On the Asymptotic Performance of the STBC Coded FWA Systems

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Abstract—The asymptotic performance of the space-time block coded systems using two transmit antennas over a broadband wireless fixed access (FWA) frequency selective channel is studied in this paper. The conducted theoretical analysis gives us an insight into the physical limitations imposed by the FWA channels and suggests solutions to improve the capacity and performance of future FWA systems.

I. INTRODUCTION

Fixed wireless access (FWA) is quickly emerging as a significant network access alternative for the delivery of voice, data, Internet, video and multimedia type applications to business and residential customers. FWA systems offer a very cost-effective way of building an access network. Easy maintainability, low incremental costs and portability are key benefits of this wireless alternative [1]. Standardization of FWA systems is currently being undertaken by the IEEE 802.16 group [2] and the ETSI HIPERMAN group [3]. Both orthogonal frequency division multiplexing (OFDM) and single-carrier solutions have been adopted in IEEE 802.16 standard as two alternatives for FWA systems operating at 2-11 GHz bands [4].

The major challenge of designing a FWA system is to provide high data rate reliable access with wire-line quality. The high requirement for quality (high data rate, low latency, low bit error rate, etc.) arises because FWA systems have to compete with cable modems and digital subscriber line (DSL) approaches which operate over fixed channels and hence are able to provide very good quality. In recent years, space-time coding has emerged as one of the most promising technologies for meeting the high data rate and high service quality requirements. The use of multiple antennas at the transmitter and receiver sides of a wireless link in combination with signal processing and coding is an effective means to satisfy quality of service requirements. Space-time codes were first introduced in [5] to provide transmit diversity in wireless fading channels using multiple antennas. There are two main types of space-time codes, namely, space-time trellis codes (STTC) [5] and space-time block codes (STBC) [6], [7]. For simplicity, we consider the use of STBC, in particular, the two-antenna transmit diversity scheme [6] proposed by Alamouti.

II. PERFORMANCE OF THE ALAMOUTI ALGORITHM

Fig. 1 shows the baseband representation of the two branch transmit diversity (2TX-1RX) scheme to be analyzed. To simplify the analysis, we assume QPSK modulation (information bits \{b_n\} are mapped into QPSK symbols \{s_n\}). However, the derived results can be easily extended to higher level PSK modulation schemes. Each complex channel coefficient is denoted as \( h_{ij} \), where the first (second) subscript \( i (j) \) is the index of the transmit (receive) antenna, the superscript \( t \) refers to the number of the channel tap. For example, \( h_{ij}^{(t)} \) denotes the channel coefficient corresponding to the first tap of the channel between transmit antenna one to receive antenna zero. The channel coefficients are complex Gaussian random variables and are assumed to remain constant during the transmission of one block of data. In this work, we select the SUI-3 channel which is appropriate for fixed wireless access (FWA) systems [8], [9]. The amplitude of the first tap \( |h_{ij}^{(1)}| \) is characterized by a Ricean distribution due to the presence of line of sight propagation. The amplitudes of the other two taps \( |h_{ij}^{(2)}|, |h_{ij}^{(3)}| \) are Rayleigh distributed. The transmitted symbols are grouped into blocks of 2 symbols at each antenna. At a given time, two symbols are simultaneously transmitted from the two antennas. At time instance \( t \), the symbol transmitted from antenna zero is denoted as \( s_0^t \), and the symbol transmitted from antenna one is denoted as \( s_1^t \). During the next symbol period \( t + T \), symbol \(-s_0^{t+1}\) is transmitted from antenna zero, and \( s_0^t \) is transmitted from antenna one. The received signals at antenna zero during these two symbol periods can be formed as

\[
\begin{align*}
\mathbf{r}_n^0 &= h_{00}^2 s_{n-1}^0 + h_{01}^2 s_{1}^0 + h_{00}^0 s_{n-1}^1 + h_{01}^0 s_{1}^1 + h_{10}^0 s_{n-1}^0 + h_{11}^0 s_{1}^0 + v_n^0 \\
\mathbf{r}_n^1 &= h_{00}^2 s_{n-1}^1 + h_{01}^2 s_{1}^1 + h_{00}^0 s_{n-1}^0 + h_{01}^0 s_{1}^0 + h_{10}^0 s_{n-1}^1 + h_{11}^0 s_{1}^1 + v_n^1 .
\end{align*}
\]

where \( v_n^0 \) is the complex additive white Gaussian noise with zero mean and variance \( N_0 \).

