Abstract— Coalitions, which allow traders to form teams, can sometimes accomplish tasks more efficiently than individuals. The core has become a popular solution concept in economic applications for researchers in Computer Science, because it provides a way to find stable sets. However, its problems hinder researchers from applying it to a real world market. This paper proposes a new Core Broking Model (CBM), which is based on the core and adopts some other solution concepts to resolve group trading problems in e-markets. It involves bundle selling of composite services from several service providers, offering amount discounts in group buying in the e-marketplace. The CBM successfully creates a win-win-win situation for customers, providers and brokers in e-markets. The comparison between the results of the new model and the core shows that the CBM is superior to the core when applied in e-markets. The results of the simulation presented in this paper demonstrate that the CBM can attract customers and deal with online group trading problems in a large coalition. An extensive evaluation of the CBM has been made and shows that it is effective and efficient.

Keywords— Coalition, Core, Stability, E-Market, Shapley Value

I. INTRODUCTION

When customers and suppliers get together and work out deals, they seize every opportunity to maximise their own profits. Forming coalitions is an effective way of striving to achieve their goals. Therefore, concepts and algorithms for coalition formation have been investigated in Multi-Agent Systems [1]. This has been extensively studied in both Computer Science and Economics communities. These research domains have approached the topic very differently. In Computer Science, the coalitions are formed in a precise structure to maximise the overall expected utility of participants. Since finding the coalition structure is NP-complete, researchers try to prescribe formation algorithms of less computational complexity [2]. On the other hand, the economics literature traditionally stresses the incentives of each selfish participant in a coalition [3]. The traders are self-interested to join a coalition only when it is to their own advantage [4]. A coalition with stability is in a condition when every member has the incentive to remain in the coalition. Thus, in addition to the issue of the coalition structure, researchers need to consider the incentives of each member so as to maintain the stability of coalitions.

According to Forrester Research, the growth of e-commerce was at one time more than 20% per year all over the world [5]. E-commerce is “where business transactions take place via telecommunications networks” [6]. E-market, also called e-marketplace, is a new trading model for e-commerce on the internet in which incentive compatibility, distributed computing, and less computational complexity, are all highly relevant. Many traders gather together with different purposes and can be grouped in varied formations in an e-market. There are two parties, the sellers and the buyers, who generally are regarded as conflicting parties. When a seller gains more profit in a transaction, this also means the buyer needs to pay more, and vice-versa. Is there any way to benefit both parties at the same time and bring them to work together? There are many practical problems that need resolving, when a model is built for the real e-marketplace.

Certain solution concepts for coalition problems related to stability notions have drawn much attention from researchers. The earliest proposed concept was called the stable set [7], which is a set with stabilities. Its definition is very general and the concept can be used in a wide variety of game formats. For many years it was the standard solution concept for cooperative games [8]. However, subsequent works by others showed that a stable set may or may not exist [9], and is usually quite difficult to find [10]. The core has become a well-known solution concept because of its incentive compatibility and because the way it finds stable sets is more appropriate. It assigns to each cooperative game the set of profits that no coalition can improve upon [11].

The aim of this paper is to introduce a new group trading model for e-markets. In this paper, the overall concept of the core is described, and the problems it has which hinder researchers from applying it to a real world marketplace are discussed. Other solution concepts which are used to deal with the problems of the core are introduced, and a new Core Broking Model (CBM) is proposed. The CBM aims to create a win-win-win situation for customers, providers and brokers in e-markets. The comparison between the results of the new model and the core has been made. The results of the simulation in a large coalition are presented. An extensive evaluation of the techniques in the CBM is also carried out.

