $Y_2$ as depicted in Fig. 2. The overall efficiency is here decomposed into the product of the two individual efficiencies, namely $E = E_1 \times E_2 = \frac{wZ}{vX_1} = \frac{uY_2}{wZ} = \frac{uY_2}{uX_1}$.

Due to the $ku'_2Y'_2$ term, model (10) is a non-linear program. For a given $k(\geq wZ)$, however, the model can be treated as a linear parametric program. The efficiencies of the first and second stages can be then obtained, respectively, through $w'Z'$ and $u'Y'$ where $w'$ and $u'$ are the optimal solutions obtained from (10). The overall efficiency is the average of the two-stage process $1/2(w'Z' + u'Y')$. Furthermore, $z = \nu^* / \nu^*_1$ prescribes how to allocate the resource ($X_1$) to the two stages so as to maximize the average efficiency of whole process.

**Remark 3.** The CRS version of the Chen and Zhu (2004) model under $\gamma_1 = \gamma_2 = 1$ is equivalent to the Kao and Hwang’s (2008) output-oriented model i.e., $E(X_1, Y_2, Z, crs)^{th} = E'(X_1, Y_2, Z, crs)^{th}$ (Chen et al., 2009).

Extending the Kao and Hwang (2008) approach, Chen et al. (2009) suggested a decomposition approach in terms of a weighted sum of the efficiencies of the individual stages for calculating the overall efficiency. In fact, Chen et al. (2009) noted that the two-stage DEA model of Kao and Hwang (2008) cannot be extended to the VRS assumption because $E = ((wZ + u_1')/\nu X_1) \times ((uY_2 + u_2')/wZ)$ could not be transformed into a linear program even when assuming the same weights on the intermediate measures for the two stages. Moreover, the approach of Chen et al. (2009) can be applied under at least CRS and VRS specifications while the method proposed by Kao and Hwang (2008) is restricted to the CRS case. Chen et al. (2009) used a weighted additive (arithmetic mean) approach to calculate the overall efficiency of the process under the VRS assumption by solving the following problem instead of combining the stages in a multiplicative (geometric) way as proposed in Kao and Hwang (2008):


\[
\begin{align*}
\text{max} & \quad E(X_1, Y_2, Z, vrs)_{vcr} = \lambda_1 \left( \frac{wZ + u_1}{vX_1} \right) + \lambda_2 \left( \frac{uY_2 + u_2}{wZ} \right), \\
\text{s.t.} & \quad \frac{(wZ + u_1)}{vX_1} \leq 1, \\
& \quad \frac{(uY_2 + u_2)}{wZ} \leq 1, \\
& \quad u, v, w \geq 0.
\end{align*}
\]

where \(\lambda_1 = \frac{vX_1}{wZ}\) and \(\lambda_2 = \frac{wZ}{vX_1}\) are the relative importances of the performance of stages 1 and 2, respectively, by means of the ‘relative sizes’ of the two stages for measuring the overall performance of the process. By putting the above weights, \(\lambda_1\) and \(\lambda_2\), assigned to the two stages in the objective function of (14) and using the Charnes–Cooper transformation, the linear model (15) can be obtained.

\[
\begin{align*}
\text{max} & \quad E(X_1, Y_2, Z, vrs)_{vcr} = wZ + u_1 + uY_2 + u_2, \\
\text{s.t.} & \quad vX_1 - wZ = 1, \\
& \quad wZ - vX_1 + u_1 \leq 0, \\
& \quad uY_2 - wZ + u_2 \leq 0, \\
& \quad u, v, w \geq 0.
\end{align*}
\]

Analogous to Kao and Hwang (2008), the weights (or multiplier) s on the intermediate measures are the same for the two stages. Once an optimal solution to (15) is obtained, the efficiency scores for the two individual stages can be calculated in the same way as in Kao and Hwang (2008) (see model (13)). In other words, Chen et al. (2009) used Kao and Hwang (2008)’s approach to find a set of multipliers which produces the largest first (or second) stage efficiency score whilst maintaining the overall efficiency score computed from model (15). In the case the first stage is to be given pre-emptive priority, the following model determines its efficiency, while maintaining the overall efficiency score at \(E^*\) computed from model (15).

\[
\begin{align*}
\text{max} & \quad E_1(X_1, Y_2, Z, vrs)_{vcr} = wZ + u_1, \\
\text{s.t.} & \quad vX_1 = 1, \\
& \quad wZ - vX_1 + u_1 \leq 0, \\
& \quad uY_2 - wZ + u_2 \leq 0, \\
& \quad (1 - E^*)wZ + uY_2 + u_1 + u_2 = E^*, \\
& \quad u, v, w \geq 0.
\end{align*}
\]

The efficiency for the second stage is then attained as \(E_2 = (E^* - \lambda_1^1 - \lambda_2^1) / \lambda_2^1\) where \(\lambda_1^1\) and \(\lambda_2^1\) are optimal weights that can be obtained using model (15). In the same way, Chen et al. (2009) approach can be easily applied under the CRS assumption to evaluate the overall efficiency and the individual stages’ efficiencies.

Wang and Chin (2010) demonstrated that a two-stage DEA model with a weighted harmonic mean of the efficiencies of two individual stages is equivalent to Chen et al. (2009)’s model.

Remark 4. The overall efficiency of Chen et al. (2009) is greater than or equal to that of Kao and Hwang (2008) under the CRS assumption i.e., \(E(X_1, Y_2, Z, vrs)_{vcr} \geq E(X_1, Y_2, Z, vrs)_{vcr}^{crs}\) (Wang & Chin, 2010).


