On the Use of Correlative Coding for OFDM

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Abstract—Recent published work suggests that correlative coding can improve OFDM performance by several dB. This work, however, does not consider fully a symbol error rate performance measure for the OFDM systems. Exploiting an equivalence between correlative coding and partial-response signaling, a bit error rate performance measure is employed to assess the performance of OFDM systems using correlative coding. It is shown that although partial-response pulse-shaping or correlative coding can reduce intercarrier interference due to carrier frequency offset, they do not necessarily improve the system bit error rate performance because of the introduction of multilevel signaling with associated reduced receiver decision distance. It is observed that bit error rate performance improvement for symbol-by-symbol detection can be achieved only in some cases of large frequency offset. In the case of smaller, practical, values of frequency offset the bit error rate is increased by correlative coding or partial-response pulse-shaping.

I. INTRODUCTION

Transmitter pulse-shaping can effectively reduce intercarrier interference (ICI) introduced by frequency offset [1] in orthogonal frequency-division multiplexing (OFDM) systems. However, when using a Nyquist pulse with roll-off factor $\alpha$, the OFDM symbol time duration will increase proportionally by $\alpha$. Thus, the data transmission rate will decrease. Partial-response signals can also be used in OFDM transmitter pulse-shaping to reduce ICI. The principle of partial-response signaling is to introduce some known ICI. At the receiver, the intentionally introduced ICI can be removed since it is known. Significantly, the data transmission rate will not be decreased by using partial-response pulse-shaping. This scenario is similar to the partial-response pulse design problem for single carrier systems in intersymbol interference (ISI) environments.

Correlative coding across subcarriers in OFDM systems was proposed, in reference [2], as an ICI reduction scheme. It was shown that, compared to a system without correlative coding, the signal-to-interference ratio (SIR) of the system with correlative coding can be improved by 3.5 dB in SIR. In further investigation, reference [3] studied the optimum frequency-domain correlative coding for ICI reduction in OFDM systems. The optimum two-tap correlative coding can reduce the ICI due to Doppler shift and the ICI due to carrier offset by about 4.0 dB and 4.5 dB in ICI power, respectively. However, as pointed out in [2], correlative coding introduces multilevel signaling which increases the system bit error rate (BER). The use of an SIR measure to assess performance in an OFDM system with ICI can be misleading because the SIR becomes infinitely large as the frequency offset tends to zero [1]. It is not clear that benefits predicted by a SIR measure will be real. While an ICI power measure does not have this problem, it still does not map directly to a quantifiable improvement in BER performance. Reference [2] did not examine quantitatively the BER performance of the correlative coding OFDM systems. Reference [3] gave simulation results for the word error rate of a Reed-Solomon coded OFDM system with correlative coding employed over a hilly-terrain channel exclusively for the case of large normalized Doppler shift of 0.1. Although the results indicate that the error rate floor due to Doppler shift is reduced from $10^{-2}$ to $10^{-3}$ with the employment of correlative coding, whether the benefit can be achieved in the case of other frequency offset values was not reported in that work.

In this paper, we derive expressions for the BER of pulse-shaped OFDM systems with symbol-by-symbol detection, and show, using these BER results as the performance measure, that in fact, only for large frequency offsets can partial-response pulse-shaping in the time domain, which is equivalent to correlative coding in the frequency domain, reduce ICI and improve BER performance. For small frequency offset values, the performance gain achieved from ICI reduction can not compensate the performance loss due to multilevel signaling, and thus there is no benefit in BER performance from using partial-response pulse-shaping, or correlative coding across subcarriers.

The remainder of this paper is organized as follows. In Section II, the system model is given. Then we derive BER expressions for symbol-by-symbol detection for the partial-response pulse-shaping OFDM system operated over AWGN channels in the presence of frequency offset in Section III. Performance comparisons between the system with partial-response pulse-shaping and the system with rectangular pulse-shaping are presented. Lastly, we draw our conclusions in Section IV.