II. THE CORE

A. Definition

The core plays an important role in the area of cooperative games and modern economics. The core was first proposed by Francis Y. Edgeworth in 1881, and it was later defined in game theory terms by Gillies in 1953. Individuals in a cooperative game are not only interested in maximizing the group’s joint efficiency, they are also
concerned with their own profits. If an individual can gain better profit by working alone without involving others, he/she will not join a group. If a group can produce a better profit by excluding certain individuals, it will certainly form a new coalition without those individuals [12]. The core is a one-point solution concept for forming stable and efficient coalitions by calculating the profits of different possible coalitions. Consequently, it is useful to adopt it into the mechanism in an e-market.

In economics, the core indicates the set of imputations under which no coalition has a value greater than the sum of its members’ profits [11]. Therefore, every member of the core stays and has no incentive to leave, because he/she receives a larger profit. A core in a cooperative game \{N, v\} with n players, denoted as C(v) of a characteristic function \(v: 2^N \rightarrow \mathbb{R}\), which describes the collective profit a set of players can gain by forming a coalition, is derived from the following function [13],

\[
C(v) = \{ \; x \in \mathbb{R}^n \mid x(N) = v(N), x(S) \geq v(S) \text{ for all } S \in 2^N \},
\]

where \(N\) is the finite set of participants with individual gains \(\{x_1, x_2, \ldots, x_n\}\). The \(x(N) = \sum_{i \in N} x_i\) represents the total profit of each individual element in \(N\) by adding \(x_i\), which denotes the amount assigned to individual \(i\), whereas the distribution of \(v(N)\) can denote the joint profit of the grand coalition \(N\). Suppose \(S, T\) are a pair of disjoint non-empty subsets of coalition \(N\). A cooperative game \(N\) is said to be convex, if

\[
v(S) + v(T) \leq v(S \cup T) + v(S \cap T)
\]

whenever \(S, T \subseteq N\) and for \(N \in 2^N\).

A convex core is always a stable set [14]. A game with \(x(N) = v(N)\) is regarded as efficient [13]. An allocation is inefficient if there is at least one person who can do better, though none is worse off. The definition can be summarised as “The core of a cooperative game consists of all undominated allocations in the game” [15]. The profit of the allocations in a core should dominate other possible solutions, meaning that no subgroup or individual within the coalition can do better by deserting the coalition.

B. Advantages

The core has been tested within some games in the previous work [16]. In these simple market games, only one type of consumer and provider deal with one kind of commodity, that is, Service. It simplifies the computational process of the core and helps us to understand the way the core works, although it seems a bit unrealistic. Undoubtedly, providing a way to find a stable set is one of the advantages of the core. It also gives that set incentive compatibility. In fact, the core is a Pareto efficiency, also called Pareto optimality. It is a central concept in Economics, proposed by an Italian economist, Vilfredo Pareto [17]. A Pareto improvement moves the current condition into a better situation that helps at least one person but harms no-one. And then, after some Pareto improvements, a situation will develop, called Pareto equilibrium, in which no Pareto improvement can be made at all. Pareto efficiency is important because it provides a weak but widely accepted standard for comparing economic outcomes. So its framework allows researchers to decide what is efficient.

C. Problems

Although it was proven that the core is always a stable set when the game is convex [14], a number of appealing sufficient conditions have to be identified to confirm whether the core is stable or not [18]. Three problems of the core need to be addressed in order to be applied in the real marketplaces.

The core may exist in different forms: a unique-core game, a multiple-core game or an empty-core game [13]. The first problem occurs when no stable set can be found in an empty-core game. It is a huge problem for the brokers, sellers and buyers -- all the efforts will be in vain if no deal can be agreed with in a market. The second problem is high computational complexity. The core tries to calculate the result of every combination of coalitions and finds the best outcome for a group and for the individuals. The process to find the core is NP-complete [19]. It is too complex to find a core in a large grand coalition. The third problem is that the core seems to deal with complete information only. And this makes it too difficult to apply in the marketplace. In order to facilitate the analysis, the players’ private utility functions become public information. Moreover, the secrecy about market information needs to be relaxed and transparency is needed to expand the market and to form a competitive equilibrium. It is often not the case in application domains that the players are willing to reveal their utility functions.