Whereas Kao and Hwang (2008) addressed a series system, Kao (2009a) proposed a model for multiple parallel units at both stage. His model minimizes the inefficiency slacks of a DMU as well as inefficiency slacks of its units in order to determine the inefficient units. Kao (2009b) developed an alternative network DEA model by defining dummy processes to transform a network system into a series system (a multi-stage system), where each stage is composed of a parallel structure with a set of processes. Kao (2009a) then built a relational network CRS-DEA (series-parallel systems) both in envelopment and multiplier forms. Hsieh and Lin (2010) applied the relational network DEA introduced by Kao (2009b) to evaluate the performance of a set of hotels.

Wang and Chin (2010) used a weighted harmonic mean of the efficiencies of the two individual stages to obtain the overall efficiency of the process by solving the following problem instead of the approaches of Kao and Hwang (2008) and Chen et al. (2009):

\[
\begin{align*}
\text{max} & \quad E(X_1, Y_2, Z, vrs)_{vcr} = \frac{1}{\lambda_1 \cdot (vX_1/wZ) + \lambda_2 \cdot (wZ/uY_2)} \\
\text{s.t.} & \quad \frac{wZ}{vX_1} \leq 1, \\
& \quad \frac{uY_2}{wZ} \leq 1, \\
& \quad u, v, w \geq 0,
\end{align*}
\]

where \(\lambda_1 = wZ / (vX_1 + uY_2 + wZ)\) and \(\lambda_2 = uY_2 / (vX_1 + uY_2 + wZ)\) are the relative importances of the performances of stages 1 and 2, respectively. These weights, similar to those of Chen et al. (2009), are the relative sizes of the two stages. By substituting \(\lambda_1\) and \(\lambda_2\) into the objective function of (17), we achieve exactly the same model proposed by Chen et al. (2009) in order to get the overall efficiency of two-stage process under the CRS assumption. Likewise, the overall efficiency of two-stage process under the VRS condition can be modeled as follows:

\[
\begin{align*}
\text{max} & \quad E(X_1, Y_2, Z, vrs)_{vcr} = \frac{1}{\lambda_1 \cdot (vX_1/wZ + u_1) + \lambda_2 \cdot (wZ/uY_2 + u_2)} \\
\text{s.t.} & \quad \frac{wZ + u_1}{vX_1} \leq 1, \\
& \quad \frac{uY_2 + u_2}{wZ} \leq 1, \\
& \quad u, v, w \geq 0.
\end{align*}
\]

Similarly, \(\lambda_1\) and \(\lambda_2\) which are the weights assigned to stages 1 and 2 can be defined as \((wZ + u_1) / (wZ + u_1 + uY_2 + u_2)\) and \((uY_2 + u_2) / (wZ + u_1 + uY_2 + u_2)\), respectively. By setting these weights into the objective function of (18), the same model defined by Chen et al. (2009) can be obtained for evaluation of the overall efficiency under the VRS assumption. Once \(E_1\) or \(E_2\) is obtained, using the proposed approach by Chen et al. (2009), the other one can be determined by \(E_2 = \lambda_2 \cdot (uY_2 / (1 - E^*)) - \lambda_1 \cdot (vX_1 / (1 - E^*))\) or \(E_1 = \lambda_1 \cdot (vX_1 / (1 - E^*)) - \lambda_2 \cdot (uY_2 / (1 - E^*))\), where \(\lambda_1\) and \(\lambda_2\) are harmonic mean weights.

Remark 5. \(E(X_1, Y_2, Z, vrs)_{vcr} = E(X_1, Y_2, Z, vrs)_{vcr}^{crs}\) where \(v = crs\) and \(v = vrs\). Chen, Du, Sherman, and Zhu (2010) proposed a DEA model under VRS for measuring the efficiency of a two-stage system with one shared input. The structure consists of \(X_1, Y_2, Z\) and \(Y_2\) (see Fig. 2). The shared inputs, \(X_1\), can be split into two parts \(X_1\) and \(X_2\), where \(L_1 \leq X \leq L_2\).

The overall efficiency was defined in Chen et al. (2006) as the average efficiency of stages 1 and 2 under CRS (see model (9)). An alternative definition in Chen et al. (2010) and Chen et al. (2009), draws on a weighted average of the efficiencies of the two stages as follows:

\[
\begin{align*}
\lambda_1 \left( \frac{wZ + u_1}{v_1X_1 + b_2X_3} \right) + \lambda_2 \left( \frac{uY_2 + u_2}{v_1(1 - X_1) + wZ} \right),
\end{align*}
\]

where \(\lambda_1 = (v_1X_1 + b_2X_3) / (v_1 + b_2)\) and \(\lambda_2 = (v_1(1 - X_1) + wZ) / (v_1 + b_2)\). The non-linear program (20) is created by substituting \(\lambda_1\) and \(\lambda_2\) in (18) and using the Charnes–Cooper transformation.
\[
\begin{align*}
\text{max} & \quad E(X_1, X_2, Z, vrs)^{dz}_w = wZ^2 + u_1 + uY_2 + u_2, \\
\text{s.t.} & \quad \nu_1X_1 + \nu_2X_1^2 + wZ^2 = 1, \\
& \quad wZ - (\nu_1X_1 + \nu_2X_1^2 + u_1) \leq 0, \\
& \quad uY_2 - (\nu_1(1 - z)X_1 + wZ) + u_2 \leq 0, \\
& \quad L_1 \leq \nu \leq L_2, u, \nu, \nu_1, \nu_2, w \geq 0.
\end{align*}
\]

Model (20) can be transformed to a linear form using the alternative variable \( \nu = \gamma \). Chen et al. (2010) applied Kao and Hwang’s (2008) approach to deal with the problem of multiple optimal solutions.