II. PARTIAL-RESPONSE PULSE-SHAPED OFDM

In this section, we develop a partial-response pulse-shaped OFDM signaling model that provides an alternative, though equivalent, view of an OFDM system with correlative coding. The alternative view is achieved by considering a duobinary...
partial-response pulse [4] for OFDM time domain pulse-shaping. In this application, the time domain response of the duobinary partial-response is

\[
p(t) = \begin{cases} \frac{2}{T} e^{j\pi t/T} \cos(\pi t/T), & |t| < \frac{T}{2} \\ 0, & \text{otherwise} \end{cases}
\]

where \( T \) is the OFDM symbol length, and \( j = \sqrt{-1} \). The Fourier transform of \( p(t) \) is

\[
P(f) = \text{sinc}(\pi f T) + \text{sinc}[\pi(f T - 1)]
\]

where sinc\((x) = \sin x/x\). The complex pulse in (1) can be transformed to a real pulse to facilitate implementation by choosing \( \frac{1}{2T} \), as the new frequency origin. Under this new frequency origin, the time-domain expression \( p_r(t) \) and the frequency-domain expression \( P_r(f) \) become

\[
p_r(t) = \begin{cases} \frac{2}{T} \cos(\pi t/T), & |t| < \frac{T}{2} \\ 0, & \text{otherwise} \end{cases}
\]

and

\[
P_r(f) = \text{sinc}[\pi(f T + \frac{1}{2})] + \text{sinc}[\pi(f T - \frac{1}{2})]
\]

respectively. A time delay \( \frac{T}{2} \) is required since the duration of the pulse starts before the pulse sampling instant. The time delayed version of \( p_r(t) \) is \( p_r^d(t) \), and

\[
p_r^d(t) = p_r(t - \frac{T}{2}) = \begin{cases} \frac{2}{T} \sin(\pi t/T), & 0 \leq t < T \\ 0, & \text{otherwise} \end{cases}
\]

The Fourier transform of \( p_r^d(t) \) is \( P_r^d(f) = P_r(f)e^{-j\pi fT} \).

We use a theoretical analysis model for pulse-shaped OFDM [1], [5], [6] in this work. In this model, one pulse-shaped OFDM symbol with length \( T \) can be written as

\[
s(t) = \sum_{n=0}^{N-1} a_n e^{j2\pi nt/T} \sqrt{p_r^d(t)}
\]

where we assume \( a_n \) is a BPSK data symbol, and \( a_n \) takes values \( \sqrt{E_b} \) or \( -\sqrt{E_b} \) with equal probability. In the case of transmission over an AWGN channel, a matched-filter receiver for the \( m \)th subcarrier gives

\[
\hat{a}_m = \int_0^T s(t)e^{-j2\pi mt/T} \sqrt{p_r^d(t)} e^{j\pi t/T} dt + w_m
\]

\[
= \int_0^T \sum_{n=0}^{N-1} a_n e^{-j2\pi(m-n-0.5)t/T} p_r^d(t) dt + w_m
\]

\[
= a_m P_r^d(-\frac{1}{2T}) + a_{m-1} P_r^d(\frac{1}{2T}) + w_m
\]

\[
= j(a_m - a_{m-1}) + w_m
\]

for \( m = 1, 2, \ldots, N-1 \) and

\[
\hat{a}_0 = j a_0 + w_0
\]

where the term \( e^{j\pi t/T} \) in (7) is used to compensate the frequency origin change in (4), \( P_r^d(-\frac{1}{2T}) = j \), and \( P_r^d(\frac{1}{2T}) = -j \). The complex additive Gaussian noise samples, \( w_m = w_m^I + jw_m^Q \), are the symbol-spaced matched filter output samples resulting from the input white Gaussian noise process.

The receiver can use the imaginary part of \( \hat{a}_m \) in (7)

\[
\Im \{\hat{a}_m\} = a_m - a_{m-1} + w_m^Q
\]

for \( m = 1, 2, \ldots, N-1 \) and \( \Im \{\hat{a}_0\} = a_0 + w_0^Q \) as its estimate of the transmitted symbol. In eq. (9), the first term represents the desired symbol, the second term, represents ICI corresponding to the previous symbol. The intentionally introduced ICI can be removed at the receiver by adding \( a_{m-1} \) from the received signal. However, in this way, the decision of \( m \)th subcarrier will depend on the \((m-1)\)th subcarrier, and thus, error propagation is introduced.