In brief, the core can be effective only when the grand coalition is small and it has a non-empty core. The computational complexity of the core may sometimes become a huge problem to deal with in real e-markets, because a large number of opportunities and plenty of time allow thousands of traders to group together on the internet. The high computational complexity of the core in a large coalition can hinder researchers from applying it to the real world marketplaces.

III. EXISTING SOLUTION FOR THE CORE

The core provides a way to find a stable set and gives that set incentive compatibility. However, the core is incapable of dealing with coalition problems in e-markets. In this paper, some solutions for these problems are introduced [20].

The first problem is about the stability of cores. In cooperative games, the stability of cores is one of the most notorious open problems. It is important to have a stable coalition, for a game with an empty core is to be understood as a situation of strong instability, as any profits proposed to the coalition are vulnerable. Determining the stability of the core is NP-complete [21]. Attempts to solve the above problem seemed to have run out of possible ideas, until a game has been proven to be non-empty core and have a stable set if and only if the game is balanced [22]. Let the game \(\langle N, v \rangle\) be balanced. Then \(C(N, v) \neq \emptyset\) [23]. From the classical Bondareva-Shapley theorem, a cooperative game has a non-empty core if and only if it is balanced [24]. A game is balanced, if for every balanced collection \(B\) with weights \(\{\lambda_\delta\}_{\delta \in B}\) and the following holds,

\[
v(N) \geq \sum_{B \in B} \lambda_\delta \; v(S).
\]
A collection \( B \) is called a balanced collection if
\[
\exists (\lambda_S)_{S \in B} \forall i \in N \sum_{S \in B} \lambda_S = 1,
\]
where \((\lambda_S)_{S \in B}\) is a vector of positive weights. It is rather easy to check the stability of coalitions with simple calculations in the above formula.

The second problem is how to find a core in a coalition. It is NP-complete [19]. It can be helpful to divide a grand coalition into several small sized sub-coalitions [25] but it is not practical and is an NP-complete problem as well [26]. Current researches show the collaborations of players can effectively reduce the computational complexity [24][27]. A broker may persuade traders to cooperate together. Suppose \( n \) brokers gather \( m \) traders. Instead of \( m \) traders, the core now only needs to deal with \( n \) brokers. Since \( n \) is much smaller than \( m \), the size of the coalition is therefore successfully reduced. When traders decide to collaborate together, it is very important to make sure everyone in the coalition can get what he/she deserves. Shapley value [28] presents a fair and unique way of solving the problem of distributing surplus among the players in a coalition, by taking into account the worth of each player in the coalition.

Assume a coalition \( N \) and a worth function \( v: N \rightarrow \mathbb{R} \), with the following properties,
\[
v(\emptyset) = 0, \quad v(S+T) = v(S) + v(T),
\]
whenever \( S \) and \( T \) are subsets of \( N \). If the members of coalition \( S \) agree to cooperate, then \( v(S) \) describes the total expected gain from this cooperation. The amount that player, gains is,
\[
Sh_i(N) = \Sigma_{S \subseteq N \setminus \{i\}} \left( |S|! |N| - |S| - 1! |N|! [v(S \setminus \{i\}) - v(S)] \right).
\]
The above expression indicates that null coalition gains nothing. It also shows the fact that collaboration can only help and can never be harmful.

The third problem is about the information which is used in the core. Collecting traders’ marginal utility function is a big problem in the real marketplace, because no one is willing to disclose to others how much they are willing to pay for the services. The traditional core can be improved by adopting seller price lists and buyer orders, which can be made available in the market.