**Remark 6.** The overall efficiency proposed by Chen et al. (2009) and Chen et al. (2010) are equal if we take the shared inputs defined in (Chen et al., 2010) away from the performance evaluation, i.e., \( E(X_1, Y_2, Z, vrs)^{dz}_w = E(X_1, Y_2, Z, vrs)^{dz}_w \), iff \( X_1 = 0 \).

Wang and Chin (2010) extended the Kao and Hwang (2008) model to the VRS assumption. We should note that the first stage is evaluated with the input-oriented VRS model and the second stage with the output-oriented VRS model. Kao and Hwang (2008)’s model under the VRS assumption can therefore be expressed as

\[
\begin{align*}
\text{max} & \quad E(X_1, Y_2, Z, vrs)^{kw}_w = uY_2^2 + u_2, \\
\text{s.t.} & \quad \nu X_1 - u_1 = 1, \\
& \quad wZ - \nu X_1 + u_1 \leq 0, \\
& \quad uY_1 - wZ + u_2 \leq 0, \\
& \quad u, \nu, w \geq 0.
\end{align*}
\]

The optimal multipliers of (21) may not be unique; hence, the decomposition of \( F = E_1 \times E_2 \) would not be unique. Hence, similar to Kao and Hwang (2008) it has been suggested to use the set of multipliers producing the largest \( E_1(E_2) \) while preserving the overall efficiency score at \( E \) calculated from (21).

**Remark 7.** The overall efficiency under the assumption of VRS is decomposed into the product of the two individual efficiencies i.e.,

\[
E(X_1, Y_2, Z, vrs)^{kw}_w = E(X_1, Y_2, Z, vrs)^{kw}_w \times F_2(Y_2, Z, vrs)^{kw}_w
\]

\[
= \frac{wZ}{X_1 - u_1} \times \frac{uY_2^2 + u_2}{wZ} = \frac{uY_2^2 + u_2}{uZ}.
\]

Wang and Chin (2010) generalized Chen et al. (2009)’s models to assess the relative performance of two individual stages. To do so, a two-stage process is transformed to a single process in which the two stages are treated equally. In other words, the single process considers stage 1’s input \( X_1 \) and an intermediate measure \( Z \) as inputs, and stage 2’s output \( Y_2 \) and an intermediate measure \( Z \) as outputs. The generalized overall efficiency of Chen et al. (2009)’s model under the VRS assumption is formulated in model (20).

\[
\begin{align*}
\text{max} & \quad E(X_1, Y_2, Z, vrs)^{kw}_w = \frac{\lambda_1(wZ^2 + u_1) + \lambda_2(uY_2^2 + u_2)}{\lambda_1/wZ^2 + \lambda_2/wZ}, \\
\text{s.t.} & \quad \frac{wZ + u_1}{X_1} \leq 1, \\
& \quad \frac{uY_2^2 + u_2}{wZ} \leq 1, \\
& \quad u, v, w \geq 0.
\end{align*}
\]

The objective function of (22) can be transformed into a weighted harmonic mean as

\[
F = \frac{1}{\gamma_1 \frac{\nu X_1}{wZ} + \gamma_2 \frac{wZ}{uY_2^2 + u_2}}
\]

where

\[
\begin{align*}
\gamma_1 &= \frac{\lambda_1(wZ^2 + u_1)}{\lambda_1/wZ^2 + \lambda_2/wZ}, \\
\gamma_2 &= \frac{\lambda_1(wZ^2 + u_1) + \lambda_2(uY_2^2 + u_2)}{\lambda_1/wZ^2 + \lambda_2/wZ}.
\end{align*}
\]

**Remark 8.** Model (14) by Chen et al. (2009) is a special case of (22) where \( \lambda_1 = \lambda_2 = 1/2 \), i.e., \( E(X_1, Y_2, Z, vrs)^{dz}_w = E(X_1, Y_2, Z, vrs)^{dz}_w \). (Proof in Wang & Chin (2010))

**Remark 9.** If \( u_1 = u_2 = 0 \) in model (22), the generalized overall efficiency \( E(X_1, Y_2, Z, crs)^{kw}_w \) under the CRS assumption can be derived.

Chen, Cook, and Zhu (2010) proposed an approach to specify the frontier points for inefficient DMUs based upon the Kao and Hwang (2008)’s model. The dual of model (12) proposed by Kao and Hwang (2008) can be expressed as

\[
\begin{align*}
\text{min} & \quad DE(X_1, Y_2, Z, crs)^{kw}_w = \theta, \\
\text{s.t.} & \quad X_1, \mu \leq 0X_1, \\
& \quad Y_2, \mu \geq 0Y_2, \\
& \quad Z(\lambda - \mu) \geq 0, \\
& \quad \mu, \theta \leq 0, \theta \leq 1.
\end{align*}
\]

Model (12) can just obtain an overall efficiency score under the assumption of CRS, but would not be able to identify how to project inefficient DMUs onto the DEA frontier. Chen et al. (2010), therefore, put forward the following model that is equivalent to model (23):

\[
\begin{align*}
\text{min} & \quad DE(X_1, Y_2, Z, crs)^{kw}_w = \theta, \\
\text{s.t.} & \quad X_1, \lambda \leq 0X_1, \\
& \quad Y_2, \mu \geq 0Y_2, \\
& \quad Z(\lambda - \mu) \geq 0, \\
& \quad \lambda, \mu, z^* \geq 0, \theta \leq 1.
\end{align*}
\]

where the decision variable \( z^* \) in the constraints \( Z(\lambda - \mu) \geq 0 \) and \( Z(\lambda - \mu) \) is treated as output and input, respectively, for the intermediate measure. According to model (24), the projection point for DMU is given by \((\theta X_1^*, z^*, Y_2^*)\) which is efficient under models (24) and (23).