The desired symbols in the above equations are underlined so that they can be distinguished from the interference symbols. According to the original Alamouti algorithm, the soft decisions on symbols \( s_0^t, s_1^t \) can be formed as

\[
\begin{align*}
\mathbf{s}_n^0 &= h_{00}^0 r_n^0 + h_{01}^0 r_n^1 + h_{00}^2 s_{n-1}^0 + h_{01}^2 s_{n-1}^1 + h_{10}^0 s_{n-1}^0 + h_{11}^0 s_{n-1}^1 + v_n^0 \\
\mathbf{s}_n^1 &= h_{01}^0 r_n^0 + h_{00}^2 s_{n-1}^1 .
\end{align*}
\]

where \( \gamma^0 = |h_{00}^0|^2 + |h_{10}^0|^2, \) and \( v_n^0(w_n^1) \) denotes the combined ISI and noise, and is a complex Gaussian random variable with...
PDF \( w_n^0 (w_n^1) \sim \mathcal{CN}(0, N_w) \), and variance
\[
N_w = \gamma^0 (P_{10}^0 + P_{10}^2 + P_{01}^2 + P_{01}^0 + N_0),
\]
where \( \gamma^0 = |h_{00}^0|^2 + |h_{10}^0|^2 \) and \( P_{ij}^0 = E[|h_{ij}^1|^2] \).

Following the procedure shown in [10] for coherent detection of the FWA single-input, single-output system, the bit error probability of the Alamouti detection algorithm conditioned on \( \gamma^0 \) can be derived similarly as
\[
P_b (\gamma^0) = Q \left( \frac{2 E_b \gamma^0}{P_{00}^1 + P_{10}^2 + P_{01}^2 + P_{10}^0 + N_0} \right). \tag{3}
\]

The average bit error probability is obtained by averaging \( P_b (\gamma^0) \) over the distributions of \( \gamma^0 \), i.e., \( \bar{P}_b = \int_{0}^{\infty} P_b (\gamma^0) p(\gamma^0) d\gamma^0 \), where \( p(\gamma^0) \) is PDF function of the random variable \( \gamma^0 \). Deriving \( p(\gamma^0) \) is straightforward if \( h_{00}^0 \) and \( h_{10}^0 \) are not correlated. However, the SUI channel models also define an antenna correlation coefficient, which has to be taken into considerations for multiple antenna FWA channels. Antenna correlation is defined as the envelope correlation coefficient between signals received at two antenna elements, i.e.,
\[
\rho = \frac{E \left[ (h_{ij}^l - E \{ h_{ij}^l \}) (h_{mn}^l - E \{ h_{mn}^l \})^* \right]}{\sqrt{E \{ (h_{ij}^l - E \{ h_{ij}^l \})^2 \} E \{ (h_{mn}^l - E \{ h_{mn}^l \})^2 \}}}.
\]
(4)

where \( i, j, m, n \in \{0, 1\} \), \( l \in \{0, 1, 2\} \). According to the IEEE802.16 specification [2], the taps with different delays are uncorrelated within a channel as well as between channels, i.e., \( E[h_{ij}^k h_{mn}^p] = 0 \), for \( k \neq p \).