IV. PROPOSED CORE BROKING MODEL (CBM)

The Core Broking Model (CBM) involves bundle selling of composite services, offering amount discount for group buying in the e-marketplaces. Several service providers are involved in transactions of bundle selling, while, on the other hand, many buyers form coalitions for the amount discount in the e-markets. The CBM inherits the core concept and a core in a coalition can also be derived from the same function as the core:
\[
C(v) := \{ x \in \mathbb{R}^N | x(N) = v(N), x(S) \geq v(S) \text{ for all } S \subseteq 2^N \},
\]
however, the CBM creates a virtual market on the Internet by involving multi e-markets. In this virtual e-market, there can be many group-trading projects in the model. Many providers join in the projects and perform joint-selling and sufficient buyers gather in e-markets because they may get the high discounts available in the projects. So there is a healthy level of competition in the virtual e-markets.

Fig. 1 represents the structure of the CBM and gives an overview of the model. It shows that core brokers initiate projects, which involve multiple providers, on the CBS website and recruit market brokers to form a team to work on a session of group-trading. The market brokers list the project on the appropriate shopping sites and form buyers’ coalitions there.

The CBM is composed of core brokers, projects, providers’ coalitions, a CBS (Core Broking System), e-markets, market brokers and buyers’ coalitions. The description of each component is as follows:

- **Core brokers:** the initiators of the trading projects.
- **Projects:** sessions of group-trading in e-marketplaces. They involve bundle selling of multiple goods from several providers, offering volume discounts to many different buyers in group-buying coalitions.
- **Providers:** provide products and services for the core brokers.
- **Market brokers:** professional brokers playing the role of team members in the core brokers’ teams helping them with the group-trading projects.
- **E-markets:** may be any existing online shopping avenues such as eBay, Groupon, Ruby Lane or the market brokers’ own sites on which they can post projects and find customers.
- **Buyers:** the market brokers’ clients, who have been attracted to the projects.
- **The CBS consists of three components as follows:**
  a. **CBS Website:** list of group-trading projects. Market brokers may come here and search for the projects which interest them. It is a place where core brokers and market brokers meet together.
  b. **Project Subsystem:** a system specially designed to assist the core broker in managing all the necessary tasks to assure quality outcomes.
  c. **Market Subsystem:** by using it, the market brokers can perform transactions for a session on a particular project; purchase electronic coupons from the core broker and send the coupons to their clients.

In the CBM, core brokers and market brokers execute group-trading sessions on the CBS website. With the help of programs in the CBS, the brokers can easily calculate the discounts for the buyers. After a project is initiated, a group-trading session can be started.
The system flow chart of the model is shown in Fig. 2. There are four stages: commencing, gathering, combining and closing. A brief description for these stages is as follows:

1. **Commencing** – the core broker recruits market brokers and begins sessions of group trading in a project.
2. **Gathering** – the market brokers attract buyers to their websites and submit the coalitions of buyers they have formed to the core broker.
3. **Combining** – the core broker combines the coalitions together, decides the final prices for the items, checks the stability of the coalitions and sends acceptance notices to the market brokers.
4. **Closing** – the market brokers close the deal with the buyers. The final orders and invoices for each customer will be prepared by the market brokers. After the buyers have paid for their purchases, the profits of the providers and the commission for all the brokers will be calculated. At this point, the core brokers may choose to have a new session of trading or stop the project for good.

Each session has a starting and an ending date. The duration of each session should be long enough for a market broker to form a coalition of buyers of a reasonable size. It should also be short enough to allow buyers to gain the items they want as soon as possible. They may be in a hurry, because some of the products or services will cease to be useful after quite a short period of time [29]. In the CBM, the suggested duration for a session is usually one week. The ending time of a session needs to be clearly communicated to the market brokers. The CBS is built to aid brokers with the ending time of a session needs to be clearly communicated to the market brokers. The CBS is built to aid brokers with the ending time of a session needs to be clearly communicated to the market brokers.

The main focus here is the comparison between the core and the CBM. Three criteria, namely the use of distributed computing, the degree of computational complexity and incentive compatibility, are used to evaluate them.