Yang, Wu, Liang, and Xu (2011) proposed a CRS DEA approach to measure the overall efficiency of the entire supply chain using the predefined PPS. By comparing the obtained supply chain frontier with other supply chains, chain-level performance can be identified as efficient or inefficient. The efficiency perspective and corresponding improvement strategies for inefficient supply chains can be given at the same time. Fig. 2 shows the two-stage structure (supplier-manufacturer) without \( X_1 \) and \( Y_1 \) in Yang et al. (2011). It is assumed that all supply chains are separable and their members can be aggregated with other supply chain members so as to make a virtual supply chain. The PPS can be characterized by all existing supply chains and some virtual supply chains. Thus, the sub-perfect supply chain CRS PPS is defined as follows:

\[
T = \{(X_1, Y_2) \mid \nu X_1 \geq X_1, \lambda Z = Z, \lambda Z \geq Z, \lambda Y_2(E_2^*) \geq Y_2, \lambda \geq 0\},
\]

where \( E_1^* \) and \( E_2^* \), which can be obtained from models (5) and (6), are the CRS efficiencies of stages 1 and 2, respectively. Note that in the proposed PPS the corresponding envelopment coefficients for each DMU are the same \( \iota \) i.e. the members of each virtual supply chain are restricted in the same actual supply chain. Note also that \((X_1^*, Z)\) and \((Z, Y_2^*)\) are projections of stage 1 and stage 2 for
DMU: Based upon the PPS, the overall efficiency of a supply chain is modeled as follows:

\[
\begin{align*}
\text{min} \quad & E^* = 0, \\
\text{s.t.} \quad & X_1^* \leq X_1, \quad Y_1^* \leq Y_2, \\
& X_1^* E_1^* \leq X_1^*, \\
& IZ \geq Z^+, \quad ZZ \leq Z^+, \\
& \lambda Y_2^*/E_2^* \geq Y_2^*, \\
& \lambda, \lambda \geq 0.
\end{align*}
\] (25)

where \((X_1^*, Y_2^*, Z^*)\) are points located at the frontier enveloped by the PPS of the sub-perfect supply chain under CRS (see T). The following model is equivalent to (25):

\[
\begin{align*}
\text{min} \quad & E^*, \\
\text{s.t.} \quad & X_1^* \leq X_1, \quad Y_1^* \leq Y_2, \\
& X_1^* E_1^* \leq X_1^*, \\
& IZ \geq Z^+, \quad ZZ \leq Z^+, \\
& \lambda \geq 0.
\end{align*}
\] (26)

Yang et al. (2011) also proved that \(E^*\) computed from model (26) is a weak lower bound to \(E^*\) obtained from model (1) under CRS. Moreover, they demonstrated that the optimal value of (26) is a weak lower bound to \(E_1^* \times E_2^*\). Their proposed approach can be applied to evaluate the efficiency of multiple-member supply chains.

In Table 4, we summarize relationships and features of the two-stage DEA studies presented in this subsection.

### 5.3. Game theoretical DEA models

Game theory allows us to explicitly model the sequence of bargaining and the strategic interaction present in decentralized decision making, such as supply chain management. Game theory has been successfully applied both to supply chain management coordination in general, and to normative applications of frontier models. Liang et al. (2006) proposed two DEA-based models for measuring the efficiency of a supply chain and its members (stages 1 and 2) using the concept of non-cooperative and cooperative games. The models are set in a seller-buyer supply chain context, when the relationship between the seller and buyer is treated first as a Stackelberg bi-level non-cooperative game, and second as a cooperative game. In the non-cooperative (leader–follower) game, the leader is first assessed, and then the follower is evaluated using the leader’s efficiency. In the cooperative structure, the overall efficiency which is modeled as an average of the two stages’ efficiencies is maximized, and both supply chain members are evaluated simultaneously. The resulting cooperative game model is a non-linear DEA model which can be solved as a parametric linear programming problem. Fig. 2 without \(Y_1\) and \(X_3\) shows the buyer-seller supply chain examined by Liang et al. (2006). Under a non-cooperative setting, they assumed that the first and second stages are the seller (leader) and the buyer (follower), respectively, and the efficiency of the first stage \((E_1)\) is obtained using the standard input oriented CRS model. Then the efficiency of stage 2 is calculated while preserving the optimal efficiency of stage 1 \((E_1^*)\). It means dominating stage 2 by stage 1. The second stage’s efficiency, therefore, can be obtained as

\[
\begin{align*}
\text{max} \quad & E_2 = \frac{u_2 Y_2^*}{v_2 X_2^* + D \times w^*}, \\
\text{s.t.} \quad & \frac{v_2 X_2 + D \times w^*}{u_2 Y_2} \leq 0, \\
& w^* - v_1 X_1 \leq 0, \\
& v_1 X_1 = 1, \\
& w^* = E_1^*, \\
& u_2, v_1, v_2, w, D \geq 0.
\end{align*}
\] (27)
where $0 < D < 1/E^*$, Model (27) can be converted into the following non-linear program:

$$\begin{align*}
\text{max} & \quad E_2 = u_2Y_2, \\
\text{s.t.} & \quad v_2X_2 + DwZ' = 1, \\
& \quad u_2Y_2 - v_1X_1 - DwZ \leq 0, \\
& \quad wZ - v_1X_1 \leq 0, \\
& \quad v_1X_1 = 1, \\
& \quad wZ' = E_1^*, \\
& \quad u_2, v_1, v_2, w, D \geq 0,
\end{align*}$$

where $0 < D < 1/E^*$ and therefore $D$ can be treated as a parameter.