The error propagation can be avoided by using a precoding technique [4]. In the case of duobinary pulse, the precoded sequence \( P_m \) is defined as

\[
P_m = D_m \oplus P_{m-1}
\]

where \( \oplus \) denotes modulo-2 addition, \( D_m \) is a source binary data sequence, and \( D_m \) taking values 0 or 1 independently with probability 0.5. The transmitted BPSK data symbol with precoding is then \( a_n = \sqrt{E_b}(2P_m - 1) \). When the precoding is used, the matched-filter output signal values are \( 2\sqrt{E_b}, -2\sqrt{E_b} \) and 0, and the decoder decides a “1” bit was transmitted if \( |D_m| > \sqrt{E_b} \), and otherwise decides a “0” bit was transmitted.

Comparison of the procedure described above with the procedure shown in [2, Fig. 1] indicates that the time-domain precoded partial-response pulse-shaping OFDM model is equivalent to the frequency domain correlative coding model using the correlation polynomial \( F(D) = 1 - D \) in [2], [3].

The OFDM system without correlative coding is equivalent to a system with rectangular pulse-shaping \( u(t) \)

\[
u(t) = \begin{cases} \frac{1}{T}, & 0 \leq t < T \\ 0, & \text{otherwise} \end{cases}
\]

It is noted that the energy of one partial response pulse-shaped OFDM symbol is

\[
E_s = \int_0^T \mathbb{E}[|s(t)|^2] dt = NE_b \int_0^T |p_r^d(t)| dt = \frac{4}{\pi} NE_b
\]

since \( \int_{-T/2}^{T/2} |p_r^d(t)| dt = \frac{4}{\pi} \) and

\[
\mathbb{E}[a_n a_m^*] = \begin{cases} E_b, & n = m \\ 0, & \text{otherwise} \end{cases}
\]

where \( \mathbb{E}[\cdot] \) denotes expectation. However, the energy of one rectangular pulse-shaped OFDM symbol is \( NE_b \). One can use a scaling factor of \( \pi/4 \) in (5) to ensure normalized pulse energy, that is \( \int_0^T |p^r_s(t)| dt = 1 \) where

\[
p^r_s(t) = \begin{cases} \frac{1}{T} \sin(\pi t/T), & 0 \leq t < T \\ 0, & \text{otherwise} \end{cases}
\]
With these definitions, the energy of one OFDM symbol with pulse-shaping \( p_r^*(t) \) is the same as the energy of one conventional OFDM symbol with rectangular pulse-shaping. In the presence of frequency offset, the received OFDM symbol can be written as

\[
r(t) = e^{j2\pi\Delta f t} \sum_{n=0}^{N-1} a_n e^{j2\pi(n-m+0.5)t/T} p_r^*(t) + w(t) \tag{15}
\]

where \( w(t) \) is a white Gaussian random process. The correlator (matched-filter) for the \( m \)th subcarrier gives

\[
\hat{a}_m = \int_{0}^{T} e^{j2\pi\Delta f t} \sum_{n=0}^{N-1} a_n e^{j2\pi(n-m+0.5)t/T} p_r^*(t) dt + w_m
\]

\[
= \sum_{n=0}^{N-1} a_n \int_{0}^{T} p_r^*(t)e^{-j2\pi(m-n-\varepsilon) t/T} dt + w_m
\]

\[
= \sum_{n=0}^{N-1} a_n P_r \left( \frac{m-n-\varepsilon - 0.5}{T} \right) + w_m \tag{16}
\]

where \( \varepsilon = \Delta f T \) and \( P_r(f) = \frac{1}{2} P_r(f)e^{-j\pi f T} \). The imaginary part of \( \hat{a}_m \), namely

\[
\hat{D}_m = \Im \{ \hat{a}_m \} = \frac{\pi}{4} a_m P_r \left( \frac{-\varepsilon - 0.5}{T} \right) \cos(\pi\varepsilon)
\]

\[
+ \frac{\pi}{4} a_{m-1} P_r \left( \frac{0.5 - \varepsilon}{T} \right) \cos[\pi(1-\varepsilon)]
\]