1. **Distributed Computing**

The core is used to find a stable set for a coalition in a traditional market. This is normally done on one computer. It might cause extra complexity if this problem were to be solved by using multi computers. The core could be used in an e-market, but it is difficult to apply it to multiple e-markets in the Internet environment. The CBM enables a core broker to use more than one e-market on the Internet and so bring more benefits to the participants. The core is normally used in a traditional marketplace, but it might be possible to put it into an e-market where it would allow customers to place orders via the Internet.

When the two systems are evaluated according to the criterion of distributed computing, (Table 1 summarises the results) the core may be applied on the Internet, but it fails when it is used with multi computers or e-markets. On the other hand, the CBM can use more than one computer being specially designed to make the best use of multi e-markets.
the core are $3^{99} = 1.71793E+47$, which is a huge search space that takes a long time to work out even for a super computer. A coalition of 99 buyers and 3 providers is definitely not a large one. However, the high computational complexity is a great difficulty to apply the core to e-markets.

On the other hand, in the CBM, the time for the result is the total time of (a) finding an equilibrium price, (b) calculating discounts for the buyers and (c) distributing the profits among the providers. The time to find an equilibrium price has a strong relationship to the search space of $2m$, where there are $m$ market brokers. Because the orders are dealt with on a ‘first come – first served’ (FCFS) basis, the search space for calculating the buyers’ discounts is 1 and the computational complexity is always low. The CBM distributes the providers’ profits by using Shapley value, so the search space is $2^p - 1$. In total, the search space for the CBM is $2^{3m + 2p}$. There are 3 market brokers and 3 providers in this case study. The small search space in the CBM demonstrates that it has less complexity when it tackles problems in a large coalition. Before the market brokers submit their orders to say Ben, they have combined their clients’ orders together. Therefore, he needs to deal with the combined orders only, and that is why the computational complexity in the coalition can be reduced so dramatically.

In the CBM, the aim is to produce outcomes for a group-trading project on an average computer within a reasonable time. For this reason, the number of market brokers or providers in a project was under 12. The search space is $2^{12} + 2^{12} = 8192$. Most computers can easily calculate the providers in a project was under 12. The search space is time. For this reason, the number of market brokers or trading project on an average computer within a reasonable complexity in the coalition can be reduced so dramatically.

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In summary, the complexity in the core can be incredibly high and this means it will not reach the required speed and will not be able to support a large coalition in a marketplace. On the contrary, the CBM can effectively and efficiently reduce the computational complexity in online trading even when multiple providers and many customers are involved.

3) Incentive Compatibility

It is crucial to give people incentives to participate in online trading. The core cannot deal with wholesale transactions in a buyers’ group, therefore individuals cannot obtain very good discounts within it when they purchase items from providers. In order to compare the incentive compatibilities, the assumption has been made that providers are willing to offer more volume discount to customers if a group of them purchases the same item from the same provider in the core, although it is rather unusual for them to give customers such discounts in a traditional market. If this is not done, there will be no means of comparing the two systems.

There are three incentives for traders, namely volume discounts, an equilibrium price and a fair distribution. The providers offer volume discounts to customers in the CBM. If the core may be empty and unstable, an equilibrium price will not be reached in a coalition. The CBM performs a stability check to make sure that there is a best price for traders. An evaluation was made to determine how fairly the profits that providers get were distributed, in both the core and the CBM, and the results are given below.

By using the Shapley value, the CBM can make a fair decision as to which items are allocated to which provider, even when customers’ requirements are less. The CBM provides fair shares to customers and providers, but the core does not. Fair distribution is crucial in teamwork. The providers might leave the team if the profits have been distributed unfairly, even though the profit they can get out of it is good. In the CBM, suppliers have a great chance to sell out their products. Even if the customers do not purchase all that is on offer, the suppliers still get their fair share. This also gives them satisfaction. Table 2 summarises incentive compatibility in the CBM and the core. The CBM has higher incentive compatibility than the core.