Finding \( p(\gamma^0) \) is tedious in presence of antenna correlation (i.e., when \( h_{00}^1 \) and \( h_{10}^1 \) are not statistically independent). To work around this problem, we form a channel vector \( h^0 = [h_{00}^0 h_{10}^0]^T \), where \( T \) denotes the transpose operation. The joint PDF of \( h^0 \) is determined by its mean vector \( m^0 \) and its covariance matrix \( C^0_h = E \{ h^0 - m^0 h^0 - m^0 \}^* \), i.e.,
\[
p(h^0) = \frac{1}{(2\pi)^{\frac{N_c}{2}}} (det C^0_h)^{\frac{1}{2}} \exp \left[ -\frac{1}{2} (h^0 - m^0)^* C^{-1}_h (h^0 - m^0) \right].
\]

Note that the antenna correlation is taken into account in the covariance matrix \( C^0_h \). Since \( \gamma^0 = h^0 m^0 h^0 \), the conditional and the average bit error probabilities can be expressed as
\[
P_b (h^0) = Q \left( \frac{2 E_b \gamma^0}{P_{00}^0 + P_{10}^1 + P_{01}^2 + P_{10}^0 + N_0} \right)
\]
\[
\bar{P}_b = \int_{h^0} P_b (h^0) p(h^0) dh^0. \tag{5}
\]

The conventional definition of the Q-function is given by
\[
Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{y^2}{2} \right) dy. \tag{6}
\]

The average bit error probability is derived by substituting (6) into (5). However, the analysis is difficult to perform in this way since the argument \( x \) appears in the lower limit of the integral in (6). The problem can be tackled by using an alternative definite integral form for the Q-function [11]
\[
Q (x) = \frac{1}{\pi} \int_{0}^{\pi/2} \exp \left( -\frac{x^2}{2 \sin^2 \theta} \right) d\theta. \tag{7}
\]

Substituting (7) into (5), \( \bar{P}_b \) can be obtained according to [12] as
\[
\bar{P}_b = \frac{1}{\pi} \int_{0}^{\pi} \left[ \det \left( \frac{\Sigma^0}{\sin^2 \theta} + I \right) \right]^{-1} \exp \left[ -m^0 (\Sigma^0 + \sin^2 \theta I)^{-1} m^0 \right] d\theta,
\]
where \( m^0 = m^0 h^0 / \alpha, \Sigma^0 = \Sigma^0_h / \alpha^2, \alpha = \sqrt{(P_{00}^0 + P_{10}^1 + P_{01}^2 + P_{10}^0 + P_{00}^2 + P_{00}^1 + N_0) / \bar{E}_b} \). This single finite-range integral can easily be evaluated numerically although it is not a closed form expression.

The performance analysis for 2TX-2RX system can be carried out similarly, but for brevity it will not be presented here.

III. ANALYSIS OF THE ASYMPTOTIC PERFORMANCE

The Alamouti algorithm was originally developed for flat fading channels and so does not take into consideration the ISI introduced by frequency-selective fading channels. As we will see in Section IV, it results in very poor performance for the FWA channels. An equalizer is an effective remedy to combat the detrimental effects caused by ISI. Equalization for STBC coded systems has been treated in several papers, e.g., in [13], where a zero-forcing (ZF) and minimum mean-square error (MMSE) schemes were proposed to combat ISI and obtain diversity gain. However, both ZF and MMSE equalizers presented in [13] involve a matrix inversion at symbol rate, which significantly increases the complexity at the receiver compared to the simple linear processing required by the orthogonally designed STBC systems over flat-fading channels.

In this section, we first analyze the asymptotic behavior of the system. The asymptotic study enlightens a new approach to designing a space-time equalizer under the constraint of linear processing at the receiver, which we shall discuss next.