That is, model (28) can be considered as a parametric linear program. Once the first and second stage's efficiency are obtained by weights on the intermediate measures are equal. The following cooperative game model, hence, seeks to maximize two efficiency functions for the first and second stages.

Chen et al. (2006) determined the Nash equilibria in the existing game between the supplier and the manufacturer. Notice that $S(E_2)$ and $M(E_1)$ are functions of $E_2$ and $E_1$, respectively. If $E_2 = M(S(E_2))$, $(E_2, E_1)$ is a Nash equilibrium, otherwise, Nash equilibria do not exist. Likewise, Nash equilibria exist if $E_1 = S(M(E_1))$. In addition, Chen et al. (2006) mentioned some properties on the two efficiency functions as well as extending their method to the centralized control system.

Cook, Zhu, Bi, and Yang (2010) extended Liang et al. (2006) to take into account multi-stage structures i.e., more than two stages in the CRS and VRS technologies. They calculated the overall efficiency as an additive weighted average of the efficiencies of the individual stages. In addition, the developed model in (Cook et al., 2010) was a LP while Liang et al. (2006) model used a heuristic search algorithm after converting the non-linear model into a parametric linear model.

Using the geometric mean of the efficiencies of the two stages, Zha, Liang, and Xu (2008) proposed a two-stage cooperative efficiency to calculate the overall efficiency under DEA-VRS model. They suggested the efficiency of the first stage to be evaluated with the input-oriented VRS model and the second stage with the output-oriented VRS model. Then, the overall efficiency is evaluated in a cooperative framework. The upper and the lower bounds are reached when a non-cooperative framework is considered. The non-linear model is transformed into a parametric one, where an optimal solution of the overall efficiency is easily reached. If an input-oriented VRS model is suggested for performance evaluation, inconsistency of the intermediate outputs exists between the two stages. Specifically, the two stages are cooperative for the reason that they are in series in an organization. They considered the non-cooperative setting in order to determine the upper and lower bounds of the efficiencies of the sub-DMUs in different stages. Two conditions are examined as follows.

(i) Sub-DMU in stage 1 dominates the system, while the sub-DMU in stage 2 follows. (ii) Sub-DMU in stage 2 dominates the system, while the sub-DMU stage 1 follows.

In both conditions, intermediate outputs need to be consistent between the two stages. So, an input-oriented VRS model is suggested when evaluating the efficiency of the sub-DMU in stage 1, and an output-oriented VRS model is suggested when evaluating the efficiency of the sub-DMU in stage 2.

The upper bound of the efficiency of stage 1 is expressed as follows:

$$\begin{align*}
\text{min} & \quad E_1^{ub} = uX_1 + u_1, \\
\text{s.t.} & \quad wZ = 1, \\
& \quad vX_1 + u_1 - wZ \geq 0, \\
& \quad v, w \geq 0.
\end{align*}$$

The lower bound of the efficiency of stage 2 can be calculated by the following model with holding the optimal value of $E_1^{ub}$ obtained from (33):

$$\begin{align*}
\text{max} & \quad E_2^{lb} = uY_2 - u_2, \\
\text{s.t.} & \quad wZ = 1, \\
& \quad vX_1 + u_1 - E_1^{ub} \geq 0, \\
& \quad u_2 - wZ - u_2 \leq 0, \\
& \quad u, v, w \geq 0.
\end{align*}$$

Note that stage 2 is entirely dominated by stage 1. Likewise, the upper bound of the efficiency of stage 2, denoted by $E_2^{ub}$, is first acquired, then, with holding $E_1^{ub}$, the lower bound of the efficiency of stage 1, denoted by $E_1^{lb}$, is calculated. Zha et al. (2008) considered...
the overall efficiency as the geometric mean of the efficiencies of the two stages. Hence, they assume that the efficiency of stage 1 and stage 2 are evaluated using the input-oriented and the output-oriented models, respectively. The geometric average cooperative efficiency of the two stages is obtained by the following model

$$\max \ E = \frac{wZ}{uX_1 + u_1} \times \frac{uy_2 - u_2}{wZ},$$

s.t. $$\frac{wZ}{uX_1 + u_1} \leq 1, \ \frac{uy_2 - u_2}{wZ} \leq 1,$$

$$\frac{wZ}{uX_1 + u_1} \leq \frac{wZ}{wZ}, \ \frac{uy_2 - u_2}{wZ} \leq \frac{1}{wZ},$$

$$u, v, w > 0.$$  

(35)

Eq. (35) can be transformed into

$$\max \ E = uy_2 - u_2,$$

s.t. $$uX_1 + u_1 = 1,$$

$$wZ - uX_1 - u_2 \leq 0, \ \frac{wZ}{uX_1 + u_1} \leq \frac{1}{wZ},$$

$$\frac{wZ}{uX_1 + u_1} \leq \frac{wZ}{wZ}, \ \frac{1}{wZ} \leq \frac{1}{wZ},$$

$$u, v, w > 0.$$  

(36)

Liang, Cook, and Zhu (2008) also developed a two-stage model using non-cooperative and cooperative concepts in game theory. In a non-cooperative approach, they assumed that the efficiency of the first stage is maximized simultaneously, while determining a set of optimal (common) weights assigned to the intermediate measures.