<table>
<thead>
<tr>
<th>TABLE 2 INCENTIVE COMPATIBILITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incentive Compat.</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Volume Discount</td>
</tr>
<tr>
<td>Equilibrium Price</td>
</tr>
<tr>
<td>Fair Share</td>
</tr>
</tbody>
</table>

4) Benefits for Traders

In a market, it is usual that if the customers get better discounts, then the providers receive less profit. To ensure the CBM has taken into account the interests of both customers and providers, the average discount that buyers can obtain and the average profit of suppliers are examined. In this way, the CBM can be evaluated to see whether it gives more benefits than the core to both customers and providers.

Within the order detail table in which every order is stored, there is a special field, which is called ‘expected discount’, that allows customers to place the orders without committing to buy the items. The buyers will wait for the final discount to be settled and decide whether the purchase should go ahead or be dropped, by comparing the final and expected discount. In the CBM, when the market broker makes the orders that come from his buyers’ coalition into one combined order, the highest of the expected discounts attached to each product, will be put on the combined order, when the CBM performs the information-hiding process. In Table 3, order O25 is an example from a typical normal customer, who does not ask for a high discount.

<table>
<thead>
<tr>
<th>TABLE 3 A NORMAL BUYER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order ID</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>O25</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

In Table 4, Order H72 is placed by a demanding buyer, who would typically ask for an extremely high expected discount for every item. The order for each item will not be put forward unless the actual discount of the item has reached or gone beyond the level of the expected discount.

<table>
<thead>
<tr>
<th>TABLE 4 A DEMANDING BUYER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order ID</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>H72</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
Table 5 shows that the customers gain higher discounts in the CBM in both scenarios: the one with normal buyers, and the one with demanding buyers. Because the CBM can really attract customers and encourage them to go through with their purchases, the providers can earn more profits in it, even after part of the profits goes to the brokers as commission. A wise provider will definitely choose to join the group-trading in the CBM rather than stay in the core of a traditional market.

### TABLE 5 TRADERS’ BENEFITS

<table>
<thead>
<tr>
<th>Incentive Compatibility</th>
<th>Normal Buyers</th>
<th>Demanding Buyers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The CBM</td>
<td>The Core</td>
</tr>
<tr>
<td>Customer’s Net Discount</td>
<td>Higher</td>
<td>Lower</td>
</tr>
<tr>
<td>Provider’s Profit</td>
<td>Higher</td>
<td>Higher</td>
</tr>
<tr>
<td>Total Benefit</td>
<td>Higher</td>
<td>Lower</td>
</tr>
</tbody>
</table>

**C. Performance of the CBM**

The CBM is built to deal with large coalitions. Without testing the CBM with a large amount of data, the evaluation would be incomplete and unconvincing.

1) **Scenario 1: Normal Buyers**

The CBM is tested with 99 orders using the scenario of normal buyers. Fig. 3 shows the total benefits for the providers and buyers. At the point of 13 customers, the graphs reach their peaks. This means that the items are fully sold out at this point. It also implies that all the customers commit their purchases because they can get the expected discounts they want.

The system divides the total number of customers into 3 equal portions and assigns a portion to each market broker. For instance, every market broker has 23 clients when there are 69 customers in the system.

Fig. 3 Total Benefits with Normal Customers

The system divides the total number of customers into 3 equal portions and assigns a portion to each market broker. For instance, every market broker has 23 clients when there are 69 customers in the system.

2) **Scenario 2: Demanding Buyers**

The CBM uses 99 orders which come from the scenario of demanding buyers. Fig. 6 shows that the graphs reach their peaks at the point of 15 customers. This is 2 buyers more than in Fig. 3. This implies that the CBM can successfully convince the demanding customers to buy, but it takes a little more time than the normal buyers need. The larger the coalition is, the bigger the total benefit should be to traders. But the total benefits drop, when the number of customers is more than 22. The unusual results in this scenario are reflected in the strange appearances of the graphs.