From the analysis in the previous section, we know that the resulting bit error probability using the Alamouti algorithm can be expressed as \( P_b = Q \left( \frac{2 E_b \gamma^0}{P_{00}^0 + P_{10}^1 + P_{01}^2 + P_{10}^0 + N_0} \right) = Q \left( \frac{2 E_b \gamma^0}{N_c} \right) \), where \( N_c \) denotes the effective noise power. Apparently, there are two factors that contribute to its high sub-optimality for the FWA channels. First, the multipath diversity is not exploited, only the desired signal components from the first path of the two channels are combined (\( \gamma^0 = |h_{00}^0|^2 + |h_{00}^0|^2 \)). On the other hand, the noise component is greatly enlarged due to the ISI. Consequently, the effective noise power is far greater than the original noise power (\( N_e = P_{00}^1 + P_{01}^2 + P_{00}^1 + P_{01}^2 + N_0 \gg N_0 \)). The Q-function is a monotonically decreasing function, meaning that in order to minimize the bit error probability \( P_b \), we should maximize the numerator (the effective signal energy) and in the meantime minimize the denominator (the effective noise power). The best performance is obtained when the full multipath diversity is achieved so that \( \gamma^0 \) is replaced by \( \gamma = \sum_{i,j} |h_{ij}^1|^2 \), which is the total received power from different channels’ different paths. The summation is carried out over all possible values of \( i \in \{0, 1\}, l \in \{0, 1, 2\}, j \in \{0 \} \) for 2TX-1RX system; \( j = \{0, 1\} \) for 2TX-2RX system. In the meantime, the ISI is completely removed so that \( N_e = N_0 \). The asymptotic performance under these two idealistic conditions can be expressed as
\[
P_{b_{\text{min}}} = Q \left( \frac{2 E_b \gamma}{N_0} \right). \tag{9}
\]
Next, we introduce some schemes with an attempt to enable the system to approach its theoretical performance limit expressed by (9).

A. Diversity combining

The first condition for the system to achieve the system’s asymptotic performance expressed by (9) can be met by using a diversity combining technique. According to (1), the received signals at antenna r are

\[ r_n = h_{00}^2 s_n^0 - h_{01}^2 s_n^1 + h_{10}^2 s_{n-1}^0 + h_{11}^2 s_{n-1}^1 + w_n^0, \]

where \( |h_{ij}|^2 \) is the channel power gain of the channel \( h_{ij} \) and \( s_{n-1}^0, s_{n-1}^1 \) are the combined received power from different paths, and combined noise and ISI terms.

\[ E[|h_{ij}|^2] > E[|h_{ij}^2|^2] \] and the asymptotic performance is approached (or fully filled given A. Diversity combining)

expressed by (9).

\[ \text{Next, we introduce some schemes with an attempt to enhance diversity gain in total received power from different paths, and combined noise and ISI terms.} \]

\[ \text{The summation in (11) is carried out over all possible values} \]

\[ \text{of } n \text{ and } l \text{ in (11) also contain more ISI terms compared to } w_n^0, w_n^1 \text{ in (2), which in turn will have a detrimental effect on the overall system performance. In order to tackle this problem, we employ the iterative interference cancellation technique, with an attempt to cancel the contribution of the ISI and achieve the ISI-free performance expressed by (9).} \]

\[ \text{Let us denote } s_n^0, s_n^1 \text{ as an estimate of } \tilde{s}_n^0, \tilde{s}_n^1 \text{ from previous stage. To simplify the notation, the iteration (stage) index is omitted whenever no ambiguity arises. Given a channel estimate } \hat{h}_{ij} \text{ and symbol estimates } \{ \tilde{s}_n^0, \tilde{s}_n^1 \}, \text{ the ISI canceled version of the received signal } r_n^0, \text{ denoted as } \tilde{r}_n^0 \text{ can be written according to (1) as } \]

\[ r_n^0 = (h_{00}^0 s_n^0 - \hat{h}_{00}^0 \tilde{s}_n^0) + (h_{01}^0 s_n^1 - \hat{h}_{01}^0 \tilde{s}_n^1) + h_{10}^0 s_{n-1}^0 + h_{11}^0 s_{n-1}^1 + w_n^0, \]

\[ = \sum_{i,j} \hat{h}_{ij}^0 s_{ij}^0 + \epsilon_n^0 \approx \gamma s_n^0 + \epsilon_n^0; \]

\[ \tilde{r}_n^1 = (h_{00}^1 s_n^0 - \hat{h}_{00}^1 \tilde{s}_n^0) + (h_{01}^1 s_n^1 - \hat{h}_{01}^1 \tilde{s}_n^1) + h_{10}^1 s_{n-1}^0 + h_{11}^1 s_{n-1}^1 + w_n^1, \]

\[ = \sum_{i,j} \hat{h}_{ij}^1 s_{ij}^0 + \epsilon_n^1 \approx \gamma s_n^1 + \epsilon_n^1, \]

where \( \hat{h}_{ij}^0 \) is an estimate of \( h_{ij}^0 \) and \( \gamma = \sum_{i,j} |h_{ij}^0|^2 \) is the combined received power from different paths, and \( \epsilon_n^0, \epsilon_n^1 \) are the combined noise and ISI terms.