\[
+ \sum_{n \neq m-1}^{N-1} \frac{\pi}{4} a_n P_r \left( \frac{m-n-\varepsilon - 0.5}{T} \right) \cos[\pi(m-n-\varepsilon)]
\]

\[
+ w_m^Q \tag{17}
\]

for \( m = 1, 2, \ldots, N-1 \), and

\[
\hat{D}_0 = \frac{\pi}{4} a_0 P_r \left( \frac{-\varepsilon - 0.5}{T} \right) \cos(\pi\varepsilon)
\]

\[
+ \sum_{n=1}^{N-1} \frac{\pi}{4} a_n P_r \left( \frac{-n-\varepsilon - 0.5}{T} \right) \cos[\pi(n+\varepsilon)] + w_m^Q \tag{18}
\]

are used as estimates of the data symbol on the \( m \)th \( (m \geq 1) \) subcarrier and the 0th subcarrier, respectively. In eqs. (17) and (18), the sum term is the ICI due to frequency offset. The term \( \hat{D}_0 \) in (18) is different from \( \hat{D}_m \) in (17) because there is no intentionally introduced ICI term.

In the case of a rectangular pulse-shaped OFDM system, the estimate of the data symbol on the \( m \)th subcarrier in the presence of normalized frequency offset \( \varepsilon \) will be

\[
\hat{D}_m = a_m U \left( \frac{-\varepsilon}{T} \right) \cos(\pi\varepsilon)
\]

\[
+ \sum_{n \neq m}^{N-1} a_n U \left( \frac{m-n-\varepsilon - 0.5}{T} \right) \cos[\pi(m-n-\varepsilon)] + w_m^Q \tag{19}
\]

where \( U(f) = \sin(\pi f T) \). Note that in the absence of frequency offset, \( \varepsilon = 0 \), eq. (19) becomes

\[
\hat{D}_m = a_m + w_m^Q. \tag{20}
\]

This is different from eq. (9) since there is no intentionally introduced ICI in the absence of frequency offset.

It is known that the ICI power depends on the sidelobe amplitudes of the pulse-shaping functions [1]. In Fig. 1, it is observed that the partial-response signaling has smaller sidelobe amplitudes. Thus, it is expected that the employment of the partial-response signal pulse-shaping (or correlative coding) will achieve smaller ICI power than the employment of the rectangular pulse (without correlative coding). The ICI reduction achieved by correlative coding was shown in [2]. Our discussion of this phenomenon here views it from a different perspective, although it reaches the same conclusion. This effect should contribute to reducing the BER for OFDM with partial-response pulse-shaping. On the other hand, however, it is seen in eq. (17), that because of the introduction of multilevel signaling, without knowledge of normalized frequency offset, \( \hat{D}_0 \) will be compared with two thresholds at \( \pm \pi \sqrt{E_b} \) and \( \mp \pi \sqrt{E_b} \) for data recovery. In contrast, in eq. (19), there is no intentionally introduced ICI term and the decision is based on threshold 0, that is, the decoder decides a “1” bit was transmitted if \( \hat{D}_0 > 0 \), and otherwise a “0” is decided. The reduction of receiver decision distance will lead to BER performance degradation. The situation is clarified in...
Fig. 2 which shows the receiver decision regions for the two cases. Ignoring other effects, the reduction in receiver decision distance will cause a performance penalty of $(\frac{2}{3})^2 = 2.1$ dB for the correlative coding OFDM system.

III. BER ANALYSIS OVER AWGN CHANNELS

We derive the BER of the two systems in AWGN channels using a characteristic function method in this section.

The characteristic function of $D_m$, conditioned on $a_m$, and $a_{m-1}$ and $\varepsilon$ can be written as

$$\Phi_m(\omega | a_{m-1}, a_m, \varepsilon) = E(e^{j\omega D_m}).$$

(21)

Conditioned on $a_m$, $a_{m-1}$ and $\varepsilon$, the sum term

$$\frac{\pi}{4}a_m P_r \left( \frac{-\varepsilon - 0.5}{T} \right) \cos(\pi \varepsilon) + \frac{\pi}{4}a_{m-1} P_r \left( \frac{0.5 - \varepsilon}{T} \right) \cos[\pi(1 - \varepsilon)] + w_m^Q$$