**Fig. 5 Provider’s Net Profit with Normal Customers**

Fig. 5 shows that the CBM is good for providers because their profits can increase rapidly when they use it.

**Fig. 6 Total Benefits with Demanding Customers**
A term ‘dead product’ is used in this paper, which is a phenomenon when a product is available but no buyers can have it. This is because every order has an over-high expected discount on it. Table 6 reveals the reason why this causes a dead product. Product Rc’s stock is 41 and its volume discount is 10%. At the point of 21 customers, MB1 expects to have an 8% discount from Rc and he gets it, while at the point of 22 customers, the expected discounts of all the market brokers are greater than 10%. So Rc becomes a dead product. Most of the buyers in the second scenario demand for extremely high discounts. As the buyers’ coalition becomes larger, the chance for the coalition to have higher expected discounts in the combined order will increase.

**TABLE 6 A DEAD PRODUCT**

<table>
<thead>
<tr>
<th>Status</th>
<th>Product</th>
<th>Stock</th>
<th>Actual Discount</th>
<th>MB1 Expected Discount</th>
<th>MB2 Expected Discount</th>
<th>MB3 Expected Discount</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>Rc</td>
<td>41</td>
<td>10%</td>
<td>8%</td>
<td>16%</td>
<td>15%</td>
</tr>
<tr>
<td>22</td>
<td>Rc</td>
<td>41</td>
<td>0%</td>
<td>11%</td>
<td>16%</td>
<td>15%</td>
</tr>
</tbody>
</table>

Fig. 7 shows the graphs of MB2 and MB3 reach the peak at the point of 20 and 13 customers respectively. The discounts for MB1’s clients keep increasing until at the point of 43 customers. MB1 gets all the items when he has 43+3+1=15 customers. This result is not significantly different from the last scenario, i.e. 14 customers. It is a good sign for the participants, as it shows that demanding buyers continue to make deals just like normal customers do.

![Fig. 7 Demanding Customers’ Net Discounts](image)

In Fig. 8, provider 3 has an unusual graph. It reaches its peak at the point of 15 customers, but it drops when the number of customers is more than 22 because of the dead product Rc. Provider 3 is the only supplier who provides the product, so the dead product reflects on his graph. The discounts for MB3’s clients are zeroes at the beginning, which is shown on the graph. It is normal that demanding customers do not buy if the price is too high for them. They begin to commit their purchases to that product again only when the provider is willing to give more discounts. The negotiations by the core broker with either of the market brokers or the provider are necessary. It is easy for Ben to deal with the dead product problem. He may inform the market brokers and ask them to alter the expected discounts. He may also ask the supplier to adjust the volume discount for product Rc.

The results in the two scenarios show that the CBM is capable of dealing with a large coalition. The CBM also shows its potential in attracting customers including demanding buyers, who want high discounts. Demanding buyers are quite common on the Internet and do not commit to their purchases easily. The CBM creates a win-win-win situation for customers, providers and brokers in the CBM.

**D. Effectiveness and Efficiency of the CBM**

The following ten adopted techniques in the CBM are examined here for their effectiveness and efficiency in e-markets:

1. **Price lists & orders with expected discount** – An equilibrium price in e-markets can be derived from them.
   - **Disadvantage** – The market brokers have to beware of putting an over-high expected discount.

2. **Brokers** – They are brought in to manage transactions and to keep the competition in e-markets healthy.
   - **Disadvantage** – Brokers’ commission may be expensive.

3. **Internet computing** – Internet technologies and programming enable the CBM to involve e-markets.
   - **Disadvantage** – The personal information of customers can be protected from others. A combined order for a coalition reduces the complexity dramatically.

4. **Information-hiding** – The personal information of customers can be protected from others. A combined order for a coalition reduces the complexity dramatically.
   - **Disadvantage** – After orders are combined, core brokers cannot specially allocate products to customers who placed their orders early.