B. Interference cancellation (IC)

The summation in (11) is carried out over all possible values of \( i \in \{0, 1\} \) and \( l \in \{0, 1, 2\} \). One can see that this combining scheme also leads to temporal diversity gain in addition to the spatial diversity gain obtained by the original Alamouti scheme. The first condition for achieving the system’s asymptotic performance is approached (or fully filled given perfect channel estimation) by applying the multipath combining scheme introduced above. On the other hand, however, \( \epsilon_n^0, \epsilon_n^1 \) in (11) also contain more ISI terms compared to \( w_n^0, w_n^1 \) in (2), which in turn will have a detrimental effect on the overall system performance. In order to tackle this problem, we employ the iterative interference cancellation technique, with an attempt to cancel the contribution of the ISI and achieve the ISI-free performance expressed by (9). Let us denote \( s_n^0, s_n^1 \) as an estimate of \( \tilde{s}_n^0, \tilde{s}_n^1 \) from previous stage. To simplify the notation, the iteration (stage) index is omitted whenever no ambiguity arises. Given a channel estimate \( \hat{h}_{ij} \) and symbol estimates \( \{ s_n^0, s_n^1 \} \), the ISI canceled version of the received signal \( r_n^0 \), denoted as \( \tilde{r}_n^0 \), can be written according to (1) as

\[ \tilde{r}_n^0 = (h_{00}^0 s_n^0 - \hat{h}_{00}^0 \tilde{s}_n^0) + (h_{01}^0 s_n^1 - \hat{h}_{01}^0 \tilde{s}_n^1) + h_{10}^0 s_{n-1}^0 + h_{11}^0 s_{n-1}^1 + w_n^0, \]

(12)

Other ISI canceled versions of the received signals, e.g., \( \tilde{r}_n^1, \tilde{r}_{n+1}^1, \tilde{r}_{n+1}^1 \) can be formed similarly, i.e., by canceling the contribution from the symbols other than \( s_n^0, s_n^1 \). Using the aforementioned combining technique, the soft decisions of \( s_n^0, s_n^1 \) can now be formed based upon the ISI canceled signals as

\[ \tilde{s}_n^0 = \hat{h}_{00}^0 \tilde{r}_n^0 + h_{10}^0 \tilde{r}_n^1 + h_{11}^0 \tilde{r}_{n+1}^1 + h_{20}^0 \tilde{r}_{n+1}^0 + h_{21}^0 \tilde{r}_{n+1}^1, \]

(13)

where \( \epsilon_n^0, \epsilon_n^1 \) denote the noise plus cancellation residual. Given correct decision feedback, all the ISI terms will be eliminated.

C. Study of asymptotic performance

With the aim of combating ISI and exploiting temporal diversity, a space-time equalization scheme is developed based on the IC and multipath combining in Section III-A and III-B. We shall now analyze the performance bound of this equalization scheme in order to gain an insight into its asymptotic performance under the condition of perfect interference cancellation and channel estimation, which would be the ideal situation leading to best achievable performance. The condition of perfect interference cancellation can be approached by proper design of equalization and decoding [14] so that ISI-free transmission can be achieved for some frequency-selective fading channels. It was shown in [15] that the error due to imperfect channel estimation can be made arbitrarily small given sufficient pilot symbols.