(22)

in eq. (17) is a Gaussian random variable with mean

$$\frac{\pi}{4}a_m P_r \left( \frac{-\varepsilon - 0.5}{T} \right) \cos(\pi \varepsilon) + \frac{\pi}{4}a_{m-1} P_r \left( \frac{0.5 - \varepsilon}{T} \right) \cos[\pi(1 - \varepsilon)]$$

(23)

and variance $\sigma^2$. The sequence $a_n$ in (17) is obtained after the precoding operation to reduce error propagation. In Appendix I, it is shown that the elements of the sequence $a_n$ are still independent. Therefore, the characteristic function of $D_m$ can be written as

$$\Phi_m(w | a_{m-1}, a_m, \varepsilon) = e^{j\omega \lambda(a_{m-1}, a_m, \varepsilon)} \beta(\omega)$$

(24a)

where

$$\lambda(a_{m-1}, a_m, \varepsilon) = \frac{\pi}{4} \cos(\pi \varepsilon) \left[ a_m P_r \left( \frac{-\varepsilon - 0.5}{T} \right) - a_{m-1} P_r \left( \frac{0.5 - \varepsilon}{T} \right) \right]$$

(24b)

and

$$\beta(\omega) = e^{-\omega^2 \sigma^2 / 2} \prod_{n \neq m, n=1}^{N-1} \cos \left( \frac{\pi}{4} \sqrt{E_b} \omega P_r \left( \frac{m - n - \varepsilon}{T} - 0.5 \right) \right) \times \cos[\pi(m - n - \varepsilon)]$$

(24c)

The $m$th ($m \geq 1$) subcarrier BER can be written as

$$P_b(m) = \frac{1}{2} P_{b1}(m) + \frac{1}{2} P_{b2}(m)$$

(25)

where $P_{b1}(m)$ and $P_{b2}(m)$ are

$$P_{b1}(m) = \text{Prob} \left\{ |\hat{D}_m| < \frac{\pi}{4} \sqrt{E_b} \mid a_m = \sqrt{E_b}, a_{m-1} = -\sqrt{E_b}, \varepsilon \right\}$$

$$= \text{Prob} \left\{ D_m < \frac{\pi}{4} \sqrt{E_b} \mid a_m = \sqrt{E_b}, a_{m-1} = -\sqrt{E_b}, \varepsilon \right\}$$

$$- \text{Prob} \left\{ D_m < -\frac{\pi}{4} \sqrt{E_b} \mid a_m = \sqrt{E_b}, a_{m-1} = -\sqrt{E_b}, \varepsilon \right\}$$

$$= \int_0^{+\infty} \frac{2\beta(\omega) \cos(\omega \gamma_1)}{\pi \omega} \sin \left( \frac{\pi}{4} \sqrt{E_b} \omega \right) d\omega$$

(26)

and

$$P_{b2}(m) = \text{Prob} \left\{ \hat{D}_m \geq \frac{\pi}{4} \sqrt{E_b} \text{ or } \hat{D}_m \leq -\frac{\pi}{4} \sqrt{E_b} \mid a_m = a_{m-1} = \sqrt{E_b}, \varepsilon \right\}$$

$$= \text{Prob} \left\{ D_m \leq -\frac{\pi}{4} \sqrt{E_b} \mid a_m = a_{m-1} = \sqrt{E_b}, \varepsilon \right\}$$

$$+ \text{Prob} \left\{ D_m \geq \frac{\pi}{4} \sqrt{E_b} \mid a_m = a_{m-1} = \sqrt{E_b}, \varepsilon \right\}$$

$$= 1 - \int_0^{+\infty} \frac{2\beta(\omega) \cos(\omega \gamma_2)}{\pi \omega} \sin \left( \frac{\pi}{4} \sqrt{E_b} \omega \right) d\omega$$

(27)

respectively, and

$$\gamma_1 = \frac{\pi}{4} \sqrt{E_b} \cos(\pi \varepsilon) \left[ P_r \left( \frac{-\varepsilon - 0.5}{T} \right) + P_r \left( \frac{0.5 - \varepsilon}{T} \right) \right]$$