Consider Fig. 2 without X2 and Y1. It is assumed that the first and second stages are the leader and the follower, respectively, the efficiency of the first stage (E1) is obtained using the standard input oriented CRS model. Next, the efficiency of stage 2 is computed while preserving the optimal value of E1. It means that stage 2 is entirely dominated by stage 1. The second stage's efficiency is then obtained as

$$\max \ E_2 = \frac{1}{E_1} - uy_2,$$

s.t. $$uy_2 - wZ \leq 0,$$

$$wZ - uX_1 \leq 0, \ \frac{1}{E_1} - uy_2 \leq wZ,$$

$$u, v, w > 0.$$  

(37)

Liang, et al. (2008) measuring the efficiency of the two stages of a cooperative setting. The cooperative approach is characterized by letting the same weights for intermediate data in the two-stage models. Note that because of the same weights for intermediate data, the overall efficiency (E1, E2) becomes uy2/ux1 which can be modeled as follows:

$$\max \ E = E_1 \cdot E_2 = \frac{uy_2}{ux_1},$$

s.t. $$E_1, E_2 \leq 1 \ \text{and} \ E_2 \leq 1, \ \forall j.$$  

(38)

The linear program of (38) is:

$$\max \ E_1 = \frac{uy_2}{ux_1},$$

s.t. $$uy_2 - wZ \leq 0,$$

$$wZ - uX_1 \leq 0,$$

$$tX_1 = 1,$$

$$u, v, w > 0.$$  

(39)

The efficiencies of the first and second stages can be then calculated as E1 = WZ1/νX1 = WZ1 and E2 = WY2/WZ.

Note that optimal multipliers from model (39) may not be unique, as a result, E1 and E2 may not be unique. To determine the unique solution, the maximum value of E1 is first calculated using the following model:

$$\max \ E_1 = wZ1,$$

s.t. $$uy_2 - wZ \leq 0,$$

$$wZ - uX_1 \leq 0,$$

$$wZ - uX_1 \leq 0,$$

$$tX_1 = 1,$$

$$u, v, w > 0.$$  

(40)

The minimum of E2 is then calculated by E2 = E1/E1. In a similar way, the maximum of E3 and the minimum of E1, denoted by E3 and E1 respectively, can be obtained. Note that E1 = E1 if and only if E3 = E3. If E1 = E1 or E2 = E2, then E1 and E2 are uniquely determined using model (39), otherwise, E1 and E2 lead to multiple optimal solutions. In the case of E1 ≠ E1 or E2 ≠ E2, Liang et al. (2008) proposed a procedure to achieve a fair and alternative distribution of E1 and E2 between the two stages.

Zha and Liang (2010) developed an approach to measure the performance of a two-stage process in a non-cooperative and cooperative manner within the framework of game theory, where the shared inputs can be allocated among different stages. Similar to Chen et al. (2006), Zha and Liang (2010) used Fig. 2 as a two-stage process with shared inputs, e.g. all inputs (denoted as X3) are directly associated with the two stages. To do this, it is assumed that X3 is divided into two parts αX3 and (1 - α)X3, Zha and Liang (2010) utilized the product of the two stages to evaluate the overall efficiency of each DMU while the average of the two stages was only used in the Liang et al. (2006)’s cooperative model. Let us assume that the first and second stages are the leader and the follower, respectively, for the non-cooperative evaluation. First the efficiency of the first stage (E1) can be calculated using the input-oriented CRS model as follows:

$$\max \ E_1 = wZ1,$$

s.t. $$wZ - tZ3X_3 \leq 0,$$

$$tZ3X_3 = 1,$$

$$tZ3, w > 0.$$  

(41)

The second stage's efficiency can be then obtained from the following program subject to the restriction that the efficiency of the first stage remains at optimal value E1:

$$\max \ E_2 = uy_2,$$

s.t. $$uy_2 - (ωWZ + νX_1) \leq 0,$$

$$wZ - tZ3X_3 \leq 0,$$

$$tZ3X_3 = 1,$$

$$wZ = E1,$$

$$tZ3X_3 + δE1 = 1,$$

$$u, ν, tZ3, w > 0.$$  

(42)

Model (42) is a non-linear program due to the term ω in the first constraint. However, this model can be treated as a parametric linear program since in specifying the optimum E1, δ ∈ [0, 1] (with regard to the interval 0 < δ < 1/E1) is considered as a parameter.
On the other hand, the second stage can be the leader and then one obtains the efficiency of the first stage (follower) model based on stage 2. In order to do so, the efficiency of stage 2 can be first calculated as in the following model:

\[
\begin{align*}
\max \quad & E_2^* = uY_2^*, \\
\text{s.t.} \quad & uY_2 - (wZ + v_2X_3) \leq 0, \\
& w^2 + v_2X_3 = 1, \\
& u, v_2, w \geq 0.
\end{align*}
\]

Note that model (43) corresponds to the conventional CRS DEA model. Assume that \(u^*, v_2^*\) and \(w^*\) are optimal solutions for (43). We must investigate three cases to obtain the efficiency of stage 1. In the first case, if there exists a given \(d = 1, \ldots, p\) satisfying \(w^2 = 0\), the efficiency of stage 1 is equivalent to

\[
\begin{align*}
\max \quad & E_1 = \delta wZ^*, \\
\text{s.t.} \quad & \delta wZ - v_2X_3 \leq 0, \\
& v_2X_3 = 1, \\
& v_2, \delta \geq 0.
\end{align*}
\]

In the second case, \(w^* = 0\), accordingly, the efficiency of stage 1 becomes zero and, lastly, if there exist multiple optima in model (41), the efficiency of stage 1 when dominated by stage 2 can be expressed as

\[
\begin{align*}
\max \quad & E_1 = \delta wZ^*, \\
\text{s.t.} \quad & \delta wZ - v_2X_3 \leq 0, \\
& v_2X_3 = 1, \\
& v_2, \delta \geq 0.
\end{align*}
\]

Note that the efficiency of stage 1 obtained from (45) is less than (44). In a special case of a single intermediate product, the optimal values of the objective function (42) and (43) are equal.