In the case of perfect channel estimation, i.e., \( \hat{h}_{ij}^0 = h_{ij}^0 \), then \( s_n^0, s_n^1 \) in (11) will be eliminated. In this case,

\[ \epsilon_n^0 = h_{00}^0 w_n^0 + h_{10}^0 w_n^1 + h_{11}^0 w_{n+1}^0 + h_{20}^0 w_{n+1}^1 + h_{21}^0 w_{n+1}^1, \]

(14)

Its variance can be easily derived as \( N_c = \sum_{i,j} |h_{ij}^0|^2 N_0 = \gamma N_0 \), using the fact that the taps with different delays are uncorrelated within a channel as well as between channels (e.g., \( h_{ij}^0 \) is uncorrelated with \( h_{ij}^0 \) and \( h_{ij}^0 \) is uncorrelated with \( h_{ij}^0 \)). The performance obtained under the assumption of perfect channel estimation and cancellation can be derived as

\[ P_b(\gamma) = Q \left( \sqrt{\frac{E_s \gamma}{N_c}} \right) = Q \left( \sqrt{\frac{2E_s \gamma}{N_0}} \right). \]

(15)
which is the exactly same as the asymptotic performance expressed by (9). We therefore come to the conclusion that the theoretical limit of the system performance can be reached with the diversity combining and the IC techniques under the condition of perfect channel estimation and perfect cancellation. In order to compute the average asymptotic bit error probability, let us form a channel vector $\mathbf{h} = [h_{00}^0, h_{01}^0, h_{10}^1, h_{11}^1, h_{00}^2, h_{01}^2, ..., h_{n-1, m-1}^{n-1,m-1}]^T$. The joint PDF of $\mathbf{h}$ is determined by its mean vector $\mathbf{m}_h$ and its covariance matrix $\mathbf{C}_h = \mathbb{E}[(\mathbf{h} - \mathbf{m}_h)(\mathbf{h} - \mathbf{m}_h)^*]$, i.e.,

$$p(\mathbf{h}) = \frac{1}{(2\pi)^{n/2} \det \mathbf{C}_h} \exp\left[-\frac{1}{2} \mathbf{h}^* \mathbf{C}_h^{-1} \mathbf{h}\right].$$

(16)

Since $\gamma = \mathbf{h}^* \mathbf{h}$, the conditional and the average bit error probabilities can be formed as

$$P_b(\gamma) = Q \left[ \frac{2 E_b \gamma}{N_0} \right];$$

$$\bar{P}_b = \int_0^\infty P_b(\gamma) p(\gamma) d\gamma = \int_0^\infty Q \left[ \frac{2 E_b \gamma}{N_0} \right] p(\gamma) d\gamma$$

$$= \frac{1}{\pi} \int_0^{2\pi} \left[ \det \left( \frac{\mathbf{\Sigma}}{\sin^2 \theta} + \mathbf{I} \right) \right]^{-1} \exp[-\mathbf{m}^* (\mathbf{\Sigma} + \sin^2 \theta \mathbf{I})^{-1} \mathbf{m}] d\theta,$$

(17)

where $\mathbf{m} = \mathbf{m}_h / \beta, \mathbf{\Sigma} = \mathbf{C}_h / \beta^2, \beta = \sqrt{N_0 / E_b}$. Equation (17) is the performance bound for the 3-path combining scheme. The asymptotic performance of the other schemes can be derived similarly.

IV. ANALYTICAL RESULTS AND PERFORMANCE COMPARISON

A. Results for the uncoded systems

Analytical and simulation results are presented in this section in order to verify the theoretical analysis conducted in the previous sections and demonstrate the efficiency of the proposed space-time equalization scheme. The simulation curves are obtained numerically by averaging the results over 1000 channel realizations. During each Monte-Carlo run, the block size is set to 10000 bits, which corresponds to 5000 QPSK symbols. The FWA channel coefficients vary from one block to another, however, they are assumed to remain constant during the transmission of one block of data. In the simulations, we assume perfect knowledge of the channel state information (CSI) unless otherwise stated. The noise variance $N_0$ and path delays are assumed to be known to the receiver. The antenna correlation coefficient defined in (4) is set to 0.4.