(28)

and

$$\gamma_2 = \frac{\pi}{4} \sqrt{E_b} \cos(\pi \varepsilon) \left[ P_r \left( \frac{-\varepsilon - 0.5}{T} \right) - P_r \left( \frac{0.5 - \varepsilon}{T} \right) \right]$$

(29)

Combining eqs. (25)-(29), one can obtain the BER for the $m$th ($m \geq 1$) subcarrier as

$$P_b(m) = \frac{1}{2} \int_0^{+\infty} \frac{2\beta(\omega) \sin \left( \frac{\pi}{4} \sqrt{E_b} \omega \right)}{\pi \omega} \times \sin \left[ \frac{\pi}{4} \sqrt{E_b} \omega \cos(\pi \varepsilon) P_r \left( \frac{-\varepsilon - 0.5}{T} \right) \right] \times \sin \left[ \frac{\pi}{4} \sqrt{E_b} \omega \cos(\pi \varepsilon) P_r \left( \frac{0.5 - \varepsilon}{T} \right) \right] d\omega.$$ 

(30)

It is noted that, in eq. (18), there is no intentionally introduced ICI term for subcarrier 0, and the decision is based on threshold 0. Applying the characteristic function method again, one can get the 0th subcarrier BER

$$P_b(0) = \text{Prob} \left\{ \hat{D}_0 < 0 \mid a_0 = \sqrt{E_b}, \varepsilon \right\}$$

$$= \frac{1}{2} - \int_0^{+\infty} \sin \left[ \frac{\pi}{4} \sqrt{E_b} \omega \right] \cos(\pi \varepsilon) \beta(\omega) \frac{1}{\pi \omega} d\omega.$$ 

(31)

The average BER over different subcarriers in the presence of normalized frequency offset $\varepsilon$ is

$$P_b(\varepsilon) = \frac{1}{N} \sum_{m=0}^{N-1} P_b(m).$$

(32)

Figs. 3 and 4 compare the BER performance of a 64 subcarrier BPSK-OFDM system with partial-response pulse-shaping and the system with rectangular pulse-shaping. The theoretical BER expression for rectangular pulse-shaped OFDM system is [7, eqs. (23) and (24)]

$$P_b^{rec}(m) = \frac{1}{2} - \int_0^{+\infty} \sin \left[ \sqrt{E_b} \omega \left( \frac{m - n - \varepsilon}{T} \right) \cos(\pi \varepsilon) \right] e^{-\frac{1}{2} \omega \sigma^2} \times \prod_{n \neq m}^{N-1} \cos \left[ \frac{\sqrt{E_b} \omega \left( \frac{m - n - \varepsilon}{T} \right) \cos(\pi \varepsilon)}{\pi \omega} \right] d\omega.$$ 

(33)
\[
\begin{align*}
\text{Prob}\{ \mathbf{X}_i \mid \mathbf{X}_j \} &= \text{Prob}\{ P_i = b_i \mid P_j = b_j \} = \text{Prob}\{ D_i \oplus D_{i-1} \oplus \cdots \oplus D_{j+1} \oplus P_j = b_i \mid P_j = b_j \} \\
&= \begin{cases} 
\text{Prob}\{ \text{the number of 1s in } D_i, D_{i-1}, \ldots, D_{j+1} \text{ is an even number} \}, & b_i = b_j \\
\text{Prob}\{ \text{the number of 1s in } D_i, D_{i-1}, \ldots, D_{j+1} \text{ is an odd number} \}, & b_i \neq b_j
\end{cases} \\
&= \left\{ \begin{array}{ll}
\left( C_{i-j}^0 + C_{i-j}^2 + \cdots + C_{i-j}^{i-j-1} \right) \frac{1}{2^{i-j}} = \frac{1}{2}, & b_i = b_j \\
\left( C_{i-j}^1 + C_{i-j}^3 + \cdots + C_{i-j}^{i-j} \right) \frac{1}{2^{i-j}} = \frac{1}{2}, & b_i \neq b_j 
\end{array} \right. \\
&= \left\{ \begin{array}{ll}
\left( C_{i-j}^0 + C_{i-j}^2 + \cdots + C_{i-j}^{i-j-1} \right) \frac{1}{2^{i-j}} = \frac{1}{2}, & b_i = b_j \\
\left( C_{i-j}^1 + C_{i-j}^3 + \cdots + C_{i-j}^{i-j} \right) \frac{1}{2^{i-j}} = \frac{1}{2}, & b_i \neq b_j 
\end{array} \right. \\
&\quad \text{when } i - j \text{ is odd} \\
&\quad \text{when } i - j \text{ is even.} \\
\end{align*}
\]