In the cooperative efficiency, while the weights of the intermediate outputs in stage 1 are equal to the weights of the corresponding intermediate inputs in stage 2, the product of stages 1 and 2 for measuring the overall efficiency can be expressed as

\[
\begin{align*}
\max \quad & E = \left( wZ^* + v_2X_3 \right) \times \frac{uY_2}{w^2 + v_2(1-x)X_3}, \\
\text{s.t.} \quad & \frac{wZ}{v_2X_3} \leq 1, \\
& \frac{uY_2}{w^2 + v_2(1-x)X_3} \leq 1, \\
& u, v_2, w \geq 0.
\end{align*}
\]

Model (46) is a non-linear program that can be rewritten as:

\[
\begin{align*}
\max \quad & E = wZ^* + uY_2, \\
\text{s.t.} \quad & wZ^* - v_2X_3 \leq 0, \\
& v_2X_3 = 1, \\
& \delta \left( h - v_2X_3 + wZ^* \right) = 1, \\
& uY_2 - \delta \left( h - v_2X_3 + wZ^* \right) \leq 0, \\
& h \geq u \geq 0, \\
& u, v_2, w \geq 0.
\end{align*}
\]

As for the non-cooperative approach, consider a bargaining parameter \(k\) such that \(L < k < U\) where \(L\) is the efficiency of stage 1 when stage 2 has all bargaining power and \(U\) is the efficiency of stage 1 when it has all bargaining power, respectively. Also, assume that \(\delta(h - v_2X_3 + wZ^*) = \delta\). Accordingly, (47) is transformed into

\[
\begin{align*}
\max \quad & E = k \times uY_2, \\
\text{s.t.} \quad & wZ^* = k, \\
& L \leq k \leq U, \\
& wZ^* - v_2X_3 \leq 0, \\
& v_2X_3 = 1, \\
& v_2X_3 + \delta wZ^* = 1, \\
& uY_2 - (v_2X_3 + \delta wZ^*) \leq 0, \\
& u, v_2, w \geq 0.
\end{align*}
\]
The leader uses two types of inputs, i.e., the shared input $X_1$ and the direct input $X_2$, to produce two different types of outputs: the intermediate output $Z$ and the direct output $Y$. The follower uses three types of inputs, i.e., the shared input $X_3$ and the direct input $X_2$ and the intermediate output $Z$, to produce the output $Y_2$. Furthermore, assume that $C_1$, $C_2$, $C_3$ and $C_4$ are the input unit cost vectors associated with $X_1$, $X_2$, $X_3$ and $Z$, respectively. In fact, the exact fixed value and maximum fixed value are two separate cases for the total amount of the shared resource that we can take into account as an extra constraint. According, when the total amount of the shared input is fixed the bi-level programming cost efficiency DEA model can be expressed as:

\[
\begin{align*}
\text{(E}_1 \text{)} & \quad \min_{\lambda, \mu} \left( C_1 \lambda X_1 + C_1 \mu X_1 + C_2 \lambda X_2 + C_2 \mu X_2 + C_3 \lambda X_3 + C_3 \mu X_3 + C_4 \lambda Z + C_4 \mu Z \right), \\
\text{s.t.} & \quad \lambda X_1 \geq X_1, \lambda X_2 \geq X_2, \lambda X_3 \geq X_3, \\
& \quad \mu \lambda X_1 \geq X_1, \mu \lambda X_2 \geq X_2, \mu \lambda X_3 \geq X_3, \\
& \quad \lambda X_1 + \lambda X_2 = F, \\
& \quad \lambda X_3 + \lambda Z = F, \\
& \quad \mu \lambda X_1 + \mu \lambda X_2 + \mu \lambda X_3 + \mu \lambda Z = F, \\
& \quad \lambda, \mu \geq 0.
\end{align*}
\]

The shared input $X_1$, the direct input $X_1$ and an optimal multiplier $\lambda$ can be calculated by the first level of model (51) so as to minimize the total costs for the leader. As a result, $X_1$ is simply obtained for the follower using $X_1 + X_1 = F$. Note that in the above bi-level programming cost efficiency DEA model the intermediate measure is the output for the leader in the upper level and also the input for the follower in the lower level. The second case is when the total amount of the shared input has the fixed maximum value. To do this, we substitute $X_1 + X_2 = F$ for $X_1 + X_2$ in model (51). Wu et al. (2010) applied the branch and bound algorithm proposed by Shi, Zhang, Lu, and Zhou (2006) to solve model (51). Once the optimal value of $(X_1^{\lambda}, X_1^{\mu}, X_1, X_2, Z, \lambda, \mu)$ is obtained from model (51) the cost efficiencies of the jth leader ($CE_1$), the jth follower ($CE_2$) and the jth system ($CE_3$) are defined as

\[
\begin{align*}
CE_1 &= C_1 \lambda X_1^{\lambda} + C_1 \mu X_1^{\mu}, \\
CE_2 &= C_2 \lambda X_2^{\lambda} + C_2 \mu X_2^{\mu} + C_2 \mu Z^{\mu}, \\
CE_3 &= \frac{C_3 \lambda X_3^{\lambda} + C_3 \mu Z^{\mu}}{C_3 X_3^{\lambda} + C_3 X_3^{\mu} + C_3 Z^{\mu}}.
\end{align*}
\]

The jth leader, the jth follower and the jth system are cost efficient if and only if $CE_1 = 1$, $CE_2 = 1$ and $CE_3 = 1$, respectively. In addition, Wu (2010) similarly to Cooper et al. (2000) used the reference units to rank the efficient DMUs.