Fig. 2 shows that the analytical results closely match the simulation results for both 2TX-1RX and 2TX-2RX systems employing Alamouti detection algorithm. An irreducible error floor is observed even with 2TX-2RX antennas. The bit error rate cannot be further reduced by increasing $E_b/N_0$. (here $E_b$ refers to the transmitted bit energy, and is not affected by the number of receive antennas). The reason is simply that the Alamouti algorithm was developed for flat fading channels and so does not take into consideration the ISI introduced by the SUI-3 FWA channel. For FWA channels, which exhibit frequency-selectivity as the transmission rate increases, equalization becomes indispensable.

![Performance of the Alamouti scheme in SUI-3 FWA multiple antenna channel. Antenna correlation $\rho = 0.4$.](image)

Fig. 2. Performance of the Alamouti scheme in SUI-3 FWA multiple antenna channel. Antenna correlation $\rho = 0.4$.

![Comparison of different schemes for the FWA system. All the curves represent the 6th stage.](image)

Fig. 3. Comparison of different schemes for the FWA system. All the curves represent the 6th stage.

Fig. 3 shows the performance of the space-time equalization algorithms introduced in Section III-A and III-B. In our simulations, the number of cancellation stages is set to 6, since it is observed that all the algorithms would converge after 6 stages. At the initial stage, we use the 3-path combining scheme (see (11)) to get a crude symbol estimate. At the following stages, the modified Alamouti algorithm with IC (see (12) and (13)) is employed to cancel ISI using decision feedback from the previous stage. As shown in Fig. 3, the IC plus 1-path non-selective scheme has the worst performance. However, it is somewhat surprising that the 2-path and 3-path schemes yield higher BER than the 1-path selective scheme. The rationale is that the IC scheme is prone to the error propagation problem. Cancellation using incorrect decision will increase interference rather than canceling interference. Decision feedback errors have more detrimental effects on the 2(3)-path schemes since they contain more cancellation residual terms.

The asymptotic performance of the proposed schemes are demonstrated in Fig. 4. The analytical bounds are given by (17);
the simulated bounds are obtained by assuming perfect knowledge of the transmitted symbols and CSI in the simulations. The plot shows fairly close agreement between the analysis and simulations, especially for the 1-path and 2-path schemes. This means that the derived theoretical bounds are tight bounds, and we apply a convolutional code to the system. The information will outperform all the other schemes. By comparing Fig. 4 according to Alamouti scheme and transmitted over the FWA channels, we apply convolutional code to the system. The information will outperform all the other schemes. By comparing Fig. 4 with Fig. 3, it is obvious that the diversity combining and IC schemes are far beyond their performance bounds. The errors in the decision feedback significantly degrade performance and prevent the algorithms from reaching their theoretical potential.

There are various ways of tackling this problem, e.g., using channel coding to reduce the feedback error probability, and using soft cancellation rather than brutal force cancellation to prevent error propagation. These ideas combined with turbo processing principle [16] lead to a new approach to space-time turbo equalization which will be described next.

**B. Space-time turbo equalization algorithm for the coded systems**

In order to reduce the error propagation and exploit the potential offered by the previously described equalization algorithm, we apply convolutional code to the system. The information sequence \( \{b_n\} \) is convolutionally encoded into coded bits \( \{u_n\} \), which are subsequently interleaved and each block of two coded and interleaved bits \( u_n^i[0], u_n^i[1] \) is mapped into one of the four QPSK symbols. The QPSK symbols are space-time coded according to Alamouti scheme and transmitted over the FWA channel. The received signal is basically the same as (1) except that the QPSK symbols are formed by coded bits rather than information bits.