5. **Distributed computing** – It can reduce the complexity in the core by using multi computers and e-markets.
   - **Disadvantage** – Core brokers cannot specially allocate products to customers who placed their orders early.

6. **Collaboration** – Without collaboration between the participants, the CBM cannot function well.
   - **Disadvantage** – Market brokers must be efficient and submit their combined orders as quickly as possible.

7. **Volume discounts** – High discounts attract both normal and demanding buyers. The results of the simulation indicate that this brings more profit to providers.

8. **Stability check** – A way to check the stability of a coalition has been given to ensure that it is not falling apart.

9. **Shapley value** – The results of the CBM suggest it is a fair distribution mechanism.

**TABLE 7 CBM’S EVALUATIONS**

<table>
<thead>
<tr>
<th>Field</th>
<th>Technique</th>
<th>Result</th>
<th>Effectiveness</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-Commerce</td>
<td>1. Price Lists &amp; Orders with Expected Discount</td>
<td>Information Collecting</td>
<td>Yes &amp; No</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>2. Broker</td>
<td>Competitive E-market</td>
<td>Yes &amp; No</td>
<td>Yes</td>
</tr>
<tr>
<td>Distributed Computing</td>
<td>3. Internet Computing</td>
<td>E-markets</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Computational</td>
<td>4. Information Hiding</td>
<td>Less Information</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Complexity</td>
<td>5. Distributed Computing</td>
<td>Multi Computers</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>6. Collaboration</td>
<td>Multi E-markets</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>7. FCFS</td>
<td>Less Complexity</td>
<td>Yes &amp; No</td>
<td>Yes</td>
</tr>
<tr>
<td>Incentive</td>
<td>8. Volume Discount</td>
<td>More Customers</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Compatibility</td>
<td>9. Stability Check</td>
<td>Stable Set</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>10. Shapley Value</td>
<td>Fair Distribution</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Table 7 summarises the results of these ten techniques in relation to effectiveness and efficiency. It shows they have all successfully produced the desired result in the CBM.

VI. LIMITATIONS
Some conditions need to be met in order for the model to function properly. There are seven assumptions used in the CBM:
- There are four levels of information for different users.
- The stock of the items may be limited.
- The CBM manages the orders on a First Come, First Served (FCFS) basis.
- The project does not offer multi-product volume discount to customers. For example, a customer has 10% and 12% discounts for buying products A and B respectively.
- When customers pay for the orders via alternative methods, they receive electronic coupons from brokers.
- No extra fee for shipping will be charged during the processing of transactions.
- The commission comes from a final value fee. The rates are suggested to be 7% of the final selling value: the market broker takes 4% and the core broker gains 3%.

VII. CONCLUSION
Building an online group buying model can be a real challenge because incentive compatibility, distributed computing, and less computational complexity have to be considered at the same time. The CBM is based on the core and adopts the additional solution concepts and successfully creates a win-win-win situation for customers, providers and brokers in e-markets. The comparison between the results of the two systems: the CS and the CBMS, shows the CBM is superior to the core in terms of the three factors mentioned above. The results of the simulation system demonstrate that the CBM can attract customers and deal with online group trading problems in a large coalition. An evaluation of the techniques in the CBM was made showing that all of them have produced the desired results in the CBM effectively and efficiently.

The new model has overcome a number of group-trading problems on the Internet. The main contribution of this research is the CBM, but during the process of creating this new model for group-trading in e-markets, three additional issues have emerged which also made a contribution to knowledge in this field: (a) the advantages and problems of the core (b) a stability check for a coalition and (c) the use of brokers in group-trading.

All systems are capable of improvement and some issues with the CBM can be identified. There will be two main targets for future research. One main target will be to create more incentives for participants. Another target will be to expand the CBM by including particular e-markets and selling a great diversity of products and services on them.

REFERENCES