6. Conclusions and future research directions

Supply chain management (SCM) covers several disciplines and is rapidly growing. Performance measurement is an important activity, both for planning and optimization purposes. DEA as a non-parametric technique for measuring efficiency continues to enjoy increasing popularity. Reviewing the multi- and two-level extensions published in the DEA literature reveals a considerable wealth of different models, based either on restrictions in the reference set, the weight system or the sequence of optimization of the DMU problems.

However, the analysis also shows several open problems in the application of DEA to supply chain performance measurement.

First, the existing models demonstrate the limitations of and the rigidity in the model specification process. Where supply chains by definition involve several stages (normally at least three) interacting independently with markets for raw materials and intermediate outputs, bulk of the extensions are limited by explicit or implicit restrictions to two-stage processes with no third-party interaction. In practice, this implies a strict dyadic buyer-seller dichotomy in which all intermediate outputs are consumed by a single entity. The assumption is very strong and in open contradiction to standard results in multi-stage supply chain planning models, where intermediate plants and distribution centers are expected to serve multiple downstream units, within and/or

Table 5

<table>
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<tr>
<th>Model's name</th>
<th>Structure</th>
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<th>Type</th>
<th>Specifications</th>
<th>Application/Example</th>
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<td>Cooperative (Average of the stages), non-cooperative (leader–follower)</td>
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<tr>
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<tr>
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<td></td>
</tr>
<tr>
<td>Liang et al. (2008)</td>
<td>$X_1$, $X_2$, Z</td>
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<td>Cooperative (product of the stages), non-cooperative (leader–follower)</td>
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<td></td>
</tr>
<tr>
<td>Zha and Liang (2010)</td>
<td>$X_1$, $X_2$, Z</td>
<td>CRS</td>
<td>Multiplier</td>
<td>Cooperative (product of the stages), non-cooperative (leader–follower)</td>
<td></td>
<td></td>
</tr>
<tr>
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without the focal enterprise. Moreover, the lack of flexibility in the model structure is commonly motivated by the solution approach, derivations of joint metrics etc. that consequently hamper the generalization of the results to a realistic situation. Further work is necessary on this fundamental point to allow applications of frontier-based methods to real multi-stage supply chains.

Second, most models lack a clear economic or technical motivation for the intermediate measures. Besides the multi-stage property, one of the underlying features distinguishing supply chain management from general operations management is the prevalence of decentralized decision making. In economics and management science, we tend to attribute these decision makers with some procedural rationality that renders them susceptible to mathematical modeling. A common assumption is that the decision makers maximize some profit or objective function subject to some rationally imposed constraints, e.g. resource allocation across a group. It is therefore necessary for any performance assessment to take into account the objectives of the underlying units in their assessment, if the resulting estimate is to have any relevance as an indication for the effectiveness of their decision making. We note that some suggested models tend to abstract from the economic or preferential reality of the evaluated units in assuming that their objectives per se should be related to, or even centered on, the very metric that analysts propose for their evaluation. In fact, most models dispose of this step by simply assuming that the objectives of the unit correspond to the maximization of some single-stage evaluation problem, such as the conventional CRS formulation. Already in a single-stage setting, the interpretation of productivity measures is associated with many limitations, cf. Agrell and West (2001).

In the supply chain setting, considering the interdependencies between the levels and the ambiguous character of the input resource restrictions, this hurdle is even more delicate to solve, both conceptually and mathematically. Here we need careful and well-justified behavioral motivation for the submodels, as well as an economically well-founded framework for the centralized models. Further consolidation of the literature based on game-theoretical approaches may be way to address this shortcoming.

Third, the existing literature largely neglects an explicit modeling of the power or governance structures within the supply chain. Given the absence of a centralized decision maker, the modeler faces a hierarchical multi-criteria problem without any clear preference structure. Whereas conventional approaches in economics would use Stackelberg-type bi-level games or Nash bargaining concepts, the supply chain management literature frequently employs non-cooperative and cooperative game theoretical approaches. Although some models are founded on elements hereof, there is need of stringent models unifying the evaluation model with the underlying assumptions about the power or governance structure within the chain. Such work, founded on economic theory and decision theory, may also eliminate the too frequent resort to ad hoc technical and scaling parameters in the models without any methodological foundation.

Fourth, the relevant literature mostly takes into account the predominance of multiplicative models. Multi-product networks, especially for dynamic approaches, involve relatively large dimensional output vectors and likely (correctly) zero-valued observations. Multiplicative approaches (radial efficiency metrics) here yield computationally poor results with efficiency scores in the presence of significant slack, i.e. weak technical efficiency. Additive models (seminal work by Charnes, Cooper, Golany, Seiford, & Stutz, 1985) are traditionally viewed as inferior, lacking translation- and unit invariance (cf. Ali & Seiford, 1990) and being difficult to decompose into relevant submeasures. However, the special structure for supply chain problems, in which measurement units often can be qualitatively homogenous (value, weight, energy contents, pieces) may enable simple, consistent and informative decompositions based on additive transformations as in Agrell and Bogetoft (2005). The use of additive approaches also opens for analyses of multi-output cases of more realistic dimensions in terms of cost- versus technical efficiency. More work is necessary to determine the properties and robustness of such models in generalized multi-stage settings. The work by Chang et al. (2011) based on the non-radial Tone and Tsutsui (2009) model is here particularly interesting, also from a conceptual viewpoint. Stating these areas of desired progress is in no way negating the positive and productive wealth of work in the areas of two-stage non-parametric frontier models. On the contrary, it is this energy and thrust that will unlock the force of the models to attack the so far unsolved, frustrating and decisive problems found in supply chain performance measurement.

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