(28)

Fig. 3. Comparisons of the system BERs with partial-response pulse-shaping (or correlative coding) and the system with rectangular pulse-shaping (without correlative coding) transmitted over an AWGN channel in the presence of normalized frequency offset 0.02, 0.05 and 0.1.

Fig. 4. Comparisons of the system BERs with partial-response pulse-shaping (or correlative coding) and the system with rectangular pulse-shaping (without correlative coding) transmitted over an AWGN channel in the presence of normalized frequency offset 0.15, 0.2, 0.25 and 0.3.

The definition of partial-response pulse in eq. (14) has ensured that the transmitted OFDM symbol with partial-response pulse-shaping has the same energy as the transmitted OFDM symbol with rectangular pulse-shaping. The solid line and the dashed line represent theoretical BER results, and the symbols (square, diamond, etc.) represent Monte Carlo simulation results. The theoretical results and the simulation results are in excellent agreement. In Fig. 3, one can see that for small normalized frequency offset values 0.02, 0.05 and 0.1, the BER performance of the system with rectangular pulse-shaping is much better than the BER performance of the system with partial-response pulse-shaping. In particular, in the presence of normalized frequency offset 0.02 and at a BER of $10^{-3}$, using partial-response pulse-shaping will lead to about 2.4 dB loss in signal-to-noise ratio (SNR). The reason for this is that for small frequency offset values, the ICI power is small. The benefit of ICI reduction by partial-response pulse-shaping can not compensate the loss due to multilevel signaling. As the normalized frequency offset increases, in Fig. 4, the benefit of using partial-response pulse-shaping can be observed. For example, in the case of normalized frequency offset 0.15, at a BER of $10^{-4}$, about 1.6 dB performance gain in SNR can be achieved by using partial-response pulse-shaping rather than the rectangular pulse-shaping. Greater performance gain, about 3.9 dB in SNR, can be observed at a BER of $2 \times 10^{-3}$ and normalized frequency offset $\varepsilon = 0.2$. However, when one further increases the normalized frequency offset value to 0.3, the BER performance favors the rectangular pulse-shaped system again. This is expected from examination of eqs. (17) and (18). In the presence of an unknown frequency offset, the desired signal amplitude is attenuated by a factor $P_r \left( \frac{-\varepsilon - 0.5}{\varepsilon} \right) \cos(\pi\varepsilon)$ which decreases as the normalized frequency offset increases. The attenuation in signal amplitude has more adverse effect on the pulse-shaped OFDM system where the decision is more strongly influenced by the amplitude than on the rectangular-shaped OFDM system where the decision is more strongly influenced by the phase of the BPSK symbol.
IV. Conclusions

Some ICI reduction in OFDM system can be achieved either by using partial-response pulse-shaping in the time-domain or by using correlative coding in the frequency-domain. The pulse-shaped OFDM system can be implemented by using digital filter techniques or a surface acoustic wave (SAW) chirp Fourier transform (CFT) [5]. Expressions for the BER of pulse-shaped OFDM systems operating in AWGN channels with symbol-by-symbol detection were derived. Both pulse-shaping and correlative coding, which were shown to be equivalent in this work, will introduce multilevel signaling and BER performance degradation. For small frequency offset values, it is not beneficial to use these two techniques to reduce ICI because the loss caused by the reduced decision distance in the multilevel signaling is greater than the benefit obtained from ICI reduction. For some large frequency offset values, these two methods can both reduce ICI and improve BER performance.