The proposed turbo equalization algorithm is illustrated in Fig. 5. It is based on the previously described combining and IC schemes. First, we use a training sequence to acquire a channel estimate \( \hat{h}_{ij} \) using some channel estimation algorithm. In the meantime, a modified Alamouti algorithm is used to obtain the soft values of the transmitted symbols in the form of log-likelihood ratio (LLR) \( \{\lambda(s_n) = \lambda(x_n) + j\lambda(y_n)\} \) where \( s_n \) denotes either \( s_n^0 \) or \( s_n^1 \). The channel estimate \( \hat{h}_{ij} \), and symbol estimates \( \{\lambda(s_n)\} \) are passed to the equalizer, which computes \( \hat{s}_n \), the soft decision of \( s_n \). The soft estimate of the symbol is then mapped to the LLR values of coded bits \( \{\lambda(u_n^i; O)\} \) by the symbol-to-bit converter (SBC), which are deinterleaved and passed through a bit-to-symbol converter (BSC) to derive a soft symbol estimate \( \lambda(s_n) \), which is used for equalization at the next iteration. We use the notations \( \lambda(\cdot; I) \) and \( \lambda(\cdot; O) \) to denote the input and output ports of a soft-input and soft-output device.

Numerical results are presented next to assess the performance of the proposed turbo equalization scheme. We employ a rate 1/3 Maximum Free Distance convolutional code with constraint length 5 and generator polynomials \((25, 33, 37)\) in octal form. During each Monte-Carlo run, the block size is set to 2000 information bits followed by 4 tails bits to terminate the trellis, which corresponds to \( 2004 \times 3 = 6012 \) coded bits or 3006 QPSK symbols, 200 of which are used as pilots to acquire a channel estimate \( \hat{h}_{ij} \). Channel estimation is conducted with the modified maximum likelihood algorithm introduced in [15]. The coded bits are interleaved by a random interleaver. The simulation curves are obtained by averaging the simulation results over at least 300 channel realizations. To study the behavior of each algorithm, the number of stages is set to 4 since it is observed that no more than 4 stages (in addition to the non-cancellation stage) are needed for the discussed schemes to converge.

Different combining (selection) strategies are compared in Fig. 6. The 1-path schemes yield the worst results. We also see that the selective scheme performs better than the non-selective one. Off all the discussed strategies, the 2-path and 3-path combining schemes are much superior to the 1-path schemes, with the 3-path scheme being the best. The results concur with our study of the asymptotic behavior for the space-time equalization scheme demonstrated in Fig. 4. Obviously, in order to fully achieve the temporal diversity from the multipath propagation, we need to combine the signals from all the paths. As illustrated in Fig. 6, the error propagation problem is effectively solved, and the error floor presented in the uncoded system is eliminated. The potential for performance improvement predicted in Fig. 4 is realized by extending the algorithm to the coded systems and by applying the turbo processing principle.

In Fig. 6, the proposed scheme is also compared with the time-reversal STBC (TR-STBC), which is a transmit diversity scheme specially designed for frequency selective channels [17]. The TR-STBC itself only decouples the symbol streams from two transmit antennas. It, however, does not resolve the ISI in each symbol stream. The ISI of course still has to be handled by an equalizer or a maximum likelihood sequence detector. In this work, we apply a 7-tap MMSE equalizer in the TR-STBC system after the symbol streams are decoupled. As shown in Fig. 6, the proposed space-time turbo
The results show that the error propagation problem causes irreducible error floor and prevents the system from reaching its theoretical limit. To tackle this problem, we extend the algorithm to a convolutionally coded STBC system and apply turbo processing principle, resulting in a space-time turbo equalization algorithm which is shown to have superior performance to the TR-STBC transmit diversity scheme for FWA channels. In the meantime, it only requires linear processing at the receiver, which makes it a feasible solution for practical implementation.

V. CONCLUSIONS

Asymptotic performance of the FWA MIMO system is investigated in this paper, and our theoretical study indicates that the temporal diversity has to be exploited and the effect of ISI has to be removed in order to approach the theoretical capacity of the system. The temporal diversity can be obtained by multipath combining, and the effect of ISI can be reduced by interference cancellation. The results show that the error propagation problem causes irreducible error floor and prevents the system from reaching its theoretical limit. To tackle this problem, we extend the algorithm to a convolutionally coded STBC system and apply turbo processing principle, resulting in a space-time turbo equalization algorithm which is shown to have superior performance to the TR-STBC transmit diversity scheme for FWA channels. In the meantime, it only requires linear processing at the receiver, which makes it a feasible solution for practical implementation.

REFERENCES