APPENDIX I

Proof of Independence of the Sequence $a_n$

We assume that the source binary data sequence and the precoded sequence are $D_n$ and $P_n$, respectively. The source binary data $D_n$, $n = 0, 1, \cdots, N - 1$, take value 0 or 1 independently with probability 0.5. The precoded sequence $P_n$ is obtained as $P_n = D_n \oplus P_{n-1}$, where $\oplus$ denotes modulo-2 plus. Without loss of generality, we assume $P_0 = D_0$, thus $\text{Prob}\{P_0 = 0\} = \text{Prob}\{P_0 = 1\} = 0.5$. Since $P_0 = D_0 \oplus P_0$, one has $\text{Prob}\{P_1 = 0\} = 0.5$. Following the same approach, we can also get $\text{Prob}\{P_1 = 1\} = 0.5$. As a result, it is proved that $P_m$ takes values 0 or 1 with probability 0.5, that is

$$\text{Prob}\{P_n = 0\} = \text{Prob}\{P_n = 1\} = 0.5. \tag{34}$$

Now we consider an arbitrary length-$N$ binary sequence $b_n$, $n = 0, 1, \cdots, N - 1$, and $b_n$ taking values 0 or 1. Let $X_i$ denote event $\{P_i = b_i\}$. From (34), we have

$$\text{Prob}\{X_i\} = \text{Prob}\{P_i = b_i\} = 0.5. \tag{35}$$

The precoded sequence $P_0$, $P_1$, $\cdots$, and $P_{N-1}$ are independent if the events $X_0$, $X_1$, $\cdots$, $X_{N-1}$ are independent [8, pp. 244].

We will first show that any two of $X_i$, $i = 0, \cdots, N - 1$, are independent. Without loss of generality, we assume that $i > j$, so the probability of event $X_i$ conditioned on $X_j$ is shown in eq. (28) at the top of this page where $C_i^j$ denotes the number of combinations of $n$ things taken $k$ at a time. On the other hand, $\text{Prob}\{X_i\} = 0.5$. Therefore $\text{Prob}\{X_i | X_j\} = \text{Prob}\{X_i\}$, that is, the occurrence of event $X_j$ does not change the probability that event $X_i$ occurs. So any two events $X_i$ and $X_j$ are independent.

Now for any three events $X_i$, $X_j$, and $X_k$, $i > j > k$, we have [8, pp.253]

$$\text{Prob}\{X_i, X_j, X_k\} = \text{Prob}\{X_i | X_j, X_k\} \text{Prob}\{X_j | X_k\} \text{Prob}\{X_k\} = \text{Prob}\{D_i \oplus \cdots \oplus D_{j+1} \oplus P_j = b_i | P_j = b_j\} \times \text{Prob}\{D_j \oplus \cdots \oplus D_{k+1} \oplus P_k = b_j | P_k = b_k\} \times \text{Prob}\{P_k = b_k\}$$

$$= \text{Prob}\{X_i | X_j\} \text{Prob}\{X_j | X_k\} \text{Prob}\{X_k\} = \frac{1}{8}. \tag{37}$$

Hence for any three events, we have

$$\text{Prob}\{X_i, X_j, X_k\} = \text{Prob}\{X_i\} \text{Prob}\{X_j\} \text{Prob}\{X_k\}. \tag{38}$$

As a result, any three events $X_i$, $X_j$, $X_k$ are shown to be independent.

Now under the assumption that any $M - 1$ ($M \leq N$) events are independent, following the same approach, we can show that

$$\text{Prob}\{X_{i1}, X_{i2}, \cdots, X_{iM}\} = \text{Prob}\{X_{i1}\} \text{Prob}\{X_{i2}\} \cdots \text{Prob}\{X_{iM}\} = \frac{1}{2^{M^2}}. \tag{39}$$

So any $M$ events are independent.

Finally, when $M = N$, we have

$$\text{Prob}\{X_0, X_1, \cdots, X_{N-1}\} = \text{Prob}\{X_0\} \text{Prob}\{X_1\} \cdots \text{Prob}\{X_{N-1}\} = \frac{1}{2^{N^2}}. \tag{40}$$

This completes the proof of the independence of the precoded sequence $P_0$, $P_1$, $\cdots$, $P_{N-1}$.

The transmitted data $a_n$, $n = 0, 1, \cdots, N - 1$, can be obtained from

$$a_n = \sqrt{E_s} (2P_n - 1) \tag{41}$$

and they are also mutually independent [8, pp.245].

References


